

[No. 25]

Perturbative approach to a non-spherical micro-lensing effect



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Gravitational lensing
by a non-spherical
compact object

Non-spherical property

is modeled by

the quadrupole moment

$$Q_{ij} = \int \rho \left(X_i X_j - \frac{1}{2} \delta_{ij} |X|^2 \right) d^3 X$$

Under the assumption

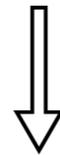
$$Q_{ij} \Rightarrow \begin{pmatrix} e & \\ & -e \end{pmatrix}$$

$$0 < e \ll 1$$

we solved the lens equation
perturbatively to obtain
approximate solutions.

Comparison with numerical solutions

Typical errors
for the “**major**” images are
less than 0.5%



very accurate!

flux ratio

Asymptotic behavior for $0 < \beta \ll 1$:

$$\frac{\mu^+}{\mu^-} \rightarrow 1 \pm \frac{4}{\beta} e$$

Asymptotic behavior for $\beta \gg 1$:

$$\frac{\mu^+}{\mu^-} \rightarrow \beta^4 \left(1 \pm 2\beta^2 e \right)$$

Anomalous flux ratio!?

The quadrupole moment can significantly change the flux ratio



a clue to the anomalous flux ratio problem?

**Please visit
our poster [No. 25].**

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Abstract: We investigate the gravitational (micro-)lensing effect by a non-spherical lens object. The non-spherical potential of the lens is modelled by use of the quadrupole moment. We calculate the bending angle and solve the lens equation for a non-spherical lens with quadrupole moment analytically by use of an perturbative approach. We show how the quadrupole moment changes the image positions and the amplification light curve.

Non-spherical compact lens model

We investigate **the gravitational lens effect** by a **non-spherical compact object**. Non-spherical property of the lens object is modeled by using a **quadrupole moment** in the Newtonian gravitational potential:

$$\phi = \phi_0 + \phi_2 = -\frac{GM}{r} - \frac{G}{2} \left(3 \frac{x^i x^j}{r^5} - \frac{\delta^{ij}}{r^3} \right) I_{ij}, \quad I_{ij} \equiv \int \rho(\vec{r}) x^i x^j d^3 r.$$

Using the above form of the potential, the deflection angle α is

$$\alpha^i = \frac{4GM}{c^2} \frac{\xi^i}{\xi^2} + \frac{8G}{c^2} \left(2Q_{jk} \frac{\xi^j \xi^k \xi^i}{\xi^6} - Q_{ij} \frac{\xi^j}{\xi^4} \right),$$

where

$$Q_{ij} = \int \rho \left(X_i X_j - \frac{1}{2} \delta_{ij} |X|^2 \right) d^3 X$$

denotes **the trace-free quadrupole moment** on the lens plane. Assuming the normalized quadrupole moment is diagonalized

$$\tilde{Q}_{ij} \equiv \frac{c^2 D_S}{2GM^2 D_L D_{LS}} Q_{ij} \Rightarrow \begin{pmatrix} e & 0 \\ 0 & -e \end{pmatrix},$$

the lens equation is

$$\beta_x = x - \frac{x}{x^2 + y^2} - e \frac{x(x^2 - 3y^2)}{(x^2 + y^2)^3},$$

$$\beta_y = y - \frac{y}{x^2 + y^2} - e \frac{y(3x^2 - y^2)}{(x^2 + y^2)^3}.$$

Comparison with numerical solutions

Next, we compare our approximate solutions with the numerical ones, and check the accuracy of our approximation. Some of the results on the y -axis case $\beta = (0, \beta_y)$, $\theta = (0, y)$ are shown in the following table.

e	β_y	numeric. y	approx. y	error	e	β_y	numeric. y	approx. y	error
0.01	0.2	1.1009	1.1009	0.00%	0.02	0.2	1.0967	1.0968	-0.01%
		-0.9456	-0.9457	-0.01%			-0.8924	-0.8928	-0.05%
		0.0995	0.1000	-0.50%			0.1408	0.1414	-0.42%
		-0.1016	-0.1000	1.54%			-0.1451	-0.1414	2.54%
0.01	0.5	1.2778	1.2778	0.00%	0.02	0.5	1.2748	1.2749	-0.01%
		-0.7726	-0.7728	-0.03%			-0.7640	-0.7649	-0.11%
		0.0981	0.1000	-1.95%			0.1380	0.1414	-2.47%
		-0.1033	-0.1000	3.17%			-0.1488	-0.1414	4.95%
0.01	1.0	1.6163	1.6163	0.00%	0.02	1.0	1.6146	1.6146	0.00%
		-0.6058	-0.6063	-0.09%			-0.5922	-0.5946	-0.40%
		0.0959	0.1000	-4.25%			0.1339	0.1414	-5.64%
		-0.1065	-0.1000	6.07%			-0.1562	-0.1414	9.48%

Typical errors for the “**major**” images are less than 0.5%, whereas for the “**minor**” images the errors are larger.

Amplification factor

Once we obtain the solutions for the image positions (x, y) in terms of the source position (β_x, β_y) , we can directly calculate the amplification factor for the “**major**” images by

$$| \frac{\partial(x^\pm, y^\pm)}{\partial(\beta_x, \beta_y)} |$$