

# Horava gravity coupled to Brans-Dicke field

(based on arXiv:1003.2840 [hep-th])

by **Tae Hoon Lee** (Soongsil University, Seoul, Korea)

in collaboration with *Joochan Lee* and *Phillial Oh*

# Abstract

We look for a Brans-Dicke type generalization of Horava-Lifshitz gravity. It is shown that such a generalization is possible within the detailed balance condition. The resulting theory reduces in the LR limit to the usual Brans-Dicke theory with a negative cosmological constant for certain values of parameters. We then consider homogeneous, isotropic cosmology and find some interesting features of the Brans-Dicke scalar field in determining the early (and late) behavior of the universe.

- **Action for Brans-Dicke theory**

$$S = \int d^4x \sqrt{-g} \left( \phi R - \frac{\omega}{\phi} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right), \text{ with}$$

$$\sqrt{-g} \phi R \simeq N \sqrt{q} \phi \left( \mathcal{R} + c^{-2} (K_{ab} K^{ab} - K^2) \right) - 2N \sqrt{q} c^{-2} K \pi - 2N \sqrt{q} D^2 \phi,$$

$$-\sqrt{-g} \omega \phi^{-1} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi = N \sqrt{q} \omega \phi^{-1} c^{-2} \pi^2 - N \sqrt{q} \omega \phi^{-1} D^a \phi D_a \phi,$$

where 4-metric  $g$  is decomposed into the lapse  $N$ , the shift  $N^a$  and 3-metric  $q_{ab}$ , and the 3-D covariant derivative  $D_a$ , and its scalar curvature  $\mathcal{R}$ .

1.  $K_{ab} \equiv \frac{1}{2N} (\dot{g}_{ab} - D_a N_b - D_b N_a)$

2.  $\pi \equiv \frac{1}{N} (\dot{\phi} - N^a \partial_a \phi).$

# Matrix form

- Brans-Dicke action can be split into the two parts

$$S_{BD} = S_{BD}^K + S_{BD}^V$$

$$1. S_{BD}^K = \int dt d^3x N \sqrt{q} \left( \phi (K_{ab} K^{ab} - K^2) - 2K\pi + \omega \phi^{-1} \pi^2 \right)$$

$$= \int dt d^3x N \sqrt{q} \begin{pmatrix} K_{ab} & \pi \end{pmatrix} \begin{pmatrix} \phi G^{abcd} & -q^{ab} \\ -q^{cd} & \omega \phi^{-1} \end{pmatrix} \begin{pmatrix} K_{cd} \\ \pi \end{pmatrix},$$

$$\text{where } G^{abcd} = \frac{1}{2} (q^{ac} q^{bd} + q^{ad} q^{bc}) - q^{ab} q^{cd}.$$

$$2. S_{BD}^V = c^2 \int dt d^3x N \sqrt{q} \left( \phi \mathcal{R} - 2D^2 \phi - \omega \phi^{-1} D^a \phi D_a \phi \right)$$

- The matrix in  $S_{BD}^K$  can be regarded as the supermetric on the space of  $(q_{ab}, \phi)$ , naturally extending the DeWitt metric on the 3-metric space.

- To construct a **Brans-Dicke type extension** of Horava-Lifshitz gravity with the **detailed balance condition**, choose the action of the form,

$S_{HLBD} = S_{HLBD}^K + S_{HLBD}^V$ , where the kinetic part is

$$S_{HLBD}^K = \int dt d^3x N \sqrt{q} \begin{pmatrix} K_{ab} & \pi \end{pmatrix} \begin{pmatrix} \phi G^{abcd}(\lambda) & -q^{ab} \\ -q^{cd} & \omega \phi^{-1} \end{pmatrix} \begin{pmatrix} K_{cd} \\ \pi \end{pmatrix}$$

and the potential part is of the form

$$S_{HLBD}^V = - \int dt d^3x N \sqrt{q} \begin{pmatrix} \frac{\delta W}{\delta q_{ab}} & \frac{1}{2} \frac{\delta W}{\delta \phi} \end{pmatrix} \begin{pmatrix} \phi G^{abcd}(\lambda) & -q^{ab} \\ -q^{cd} & \omega \phi^{-1} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\delta W}{\delta q_{cd}} \\ \frac{1}{2} \frac{\delta W}{\delta \phi} \end{pmatrix}$$

for some function  $W(q, \phi)$

- **Inverse supermetric with new parameters**

[With  $G^{abcd}(\lambda) \equiv \frac{1}{2} (q^{ac}q^{bd} + q^{ad}q^{bc}) - \lambda q^{ab}q^{cd}$  ]

Inverse supermetric comes out to be of form  $\begin{pmatrix} \phi^{-1}\mathcal{G}_{abcd} & -Aq_{ab} \\ -Aq_{cd} & B\phi \end{pmatrix}$ ,

where  $\mathcal{G}_{abcd} = \frac{1}{2} (q_{ac}q_{bd} + q_{ad}q_{bc}) - \bar{\lambda}q_{ab}q_{cd}$ ,

with  $A = \frac{1}{\omega(3\lambda-1)+3}$ ,  $B = \frac{3\lambda-1}{\omega(3\lambda-1)+3}$ ,  $\bar{\lambda} = \frac{1+\omega\lambda}{\omega(3\lambda-1)+3}$ .

# Main Results

- Choose

$$W = c_1 \int d^3x \sqrt{q} \phi (\mathcal{R} - 2\Lambda_b) - c_2 \int d^3x \sqrt{q} \omega \phi^{-1} D^a \phi D_a \phi,$$

then from Equations in Page 5-6 you have

$$S_{HLBD}^V = \int dt d^3x N \sqrt{q} \left\{ \alpha \phi + \beta (\phi \mathcal{R} - \frac{c_2}{c_1} \omega \phi^{-1} D^a \phi D_a \phi) + \gamma (-2D^2 \phi) \right\} \\ - \int dt d^3x N \sqrt{q} \left( Q^{ab} \phi^{-1} \mathcal{G}_{abcd} Q^{cd} - 2A Q^{ab} q_{ab} Q + B \phi Q^2 \right),$$

with parameters

$$\alpha = (c_1 \Lambda_b)^2 \frac{3\omega + 7 - 3\lambda}{\omega(3\lambda - 1) + 3},$$

$$\beta = -(c_1)^2 \Lambda_b \frac{\omega + 5 - 3\lambda}{\omega(3\lambda - 1) + 3}$$

$$\gamma = -(c_1)^2 \Lambda_b \frac{2(\omega + 1) - \frac{c_2}{c_1} \omega(4 - 3\lambda)}{\omega(3\lambda - 1) + 3}, \text{ and with}$$

$$Q^{ab} \equiv c_1 \left( -\phi (\mathcal{R}^{ab} - \frac{1}{2} \mathcal{R} q^{ab}) + D^a D^b \phi - q^{ab} D^2 \phi \right),$$

$$Q \equiv c_1 \frac{\mathcal{R}}{2} - c_2 \left( -\omega \phi^{-1} D^2 \phi + \frac{\omega}{2} \phi^{-2} D^a \phi D_a \phi \right).$$

- **IR Limit**

In the infrared limit the potential part of the action becomes

$$S_{BDHL}^V|_{IR} = -(c_1)^2 \Lambda_b \frac{\omega+2}{2\omega+3} \int dt d^3x N \sqrt{q} \left( \phi(\mathcal{R} - 2\Lambda) - 2D^2\phi - \omega\phi^{-1} D^a\phi D_a\phi \right),$$

where

$$\Lambda = \frac{3\omega+4}{2(\omega+2)} \Lambda_b.$$

This expression coincides with that of the Brans-Dicke theory except that the cosmological constant term is present. The speed of light is

$$c^2 = -(c_1)^2 \Lambda_b \frac{\omega+2}{2\omega+3}.$$

- **Equations in Robertson-Walker spacetime**

$$3H^2 + 3H\frac{\dot{\phi}}{\phi} - \frac{1}{2}\omega\left(\frac{\dot{\phi}}{\phi}\right)^2 = \frac{1}{2}\phi^{-1}\rho_m - \left(\frac{3k}{a^2} - \Lambda\right) - \frac{1}{2}\left(\frac{B^2}{a^4}\right),$$

$$(2\omega + 3) \left(\frac{\ddot{\phi}}{\phi} + 3H\frac{\dot{\phi}}{\phi}\right) = \frac{1}{2}\phi^{-1}(\rho_m - 3p_m) + 2\Lambda + \frac{B^2}{a^4},$$

with the usual form of the continuity equation for the matter density  $\rho_m$ ,

$$H \equiv (\dot{a}/a),$$

$$B^2 = \frac{3\omega}{2\omega+3}(kc_1)^2 = \frac{3\omega(3\omega+4)}{2(\omega+2)} \frac{k^2}{(-\Lambda)}.$$

- **Cosmology**

Consider two limiting cases for vacuum,  $\rho_m = p_m = 0$ .

First, for small  $a$ , the dark radiation term dominates:

$$a^2(t) = 2ht, \quad \phi(t) = \frac{\phi_0}{t}, \quad (1)$$

Secondly, in the large  $a$  limit, cosmological term dominates over the curvature and dark radiation terms:

$$H = h, \quad \frac{\dot{\phi}}{\phi} = g,$$
$$h = -\frac{g}{2} = \pm \sqrt{\frac{-4\Lambda}{2\omega+3}}.$$

This solution represents the universe exponentially expanding (recently).

- **References**

P. Horava, *Phys. Rev.* **D79** (2009) 084008.

T. Takahashi and J. Soda, *P. R. L.* **102**, (2009) 231301.

S. Mukohyama, *JCAP* **0906**, (2009) 001.

R.-G. Cai, L.-M. Cao, and N. Ohta, *P. R.* **D80**, (09) 024003.

R.-G. Cai, Y. Liu, and Y.-W. Sun, *JHEP* **0906**, (2009) 010.

E. Kiritsis and G. Kofinas, *Nucl. Phys.* **B821**, (2009) 467.

C. Brans and R. H. Dicke, *Phys. Rev.* **124**, (1961) 925.

K. Uehara, C.W. Kim, *Phys. Rev.* **D26**, (1982) 2575.