Poster 08: Deformation of Codimension-2 Surface and Horizon Thermodynamics

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- **Deformation equation of a spacelike submanifold**

(1). The case with an arbitrary codimension

$$\mathcal{L}_X\theta^{(Y)} = q^{cd}X^eY^f\mathscr{R}_{ecdf} - K^{(Y)ab}K^{(X)}_{ab} - Y^c\tilde{D}_a\tilde{D}^aX_c - K_c\left(X^d\nabla_dY^c\right) \,. \quad (1)$$

(2).Codimension-2 case

$$\begin{aligned}\mathcal{L}_X\theta^{(Y)} &= -\left(\mathscr{G}_{ab} + K_{cda}K^{cd}{}_b\right)\left[X^aY^b - h^{ab}\left(X_eY^e\right)\right] \\ &\quad + \frac{1}{2}\left(R - K_{abc}K^{abc} - K_cK^c\right)\cdot\left(X_eY^e\right) \\ &\quad - Y^e\tilde{D}_c\tilde{D}^cX_e - K_c\left(X^e\nabla_eY^c\right) \,. \qquad (2)\end{aligned}$$

By selecting a null frame, we get focusing and cross focusing equations.

- **Dynamics of trapping horizon by introducing a quasilocal energy**

(1). We generalize Hawking mass to higher dimension:

$$\mathscr{E} = \frac{\left(\int \epsilon_q\right)^{\frac{n-3}{n-2}}}{16\pi G \left(\Omega_{n-2}\right)^{\frac{1}{n-2}} (n-3)} \left\{ \frac{\int \epsilon_q R}{\left(\int \epsilon_q\right)^{\frac{n-4}{n-2}}} - \left(\frac{n-3}{n-2}\right) \frac{\int \epsilon_q K_c K^c}{\left(\int \epsilon_q\right)^{\frac{n-4}{n-2}}} \right\}. \qquad (3)$$

(2). The deformation of this generalized Hawking mass:

$$\mathcal{L}_X \mathscr{E} = \int \epsilon_q \left\{ \left(\frac{\mathcal{E}}{n-2}\right) \left(\mathcal{L}_X \mathscr{A} / \mathscr{A} + K_e X^e\right) \right\}$$
$$+ \frac{1}{8\pi G} \left(\frac{L}{n-2}\right) \int \epsilon_q \left\{ - K^e \tilde{D}_c \tilde{D}^c X_e + \frac{1}{2} \left(\mathscr{G}_{ab} h^{ab}\right) \cdot \left(K_e X^e\right) \right.$$
$$\left. - \left(\mathscr{G}_{ab} + C_{cda} C^{cd}{}_b\right) \left[K^a X^b - \frac{1}{2} h^{ab} \left(K_e X^e\right)\right] \right\}. \qquad (4)$$

The deformation of the energy has similar structure as the case in four dimension. Further, on the trapping horizon, the evolution of this energy can be decomposed into two parts: matter fields and gravitational radiation, i.e.,

$$\mathscr{T}_{ab}; \quad \|\sigma\|^2 \quad \text{and} \quad \|\zeta\|^2$$

- **Dynamics of trapping horizon without introducing a quasilocal energy**

We generalize the slowly evolving future outer trapping horizon (Booth) to more general cases: for example, past trapping horizon. The slowly evolving parameter $\epsilon \ll 1$ is defined as

$$\frac{\epsilon^2}{L^2} = \max\left[|C|\left(\|\sigma^{(\ell)}\|^2 + (8\pi G)\mathscr{T}_{ab}\ell^a\ell^b + \frac{1}{n-2}\theta^{(\ell)}\theta^{(\ell)}\right)\right].$$

- **Slowly evolving past trapping horizon in FRW universe**

The $\epsilon \ll 1$ automatically implies

$$-\frac{\dot{H}}{H^2} \approx \frac{\epsilon^2}{2} \ll 1$$

This is just the condition of slow-roll inflation.

The requirement on the surface gravity $|\mathcal{L}_X \kappa_X| \preceq \epsilon/L^2$ gives

$$\left|\frac{\ddot{H}}{H^3}\right| \preceq \epsilon$$

This condition ensure the temperature evolves slowly, and the system is near equilibrium.

$$T = \left|\frac{\kappa_X}{2\pi}\right| \sim \frac{H}{2\pi}\left(1 - \frac{\epsilon^2}{4}\right) + \mathscr{O}(\epsilon^4)\,.$$

Different temperatures in different theories are essentially same up to the second order of the ϵ.

Thank you!