Fractal universe

based on

Phy. Rev. Lett. 104, 251301 (2010); JHEP 03, 120 (2010).

Gianluca Calcagni

Max Planck Institute for Gravitational Physics (AEI)

September 21, 2010

Dimensional flows in quantum gravity

- Perturbative QG, asymptotic safety, CDT, HL gravity, non-commutative FT, spin foams: $4 > d \sim 2$ in the UV.

Dimensional flows in quantum gravity

- Perturbative QG, asymptotic safety, CDT, HL gravity, non-commutative FT, spin foams: $4 > d \sim 2$ in the UV.
- Proposal: Dimensional flow at structural level via a change of measure: Lorentz invariant and (power-counting) renormalizable model.

Dimensional flows in quantum gravity

- Perturbative QG, asymptotic safety, CDT, HL gravity, non-commutative FT, spin foams: $4 > d \sim 2$ in the UV.
- Proposal: Dimensional flow at structural level via a change of measure: Lorentz invariant and (power-counting) renormalizable model.
- Motivation: at least to capture the effective dynamics of some QG models.

Stieltjes integrals and fractal dimension

Stieltjes integrals and fractal dimension

$$\mathrm{d}^D x \to \mathrm{d}\varrho(x)\,, \qquad [\varrho] = -D\alpha \neq -D\,.$$

Stieltjes integrals and fractal dimension

$$\mathrm{d}^D x \to \mathrm{d}\varrho(x)\,, \qquad [\varrho] = -D\alpha \neq -D\,.$$

Scaling of a Euclidean volume of size R, $V(R) \sim R^{d_{\mathrm{H}}}$

Stieltjes integrals and fractal dimension

$$\mathrm{d}^D x \to \mathrm{d}\varrho(x)\,, \qquad [\varrho] = -D\alpha \neq -D\,.$$

Scaling of a Euclidean volume of size R, $V(R) \sim R^{d_\mathrm{H}}$:

$$V(R) \sim \int_{D\text{-ball}} \mathrm{d}\varrho_\mathrm{Eucl}(x)$$

Stieltjes integrals and fractal dimension

$$\mathrm{d}^D x \to \mathrm{d}\varrho(x)\,, \qquad [\varrho] = -D\alpha \neq -D\,.$$

Scaling of a Euclidean volume of size R, $V(R) \sim R^{d_\mathrm{H}}$:

$$V(R) \sim \int_{D\text{-ball}} \mathrm{d}\varrho_\mathrm{Eucl}(x) \sim \int_0^R \mathrm{d}r\, r^{D\alpha-1}$$

Stieltjes integrals and fractal dimension

$$\mathrm{d}^D x \to \mathrm{d}\varrho(x)\,, \qquad [\varrho] = -D\alpha \neq -D\,.$$

Scaling of a Euclidean volume of size R, $V(R) \sim R^{d_\mathrm{H}}$:

$$V(R) \sim \int_{D\text{-ball}} \mathrm{d}\varrho_\mathrm{Eucl}(x) \sim \int_0^R \mathrm{d}r\, r^{D\alpha-1} \sim R^{D\alpha},$$

Stieltjes integrals and fractal dimension

$$\mathrm{d}^D x \to \mathrm{d}\varrho(x)\,, \qquad [\varrho] = -D\alpha \neq -D\,.$$

Scaling of a Euclidean volume of size R, $V(R) \sim R^{d_\mathrm{H}}$:

$$V(R) \sim \int_{D\text{-ball}} \mathrm{d}\varrho_\mathrm{Eucl}(x) \sim \int_0^R \mathrm{d}r\, r^{D\alpha-1} \sim R^{D\alpha},$$

$$\alpha = \frac{d_\mathrm{H}}{D}.$$

Stieltjes integrals and fractal dimension

$$d^D x \to d\varrho(x)\,, \qquad [\varrho] = -D\alpha \neq -D\,.$$

Scaling of a Euclidean volume of size R, $V(R) \sim R^{d_H}$:

$$V(R) \sim \int_{D\text{-ball}} d\varrho_{\rm Eucl}(x) \sim \int_0^R dr\, r^{D\alpha-1} \sim R^{D\alpha},$$

$$\alpha = \frac{d_H}{D}.$$

Example: Lebesgue integral (ϱ absolutely continuous), $d\varrho(x) = v(x)d^D x$, v Lorentz scalar.

Stieltjes integrals and fractal dimension

$$\mathrm{d}^D x \to \mathrm{d}\varrho(x)\,, \qquad [\varrho] = -D\alpha \neq -D\,.$$

Scaling of a Euclidean volume of size R, $V(R) \sim R^{d_\mathrm{H}}$:

$$V(R) \sim \int_{D\text{-ball}} \mathrm{d}\varrho_\mathrm{Eucl}(x) \sim \int_0^R \mathrm{d}r\, r^{D\alpha-1} \sim R^{D\alpha}\,,$$

$$\alpha = \frac{d_\mathrm{H}}{D}\,.$$

Example: Lebesgue integral (ϱ absolutely continuous), $\mathrm{d}\varrho(x) = v(x)\mathrm{d}^D x$, v Lorentz scalar. We can choose

$$v(x) \sim X^{D(\alpha-1)} + M^{D(1-\alpha)}$$

Dynamics

- It can be applied to any QFT, including gravity.

Dynamics

- It can be applied to any QFT, including gravity.
- Friction terms, as in cosmology or **dissipative** systems.

Dynamics

- It can be applied to any QFT, including gravity.
- Friction terms, as in cosmology or **dissipative** systems.
- Poincaré algebra deformed! ("Fractals break translation invariance")

Dynamics

- It can be applied to any QFT, including gravity.
- Friction terms, as in cosmology or **dissipative** systems.
- Poincaré algebra deformed! ("Fractals break translation invariance")
- Interesting possibilities for early-universe cosmology.

Dynamics

- It can be applied to any QFT, including gravity.
- Friction terms, as in cosmology or **dissipative** systems.
- Poincaré algebra deformed! (“Fractals break translation invariance”)
- Interesting possibilities for early-universe cosmology.
- Evidence of a duality between fractal FT’s and non-commutative FT’s.