

BLACK HOLE REMNANTS, PRE INFLATION MATTER ERA AND CMB POWER SPECTRUM

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Introduction and Outline

- Micro black holes in the pre-inflation era ($10^{-43} - 10^{-37} \text{ s}$)
 - matter-dominated universe
 - influence on the evolution of a scalar field
 - becomes visible in the CMB power spectrum as
Suppression of the large scale multipole (quadrupole) moments

Nucleation of Micro Black Holes: Gross, Perry, Jaffe (1982); Kapusta (1984): gravitational instabilities of flat space. Expression for the probability of spontaneous formation (bubbling) of black holes out of the gravitational (metrical) instabilities of spacetime.

$$\Gamma_N(\Theta) = \frac{1}{15 \cdot 8\pi^2} \Theta^{-\frac{167}{45}} \exp\left(-\frac{1}{16\pi\Theta^2}\right)$$

Generalized Uncertainty Principle and Black Hole Remnants

GUP: String theory

$$\Delta x \geq \frac{\hbar}{2\Delta p} + 2\beta l_{4n}^2 \frac{\Delta p}{\hbar},$$

Heisenberg argument: the smallest detail theoretically detectable with a beam of photons of energy E is roughly given by

$$\delta x \simeq \frac{\hbar c}{2E}.$$

GUP version of the standard Heisenberg formula

$$\delta x \simeq \frac{\hbar c}{2E} + \beta l_p \frac{E}{\epsilon_p}.$$

Uncertainty in photon position just outside a BH

$$\delta x \simeq 2\mu R_S = 2\mu \ell_p m$$

Equipartition law: energy of unpolarized photons of outgoing Hawking radiation

$$E_\epsilon \simeq k_B T.$$

Measuring all temperatures in Planck units as $\Theta = T/T_p$, we have

$$2m = \frac{1}{2\pi\Theta} + \zeta 2\pi\Theta$$

where $\zeta = \beta / \pi^2$

Assume $\zeta \approx 1$

→→→

$$\Theta_{\max} = \frac{1}{2\pi\sqrt{\zeta}}$$
$$m_{\min} = \sqrt{\zeta}$$

Minimum Mass & Maximum Temperature

→→ ! **BLACK HOLE REMNANTS !**

Once nucleated, even after BH evaporation, REMNANTS stay there [GUP !]. They are sufficient to put the Universe in matter era until the onset of Inflation

Equation of motion

Pre inflationary Universe containing:

Matter (nucleated micro black holes)(A), Radiation (B),

Constant Vacuum Energy (= responsible for the inflation) (C)

Flat FRW metric: $ds^2 = -c^2dt^2 + a(t)^2 (dr^2 + r^2d\Omega^2)$.

Einstein
equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \kappa \left(\frac{A}{a^3} + \frac{B}{a^4} + C\right)$$

Matter era
Condition

$$\frac{B}{Aa} \ll 1$$

Pre inflation
radiation era solution

$$a(t) = \left(\frac{B}{C}\right)^{1/4} \left[\sinh\left(2\sqrt{\kappa C} t\right)\right]^{1/2}$$

$$a(t) \sim t^{1/2}$$
$$A = 0$$

Pre inflation
matter era solution

$$a(t) = \left(\frac{A}{C}\right)^{1/3} \left[\sinh\left(\frac{3}{2}\sqrt{\kappa C} t\right)\right]^{2/3}$$

$$a(t) \sim t^{2/3}$$
$$B = 0$$

Time in Planck units:

$$\tau = t/t_p$$

Constants A, B:

$$A = \rho_m(\tau_c) \cdot a^3(\tau_c),$$
$$B = \rho_r(\tau_p) \cdot a^4(\tau_p),$$

Numerical simulation: computation of A, B

Assuming $\rho_{\text{rad}} = \text{Planck density}$ at $\tau \approx 1 t_p \rightarrow$

$$B = 1$$

Adiabatically expanding Universe:

Nucleations rate as
functions of time

$$\Gamma_{N,r}(\tau) = \frac{1}{15 \cdot 8\pi^2} \cdot \tau^{167/90} e^{-\tau/16\pi}$$

CUTOFFS:

From GUP: $m \approx 1 \Rightarrow \tau \approx 160$ (not enough to avoid BH superpositions)

From **HOLOGRAPHIC PRINCIPLE:**

$$S[L(B)] \leq k_B \frac{A(B)}{4\ell_p^2} \rightarrow S_{bh}(\tau) \leq S_{HS}(\tau) \rightarrow$$

BH Nucleation
effective at

$$\tau_c \approx 990 t_p$$

- Universe starts at $\tau = 1$ in Radiation dominated era; and so evolves until $\tau_c \approx 990$
- **BH nucleation starts at $\tau_c \approx 990$, goes on for $\approx 10 t_p$, then is exponentially suppressed**
- **Inflation starts at $\tau \approx 10^6 - 10^7 t_p$**
- **About $N = 10^4$ BHs are produced.** The average mass is $m(\tau_c) = 2.5 M_p$. They evaporate down to $\approx 1 M_p$ in about $10^4 t_p$

Matter density at end of BH Nucleation era:

$$\rho_m(\tau_c) \sim \frac{10^4 \text{ black holes}}{R_H^3(\tau_c)} \sim \frac{10^4 \sqrt{\zeta} \epsilon_p}{10^9 V_p}$$



$$A = \frac{10^4 \sqrt{\zeta} \epsilon_p}{10^9 V_p} \cdot 10^{9/2} \sim 10^{-1/2} \sqrt{\zeta} \frac{\epsilon_p}{V_p}$$

$$\frac{3}{2} \frac{B}{A \cdot a(\tau)} = \frac{1}{10^{-1/2} \cdot a(\tau)} \sim 10^{-1} - 10^{-3} \ll 1$$

**Condition for matter dominance:
SATISFIED !**

Influences of a pre-inflation matter era on scalar field fluctuations

Scalar field fluctuations: $\Phi(t, \vec{y}) = \Phi_0(t, \vec{y}) + \varphi(t, \vec{y})$

Equation of motion:

$$\square \varphi(t, \vec{y}) = 0$$

$$\ddot{\phi}_k(t) + 3\frac{\dot{a}}{a}\dot{\phi}_k(t) + \left(\frac{k^2}{a^2}\right)\phi_k(t) = 0$$

THE RE-ENTERING K-MODES:

k-modes leaving the horizon just at the onset of inflation, are just now re-entering our Hubble radius. They bring imprints of a possible pre-inflation (matter) era.

Mode of largest visible perturbation

$$k_{\min} \simeq aH$$

Pre-inflation radiation era

$$k_{\min} = a\sqrt{\kappa} \left(\frac{B}{a^4} + C \right)^{1/2}$$

Pre-inflation matter era

$$k_{\min} = a\sqrt{\kappa} \left(\frac{A}{a^3} + C \right)^{1/2}$$

ANALYTICAL COMPUTATION:

Solve EoM for $\phi(a, k)$ k -parameter. Express $a=a(k)$

Construct Primordial Power Spectrum of the quantum fluctuations of the field ϕ

$$P(k) = k^3 |\phi(a(k), k)|^2$$

$P(k)$ feeds CMBFAST code $\rightarrow \rightarrow \rightarrow$ CMB anisotropy power spectrum

EoM for $\phi(a, k)$ in **pre inflation matter** era

$$\phi_k'' + \frac{1}{a} \left(\frac{4Ca^3 + \frac{5}{2}A}{Ca^3 + A} \right) \phi_k' + \left(\frac{k^2}{\kappa a(Ca^3 + A)} \right) \phi_k = 0$$

For comparison we consider also $\phi(a, k)$ in **pre inflation radiation** era

$$\phi_k'' + \frac{2}{a} \left(\frac{Ca^4}{Ca^4 + B} + 1 \right) \phi_k' + \left(\frac{k^2}{\kappa(Ca^4 + B)} \right) \phi_k = 0$$

- Boundary conditions in full inflationary era:

From Last WMAP data: almost **scale invariant** (flat), **slightly tilted**, primordial power spectrum $P(k)$

$$P(k) \sim k^{n_s-1}$$

with

$$n_s = 0.963 \pm 0.012 \text{ (68\% CL)} .$$

Therefore the field ϕ must behave as

$$|\phi(a(k), k)| \sim \frac{k^{\frac{1}{2}(n_s-1)}}{k^{3/2}}$$

for large k .

- WKB solutions of equation of motion:

Pre inflation
matter era

$$\phi_k(a) = \frac{2 \sqrt[4]{\kappa C^2} [c_+(k)e^{iG(a)} + c_-(k)e^{-iG(a)}] e^{-i\pi/4}}{[32\kappa C^2 a^6 - 16k^2 C a^4 + 9\kappa A^2]^{1/4} \cdot \exp[A/(4C a^3)]}$$

Arbitrary constants $c(k)$ can be chosen so that

$$a(k) \simeq \frac{k}{\sqrt{\kappa C}} - \frac{A\kappa}{2k^2}$$

$$[c_+(k)e^{iG(a(k))} + c_-(k)e^{-iG(a(k))}] \sim k^{(n_s-1)/2}$$

Analogous procedure for the pre inflation radiation era

Qualitative plots for $P(k)$ ($A = B = C = 1$)

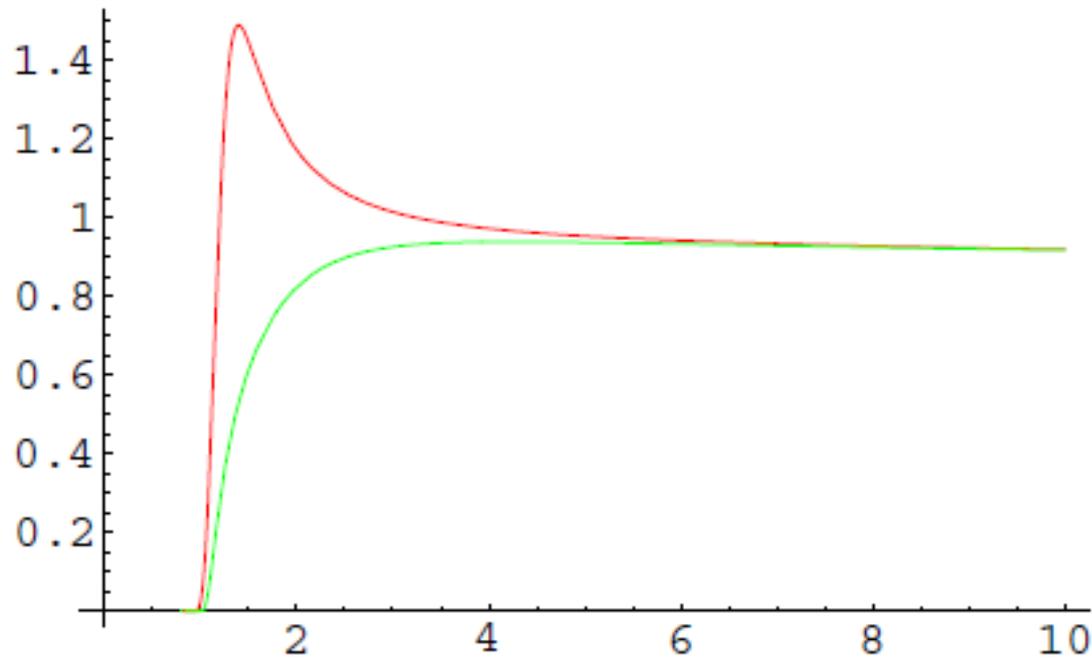


FIG. 14: Primordial power spectra $P(k)$ versus k , for pre-inflation matter (red line) and radiation (green line) eras.

EXACT NUMERICAL COMPUTATION OF THE PRIMORDIAL POWER SPECTRUM

- MATTER ERA:** Friedmann parameters

$$A = 10^{-1/2} \frac{\epsilon_p}{V_p}$$

Condition for the onset of inflation \rightarrow Computation of C :

$$C = \frac{A}{2 a_{inf}^3} = \frac{1}{2} 10^{-11} \frac{\epsilon_p}{V_p}$$

$$a_{inf} = 1 \cdot (10^3)^{-1/6} \cdot (10^6)^{2/3} = 10^{7/2}$$

Numerical solution for $\phi(a(k_i), k_i)$ and $P(k_i)$:
just a COLLECTION of DATA POINTS

Fitting Function

$$P(k) = a - \frac{b}{1 + \frac{k^2}{c}} + \frac{d}{1 + \frac{k^4}{e}} - \frac{f}{1 + \frac{k^6}{g}}$$

to feed
CMBFAST

parameter	a	b	c	d
value	$2.205 \cdot 10^{-12}$	$3.233 \cdot 10^{-12}$	0.03	$2.578 \cdot 10^{-12}$
parameter	e	f	g	
value	$1.680 \cdot 10^{-9}$	$1.593 \cdot 10^{-12}$	$6.584 \cdot 10^{-14}$	

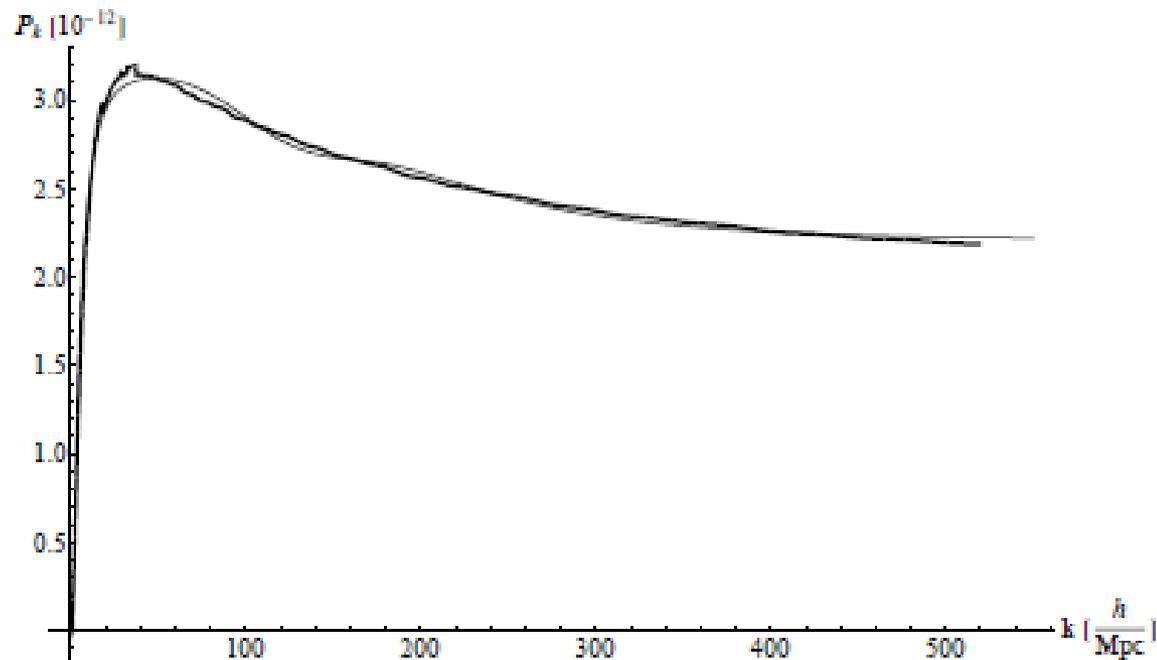


FIG. 15: The numerical solution for the primordial power spectrum (dashed line) and its fitting function (full line) in the pre-inflation matter era scenario.

- Radiation era with totally evaporating black holes (NO GUP !)

Friedmann parameters:

$$B = 10^{2/3} \sqrt{\zeta} \frac{\epsilon_p}{V_p}$$

$$\zeta \sim 1$$

$$a(\tau_{inf}) = 10^{25/6} \sim 10^{4.16}$$

$$C \simeq \frac{B}{a_{inf}^4} \sim 10^{-16}$$

Fitting Function:

$$P(k) = a + b \cdot \text{Arctan}(c \cdot k) + d \cdot \text{Arctan}^2(c \cdot k)$$

parameter	a	b	c	d
value	$-3.701 \cdot 10^{-18}$	$1.261 \cdot 10^{-16}$	3.116	$-5.935 \cdot 10^{-17}$

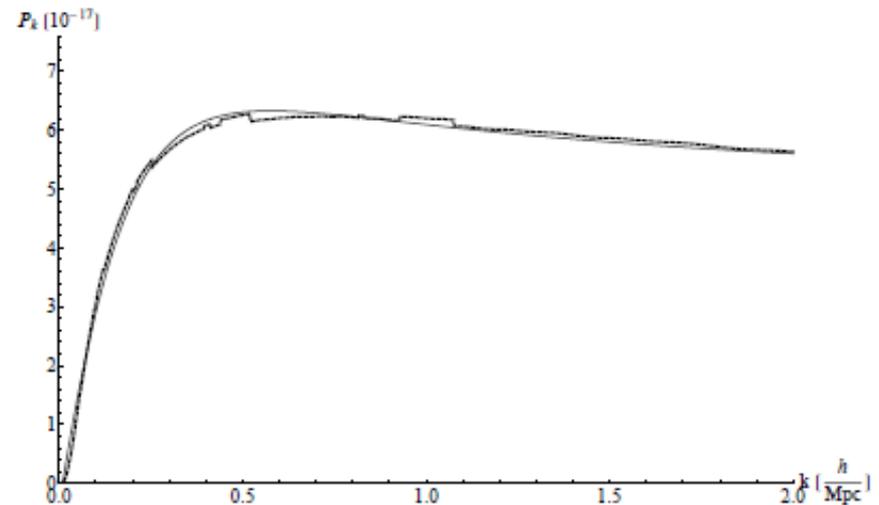


FIG. 16: The numerical solution for the primordial power spectrum (dashed line) and its fitting function (full line) in the pre-inflation radiation era scenario, with completely evaporating black holes (no GUP active).

- Radiation era without black holes

$$a(\tau_{inf}) = 10^{7/2}$$

$$B = \rho_p(\tau_p) \cdot a^4(\tau_p) = 1 \frac{\epsilon_p}{V_p}$$

$$C = \frac{B}{a_{inf}^4} \sim 10^{-14}$$

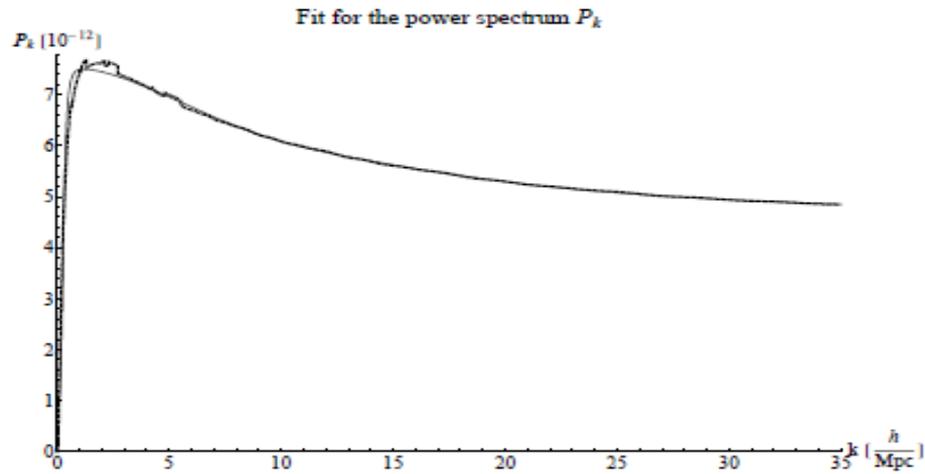
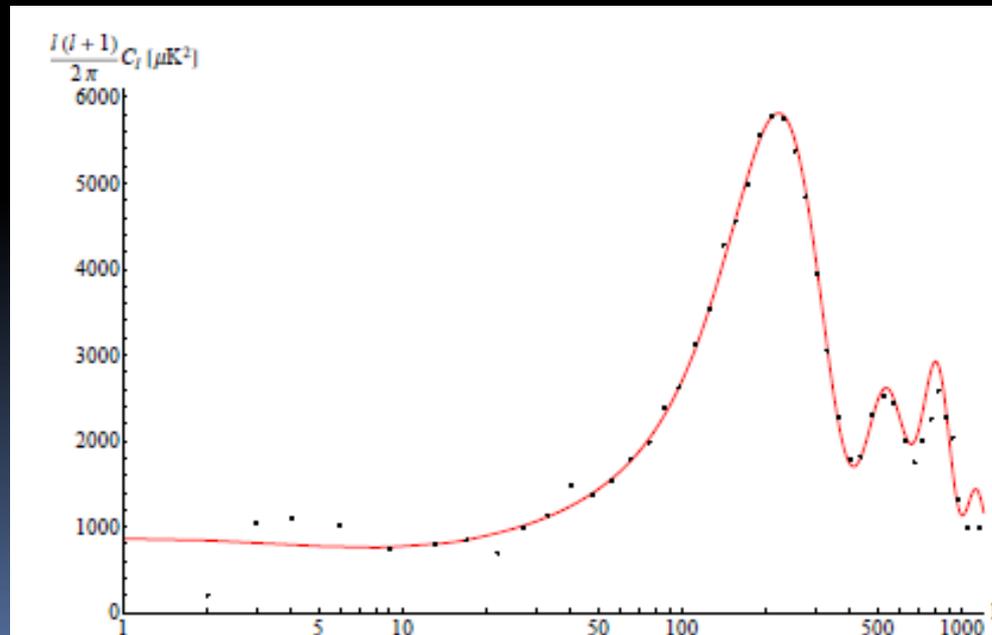
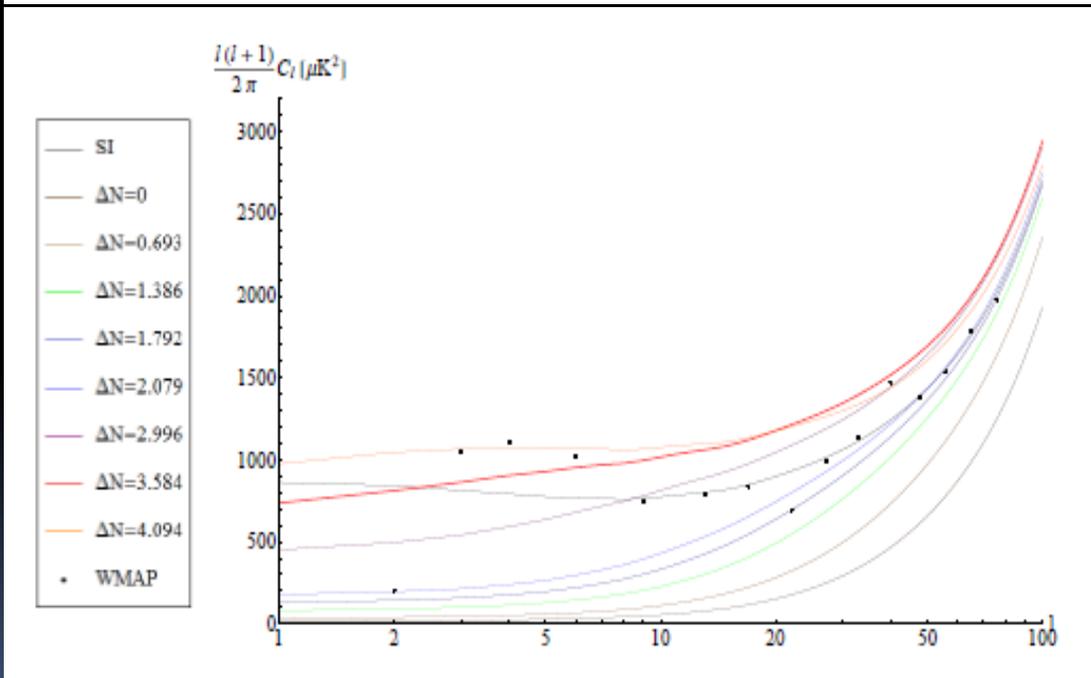
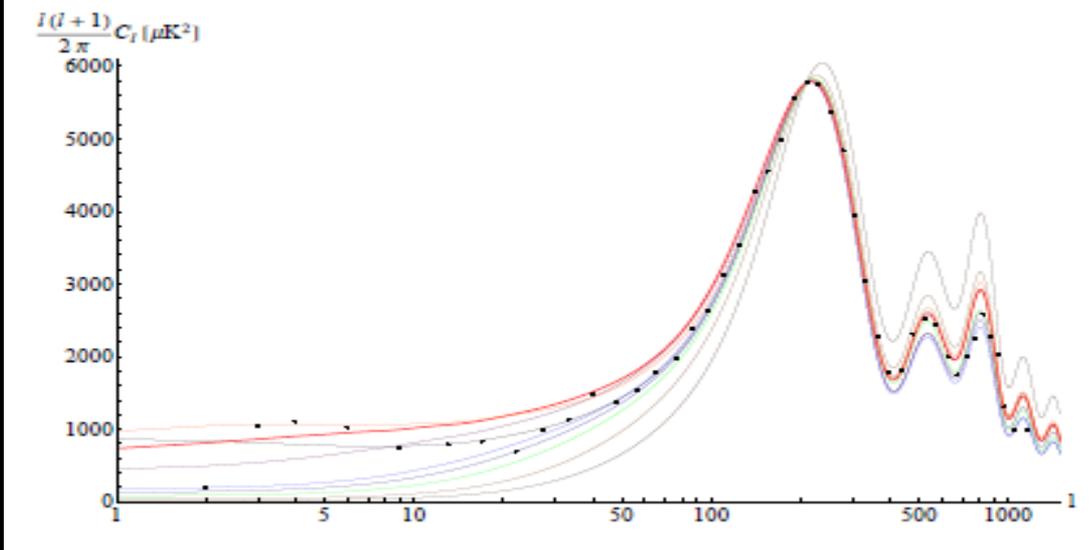


FIG. 17: The numerical solution for the primordial power spectrum (dashed line) and its fitting function (full line) in the pre-inflation radiation era scenario without black holes.

CMB POWER SPECTRUM

- Primordial power spectra (with GUP, without GUP, without black holes)
→ CMBFAST code [Seljak, Zaldarriaga 1996] → CMB temperature anisotropy spectrum.
- Compare with WMAP 7 year data, and with the standard CMB spectrum of standard inflationary Λ CDM
- Λ CDM model: anomalies in the suppressed quadrupole moment: the $l = 2$ mode is very low in comparison to the CMB spectrum .





The CMB power spectrum for a **pre-inflation matter era**, for various cases of ΔN *e*-folds added. Overall view and zoom.

- In the numerical computation for CMB power spectrum, **THE TOTAL NUMBER OF E-FOLDS OF INFLATION** (from when the mode k_i left the horizon, to the end of inflation) **CAN BE VARIED** [nobody knows EXACTLY when inflation started].

$$N_{tot} = N(k_0) + \ln\left(\frac{k_0}{k_i}\right)$$

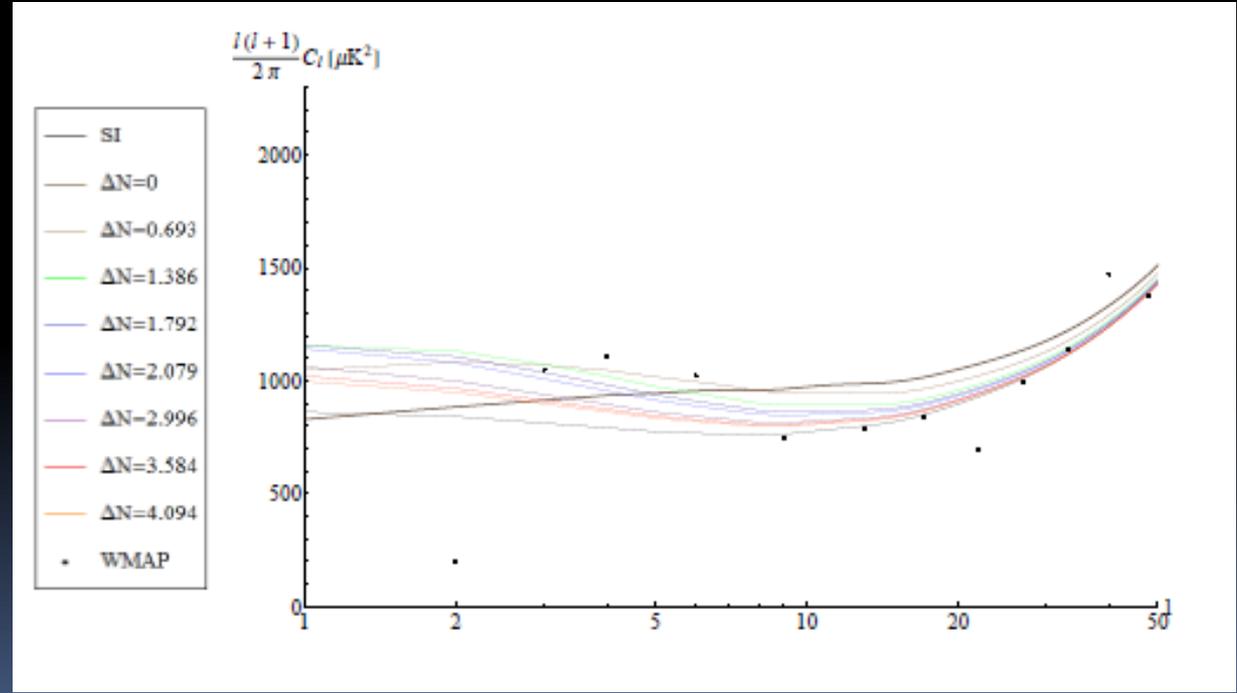
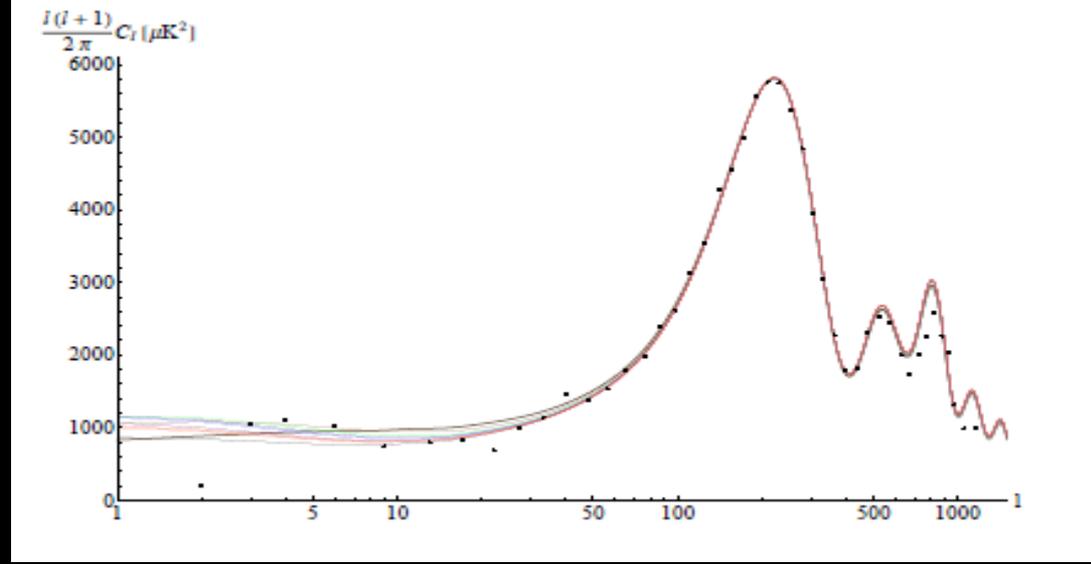
- k_0 is the currently largest mode within the horizon

$$k_0 = 0.002 \text{ hMpc}^{-1}$$

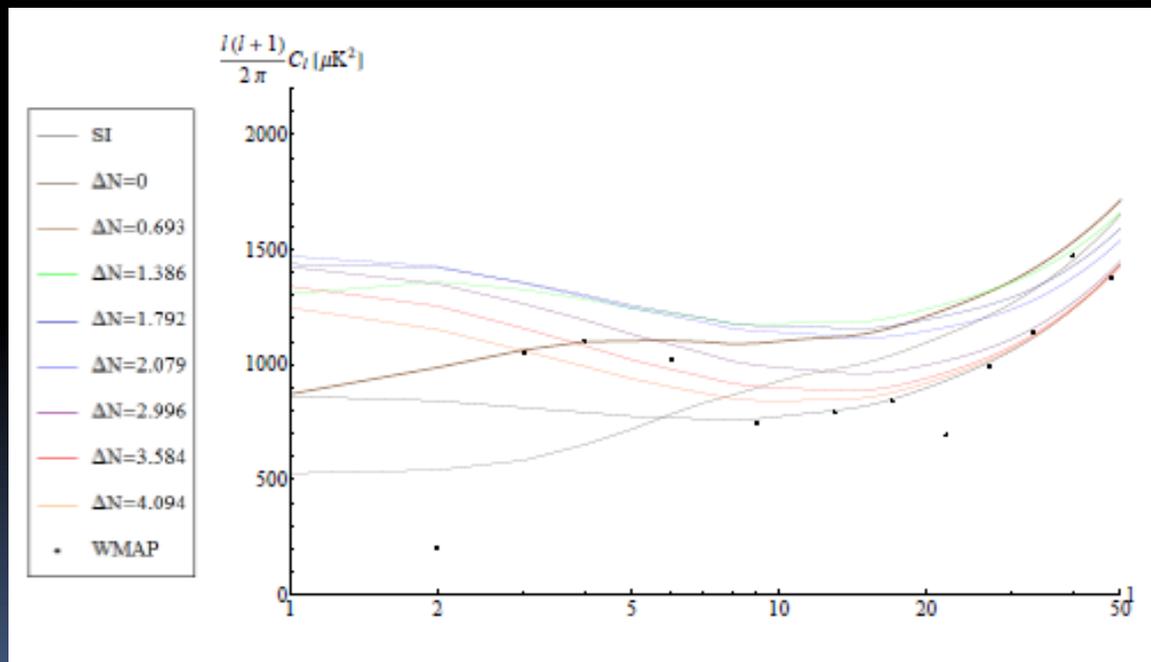
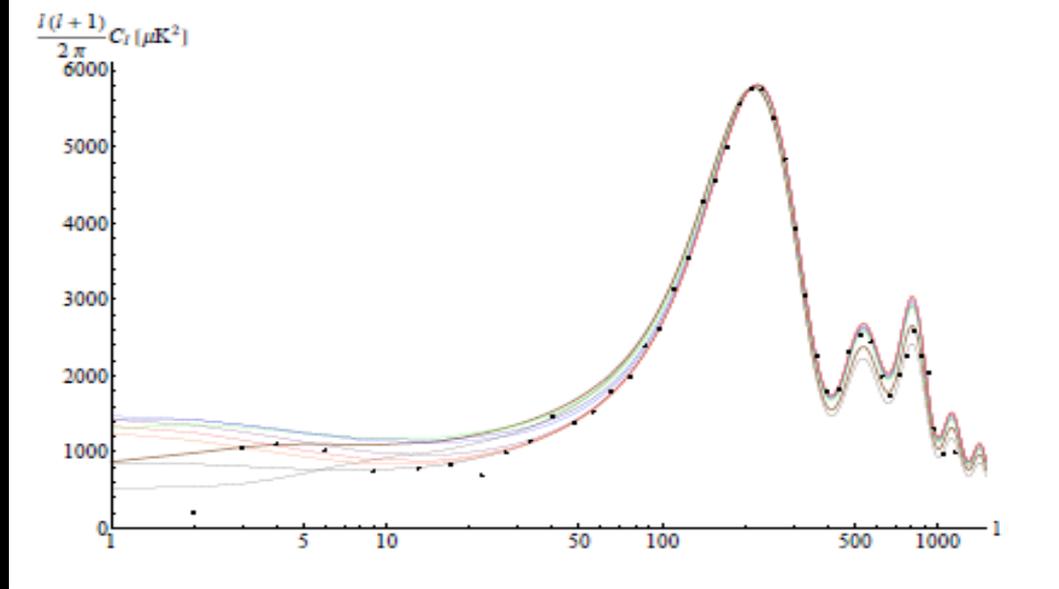
Observations constrain the number $N(k_0)$
no information on the number of e-folds ΔN of inflation before k_0 exited the horizon during inflation. The constraint on $N(k_0)$ is

$$N(k_0) = 54 \pm 7$$

So, varying k_i is equivalent to adding e-folds ΔN to the experimentally constrained number $N(k_0) = 54 \pm 7$.



The CMB power spectrum for a **pre-inflation radiation era (NO GUP)**, for various cases of ΔN e-folds added. Overall view and zoom.



The CMB power spectrum for a **pre-inflation radiation era without any black holes**, for various cases of ΔN . Overall view and zoom.

Conclusions and outlook

- Investigated effects of **pre inflation era** on CMB power spectrum
- **BH nucleation** induces a **pre inflation matter** dominated era
- Computed (analytically and numerically) the power spectrum of primordial **fluctuations of a scalar field Φ** living in this scenario
- The **primordial power spectrum processed by CMBFAST** code to yield the **CMB temperature anisotropy power spectrum**, compared with observations
- Alternative scenarii investigated: pre inflation radiation dominated era (with no remnants or with no BH at all)
- **The pre inflation matter model seems to be the only one, among those studied, able to capture and describe the $l = 2$ mode suppression, although the radiation model still presents a better fitting of the data at high l values.**