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A new plane symmetric solution

Zhang, Hongsheng
(张宏升)
with collaborators Zhu, Zong-Hong and Noh, Hyerim
Shanghai Normal University

Exact solutions

- Non-linear, superposition principle fails. Every solution is unique.
- Exact solutions----- More than a thousand (Exact Solutions of Einstein's Field Equations, H. Stephani, D. Kramer, M. A. H. Ma'cCallum, C. Hoenselaers and E. Herlt)
- But physical meanings?-----seldom

"The key to physical interpretation is to find out the nature of the sources which generate the vacuum spaces."-----Bonner

Taub solution

$$ds^2 = -z^{-2/3}k^2dt^2 + dz^2 + z^{4/3}l^2\left(dx^2 + dy^2\right).$$

Taub, 1951

No-go theorem

across the boundary. We then have the result:

An incompressible fluid cannot bound a vacuum in a space-time with plane symmetry unless the boundary condition of the continuity of the derivatives of the metric tensor is violated. As is well known, a similar

from the differential equations satisfied by Y on both sides of the point $H=-2\phi_0$. For the empty region, we have

$$dY/dH=-\tfrac{3}{2}Y,$$

whereas for the region occupied by the incompressible fluid we have the equation before (9.1). Since $\delta\neq 0$, dY/dH cannot be continuous across the boundary and hence we cannot satisfy the boundary condition that the derivatives of the metric tensor be continuous across the boundary. We then have the result:

An incompressible fluid cannot bound a vacuum in a space-time with plane symmetry unless the boundary condition of the continuity of the derivatives of the metric tensor is violated. As is well known, a similar result holds in the spherically symmetric case.

$$Y'=-Y\frac{\rho'}{\rho}\left(\frac{p}{p_0}+3Y^2\right)\Big/\left(\frac{p+p'}{p_0}-3Y^2\right) \qquad (10.6)$$

and

$$2e^{-\phi}\phi_\tau=\frac{\rho}{\rho' Y}\left(\frac{p+p'}{p_0}-3Y^2\right). \qquad (10.7)$$

Equation (10.6) is an Abel differential equation for the variable Y^2. When it is solved for $Y=Y(\phi)$ we may substitute its solution into Eq. (10.7) and determine $\phi(\tau)$. We shall illustrate the method of dealing with these equations for the special case of the degenerate gas where

$$p/p_0=e^{-4\phi}$$

and

$$\rho/\rho_0=e^{-3\phi}.$$

Taub, 1956

Our solution

We find that the following metric solves Einstein filed equation with source in perfect fluid form,

$$ds^2 = -f(z)^2 dt^2 + dz^2 + e^{2az} f(z)^2 e^{-2\left[az + \frac{h(z)\arctan\left(e^{az}\sqrt{\frac{w}{c}}\right)}{\sqrt{wc}} - dh(z)\right]} \left(dx^2 + dy^2\right),$$

where,

$$f(z) = ce^{-az} + we^{az} \quad ,$$

$$h(z) = -ce^{-az} + we^{az}.$$

Clearly there are four Killing fields $\frac{\partial}{\partial t}$, $\frac{\partial}{\partial x}$, $\frac{\partial}{\partial y}$ and $-y\frac{\partial}{\partial x} + x\frac{\partial}{\partial y}$.

Density and pressure

$$\rho = -\Big[-16b^3c^4du^3 + 16b^4c^3du^5 + 12b^3c^4du^3 + 45b^2c^4u^2v^2 + 3b^7d^2cu^{10} + 12b^{7/2}c^{5/2}u^5v + 18b^2c^6d^2 - 12b^4c^3du^5 + 45b^3c^5d^2u^2 + 18b^6c^2d^2u^8 + 45b^5c^3d^2u^6 + 18bc^5v^2 - 6b^6\sqrt{bc}du^{10}v + 18b^{9/2}c^{3/2}u^7v - 16b^2c^4e^{2ar} - 120(bc)^{7/2}du^4v + 6b^5cu^8 + 6b^{11/2}c^{1/2}u^9v - 24b^2c^5du - 8bc^6/u + 24b^{3/2}c^{9/2}v + 8\sqrt{bc}c^5u^{-1}v - 12b^{5/2}c^{7/2}u^3v + 45b^4c^2u^6v^2 + 8b^4c^2u^6 + 24b^5c^2du^7 + 60b^4c^4d^2u^4 - 90b^{9/2}c^{5/2}dv + b^{5/2}c^{9/2}du + 8b^6cdu^9 - 8bc^5 - 6\sqrt{bc}c^6u^{-2}v + 18b^5cu^8v^2 + 3c^6u^{-2}v^2 + 6bc^5 - 16b^2c^3u^4 + 3bc^7d^2u^{-2} - 18b^{3/2}c^{9/2}uv - 6b^{1/2}c^{11/2}u^{-1}v - 8b^5cu^8 + 4b^3c^3u^4 - 18b^5c^2du^7 + 6bc^6d/u - 16b^4c^2u^6 + 3b^6u^{10}v^2 - 36b^{3/2}c^{11/2}v + 16b^2c^3\sqrt{bc}u^3v - 16b^4c^3u^5v - 90b^2c^4\sqrt{bc}du^2v - 24b^5c^2u^7v + +8b^2c^4u^2 - 36b^5c\sqrt{bc}du^8v - 8b^5\sqrt{bc}v + 60b^3c^3u^4v^2 - 6b^6cdu^9\Big]\, a^2(bc)^{-1}(c+bu^2)^{-4}\ ,$$

$$p = -\Big[4bc^6du^{-1} - 6b^2c^6d^2 - 2b^5\sqrt{bc}u^7v + 40b^3c^3\sqrt{bc}dv/u - 8b^3c^3u^2 + 2\sqrt{bc}c^6u^{-2}v - 6b^2c^5d/u - 4bc^5 + 6b^5c^2du^5 - 15b^4c^2u^4v^2 - 8b^4c^3du^3 - 12b^5c^2du^5 - 6b^4c\sqrt{bc}u^5v - 4b^5cu^6 + \sqrt{bc}c^5vu^{-3} + 2b^6cdu^7 + 2b^6\sqrt{bc}du^8v + 12b^4c\sqrt{bc}u^5v + 12b^5c\sqrt{bc}u^6v + 8b^3c^4du - 15b^2c^4v^2 - 6b^6c^2d^2u^6 - 6bc^5u^{-2}v^2 - b^6u^8v^2 - 4b^6cdu^7 + 30b^2c^4\sqrt{bc}dv + 4b^4c^3du^3 + 8b^3c^3u^2 + 4b^5cu^6 + 6bc^4\sqrt{bc}v/u + 12bc^5\sqrt{bc}u^{-2}v - 15b^3c^5d^2 + 30b^4c^2\sqrt{bc}u^4v + 8b^3c^2\sqrt{bc}u^3v + 4bc^5\sqrt{bc}u^{-2} - 2bc^6\sqrt{bc}du^{-3} - c^6u^{-4}v^2 - b^7cd^2u^8 - 4b^3c^4du + 4b^5\sqrt{bc}u^7v + 12b^2c^5d/u + 4b^2c^3\sqrt{bc}uv - bc^7d^2u^{-4} - 12bc^4\sqrt{bc}v/u + 8b^2c^4 - 20b^4c^4d^2u^2 - 20b^3c^3u^2v^2 - 8b^3c^3\sqrt{bc}uv - 4c^5\sqrt{bc}u^{-3}v - 6b^5cu^6v^2\Big]\, a^2(bc)^{-1}(c+bu^2)^{-4}\ ,$$

where $u = e^{az}$, $v = \arctan\left(u\sqrt{\frac{b}{c}}\right)$.

General requirement to be an interior solution of a vacuum solution

General boundary conditions

1. The metric is continuous
2. The extrinsic curvatures measured by the different sides of the boundary surface satisfy

$$[K - h\mathrm{tr}(K)]^{\pm} = -\tau,$$

The source of Taub space

- In a special family of the above solution, under the condition $az_0 = -1/3$ the large branch of plane symmetric solution we proposed can perfectly match to the vacuum Taub space. Hence it is the proper source of Taub space.

What’s the fundamental new idea?

- Negative pressure **is** possible.
- Matter which violates energy conditions may **exist**.
- The essential physics: negative pressure **props up** this thick plane

The properties of this space

- Geodesics
- Essential contents

N-dim solution

$$ds^2 = -e^{2az}\, dt^2 + dz^2 + e^{2[az+be^{az/(n-3)}]}\, d\Sigma^2,$$

$$\rho = -\frac{a^2}{2}\frac{n-2}{(n-3)^2}\left[(n-3)^2(n-1) + 2(n-2)^2 b e^{az/(n-3)} + (n-1)b^2 e^{2az/(n-3)}\right],$$

$$p = \frac{a^2}{2}\frac{n-2}{(n-3)}\left[(n-3)(n-1) + 2(n-2) b e^{az/(n-3)} + b^2 e^{2az/(n-3)}\right].$$

Reference

- H Zhang et al, Arxiv: 0804.2931, 0904.0063, 0904.0065, 0904.0067

Thank you!