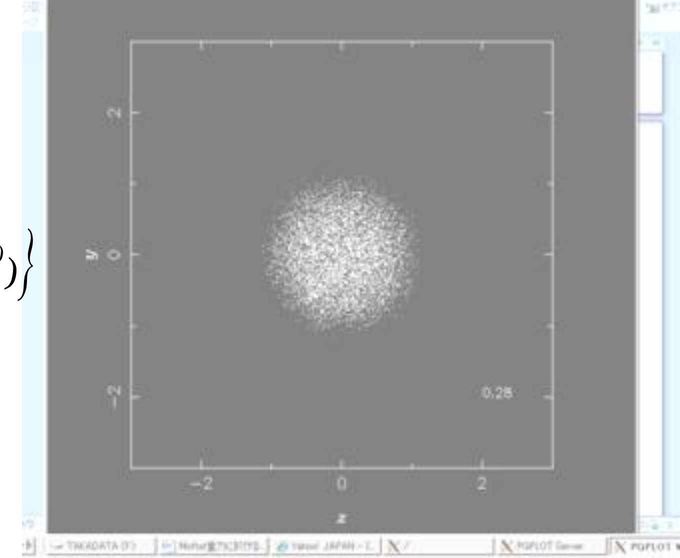
$$a_i = -G_N \sum_j \frac{(r_j - r_i)m_i}{|r_j - r_i|^3}\left\{1 + \alpha\left(1 - (1 + \mu|r_j - r_i|)e^{-\mu(|r_j - r_i|)}\right)\right\}$$



N-Body simulation on MOdified Gravity

$$S_G = -\frac{1}{16\pi}\int \frac{1}{G}(R + 2\Lambda)\sqrt{-g}\,d^4x,$$

$$S_\phi = -\int \omega\left[\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{2}\mu^2\phi_\mu\phi^\mu + V_\phi(\phi)\right]\sqrt{-g}\,d^4x,$$

$$S_S = -\int \frac{1}{G}\left[\frac{1}{2}g^{\mu\nu}\left(\frac{\nabla_\mu G\nabla_\nu G}{G^2} + \frac{\nabla_\mu \mu\nabla_\nu \mu}{\mu^2} - \nabla_\mu\omega\nabla_\nu\omega\right) + \frac{V_G(G)}{G^2} + \frac{V_\mu(\mu)}{\mu^2} + V_\omega(\omega)\right]\sqrt{-g}\,d^4x.$$

(MOG=MOdified Gravity? or MOffat Gravity?)

YAMAGUCHI UNIVERSITY
山口大学

Yamaguchi University

Laboratory of Theoretical particle physics and Cosmology

PhD student 2nd grade Takayuki Suzuki 鈴木隆之

○ Basic theory and review of Moffat gravity

Scalar Tensor Vector Gravity(STVG)

ACTION developed by John Moffat 2005

$$S_G = -\frac{1}{16\pi}\int \frac{1}{G}(R+2\Lambda)\sqrt{-g}\,d^4x,$$

$$S_\phi = -\int \omega\left[\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{2}\mu^2\phi_\mu\phi^\mu + V_\phi(\phi)\right]\sqrt{-g}\,d^4x,$$

$$S_S = -\int \frac{1}{G}\left[\frac{1}{2}g^{\mu\nu}\left(\frac{\nabla_\mu G\nabla_\nu G}{G^2} + \frac{\nabla_\mu \mu\nabla_\nu \mu}{\mu^2} - \nabla_\mu\omega\nabla_\nu\omega\right) + \frac{V_G(G)}{G^2} + \frac{V_\mu(\mu)}{\mu^2} + V_\omega(\omega)\right]\sqrt{-g}\,d^4x.$$

$$J^\nu = -\frac{1}{\sqrt{-g}}\frac{\delta S_M}{\delta \phi_\nu}.$$

It have a massive vector field $\varphi_\mu(x)$ which couples with matter directly, and 3 scalar fields $G(x)$, $\omega(x)$ and $\mu(x)$. $B_{\mu\nu} = \partial_\mu\varphi_\nu - \partial_\nu\varphi_\mu$ and

$V_\varphi(\varphi)$, $V_G(G)$, $V_\omega(\omega)$ and $V_\mu(\mu)$ denote self-interaction potentials.

G(x) is gravitational constant.

Weak field approximation

- The equation of motion of a test particle is given by

$$m\left(\frac{du^{\mu}}{ds}+\Gamma^{\mu}_{\alpha\beta}u^{\alpha}u^{\beta}\right)=-\alpha\,\kappa\,\omega\,m\,B^{\mu}{}_{\nu}u^{\nu}.$$

- For weak fields the MOG acceleration law is

$$\frac{d^2r}{dt^2}=-\frac{G_N M}{r^2}\left[1+\alpha-\alpha(1+\mu r)e^{-\mu r}\right],$$

$$\alpha=\frac{M}{\left(\sqrt{M}+E\right)^2}\left(\frac{G_\infty}{G_N}-1\right),\qquad \mu=\frac{D}{\sqrt{M}},\qquad \begin{aligned} D &\cong 6250 M_\Theta^{1/2}\mathrm{kpc}^{-1},\\ E &\cong 25000 M_\Theta^{1/2},\\ G_\infty &\cong 20 G_N,\end{aligned}$$

$$\frac{d^2r}{dt^2}=-\frac{G_{\mathrm{eff}}M}{r^2},\qquad G_{\mathrm{eff}}=G_N\left[1+\alpha-\alpha(1+\mu r)e^{-\mu r}\right].$$

key point : Vector field is
couple with matter → Geodesic equation has external force term
massive → It has effective range → *Yukawa* like force

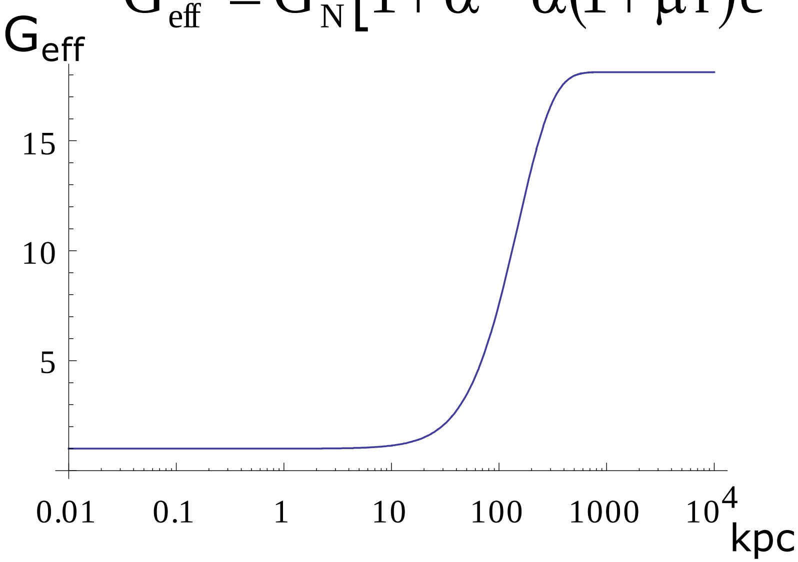
In fact,
Newton gravity we recognize = (1+α) × Gravitation of the inverse square law ー Yukawa like Repulsive force

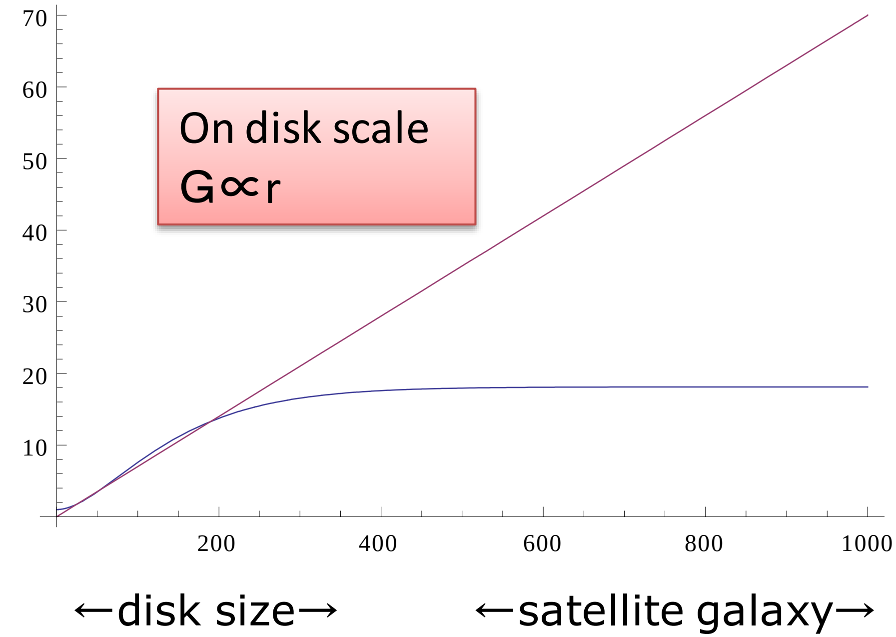
Why can MOG explain galaxy flat rotate curve without dark matter?

if galaxy mass ~10^{11} $M_\odot$

→linear plot

$$G_{eff} = G_N\left[1+\alpha-\alpha(1+\mu r)e^{-\mu r}\right]$$





Equation of Kepler motion

$$\frac{GM}{r^2} = \frac{v^2}{r} \quad \Rightarrow \quad v = \sqrt{\frac{GM}{r}}$$

if G∝r, v is constant!

How are MOG different from MOND?

As it is said from old days,

MOND can explain galactic flat rotate curve without dark matter.

MOND=MOdified Newton Dynamics :Mordehai Milgrom 1983

BUT, MOG is・・・

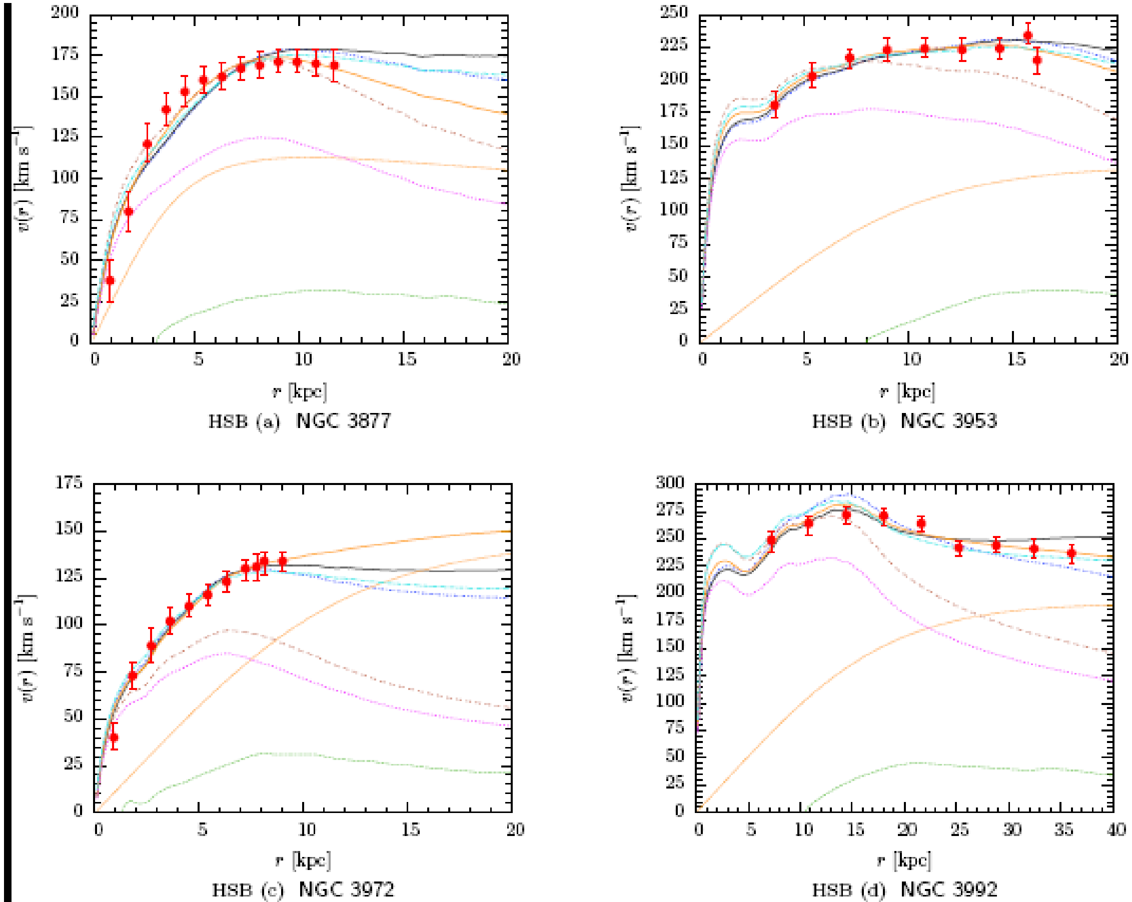
- It is not simple-minded phenomenalism.
  - ➢ It is relativistic gravity theory.
  - ➢ It can be derived from an action principle.
- It can explain without dark matter from small scale(galaxy) to large scale(cosmology).
- It can explain dark energy too. (**arXiv:0710.0364**)

Moffat says —

- A fitting routine has been applied to fit a large number of galaxy rotation curves **(101 galaxies),** using photometric data (58 galaxies) and a core model (43 galaxies) (J. R. Brownstein and JWM, 2005; J. R. Brownstein, 2009).

- The fits to the data are remarkably good for STVG.

For the photometric data, only one parameter, the mass-to-light ratio M/L, is used.



J. R. Brownstein's doctoral thesis

$v(r)$	red filled circles with error bars
Component	**with lines**
Actual HI gas	green dashed line
Stellar disk [$\Upsilon = 1$]	magenta dotted line
Dark matter halo	orange fine-dotted line

Theory	**with lines**
Newtonian core	brown dot-dotted line
STVG	black solid line
MSTG	blue short dash-dotted line
MOND	cyan long dash-dotted line
Dark matter	orange solid line

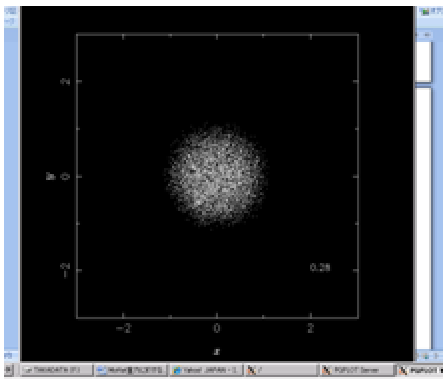
But, above verification is the viewpoint from "statics".
It is necessary to examine it about the real galactic "kinetic" evolution more.

○the summary and motivation of my study

My study is verification of Moffat gravity from the viewpoint of N-body simulation.

$$a_i = -G_N \sum_j \frac{(r_j - r_i)m_i}{|r_j - r_i|^3}\left\{1 + \alpha\left(1 - (1 + \mu|r_j - r_i|)e^{-\mu(|r_j - r_i|)}\right)\right\}$$

Method is very simple.
performing normal N-body simulation
after having changed equation of motion.



cold collapse ・・・

The gravitational collapse of the isodensity ball.
Very simple N-body simulation. This seems to be "an exercise simulation" for beginners
But,It's a bare process of the elliptical galaxy formation.

If you want to know the details

please watch my poster(P31).

○ Acknowledgments

Numerical computations were carried out on the general-purpose PC farm at Center for Computational Astrophysics, **CfCA**, of National Astronomical Observatory of Japan.
数値計算には国立天文台**天文シミュレーションプロジェクトの汎用計算機を使わせて頂きました。**

Thanks to