Determination of fragmentation functions
and proposal for exotic-hadron search

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Motivation for studying fragmentation functions

1. Basic quantity for describing hadron production in high-energy processes
   - Description of hadrons by quark and gluon degrees of freedom
   - Important applications
     Some functions are not so determined well.
     → affects nucleon-spin and heavy-ion studies

2. “Favored” and “disfavored” (unfavored) fragmentation functions
   - Possibility of finding exotic hadrons in high-energy processes

   e.g. if $f_0(980) = ss$: favored $s \rightarrow f_0$, $\bar{s} \rightarrow f_0$
   disfavored $u \rightarrow f_0$, $d \rightarrow f_0$, $u \rightarrow f_0$, $d \rightarrow f_0$, …

   $f_0(980) = \frac{1}{\sqrt{2}} (uu + dd)$, $ss$, $\frac{1}{\sqrt{2}} (u\bar{u}s\bar{s} + d\bar{d}s\bar{s})$, $K\bar{K}$, or $gg$
(1) Determination of fragmentation functions
   • For $\pi$, $K$, $p$
   • Roles of future Belle data

(2) Fragmentation functions for finding exotic hadrons
   • For example, $f_0(980)$


Major contribution has been made especially on (2) by
Masanori Hirai (Tokyo University of Science).
Fragmentation Functions for $\pi, K, p/p$
(1) Introduction to fragmentation functions (FFs)
  • Definition of FFs
  • Motivation for determining FFs

(2) Determination of FFs
  • Analysis method
  • Results
  • Comparison with other parameterizations

(3) Summary I
Fragmentation Function

Fragmentation: hadron production from a quark, antiquark, or gluon

\[ z \equiv \frac{E_h}{\sqrt{s}/2} = \frac{2E_h}{Q} = \frac{E_h}{E_q}, \quad s = Q^2 \]

Variable \( z \)
- Hadron energy / Beam energy
- Hadron energy / Primary quark energy

A fragmentation process occurs from quarks, antiquarks, and gluons, so that \( F^h \) is expressed by their individual contributions:

\[
F^h(z,Q^2) = \frac{1}{\sigma_{tot}} \frac{d\sigma(e^+e^- \rightarrow hX)}{dz}
\]

\( \sigma_{tot} \) = total hadronic cross section

Calculated in perturbative QCD

\[
C_i(z,Q^2) = \text{coefficient function}
\]

\[
D^h_i(z,Q^2) = \text{fragmentation function of hadron } h \text{ from a parton } i
\]
**Momentum (energy) sum rule**

\[ D_i^h \left( z, Q^2 \right) = \text{probability to find the hadron } h \text{ from a parton } i \]

with the energy fraction \( z \)

Energy conservation:

\[ \sum_h \int_0^1 dz \, z \, D_i^h \left( z, Q^2 \right) = 1 \]

\[ h = \pi^+, \pi^0, \pi^-, K^+, K^0, \bar{K}^0, K^-, p, \bar{p}, n, \bar{n}, \ldots \]

**Favored and disfavored fragmentation functions**

Simple quark model:

\[ \pi^+ (u \bar{d}), \quad K^+ (u \bar{s}), \quad p (uud), \quad \ldots \]

**Favored fragmentation:**

\[ D_u^{\pi^+}, \quad D_{\bar{d}}^{\pi^+}, \quad \ldots \]

(from a quark which exists in a naive quark model)

**Disfavored fragmentation:**

\[ D_d^{\pi^+}, \quad D_{\bar{u}}^{\pi^+}, \quad D_s^{\pi^+}, \quad \ldots \]

(from a quark which does not exist in a naive quark model)
**Purposes of investigating fragmentation functions**

Semi-inclusive reactions have been used for investigating

- **origin of proton spin**
  \[ \tilde{e} + \tilde{p} \rightarrow e' + h + X, \quad \tilde{p} + \tilde{p} \rightarrow h + X \text{ (RHIC-Spin)} \]

  Quark, antiquark, and gluon contributions to proton spin
  (flavor separation, gluon polarization)

- **properties of quark-hadron matters**
  \[ A + A' \rightarrow h + X \text{ (RHIC, LHC)} \]

  Nuclear modification
  (recombination, energy loss, …)

\[
\sigma = \sum_{a,b,c} f_a(x_a, Q^2) \otimes f_b(x_b, Q^2) \otimes \hat{\sigma}(ab \rightarrow cX) \otimes D_c^\pi(z, Q^2)
\]
Pion production at RHIC: \( p + p \rightarrow \pi^0 + X \)

S. S. Adler et al. (PHENIX), PRL 91 (2003) 241803

- Consistent with NLO QCD calculation up to \( 10^{-8} \)
- Data agree with NLO pQCD + KKP
- Large differences between Kretzer and KKP calculations at small \( p_T \)

\( \sqrt{s} = 200 \text{ GeV} \)

\[ p \rightarrow \pi \rightarrow p_T \]

Blue band indicates the scale uncertainty by taking \( Q=2p_T \) and \( p_T/2 \).
Situation of fragmentation functions (before 2007)

There are two widely used fragmentation functions by Kretzer and KKP.

An updated version of KKP is AKK.

(Kretzer) S. Kretzer, PRD 62 (2000) 054001

(KKP) B. A. Kniehl, G. Kramer, B. Pötter, NPB 582 (2000) 514

(AKK) S. Albino, B.A. Kniehl, G. Kramer, NPB 725 (2005) 181

The functions of Kretzer and KKP (AKK) are very different.

See also Bourhis-Fontannaz-Guillet-Werlen (2001) for FFs without hadron separation.
### Status of determining fragmentation functions (before 2007)

<table>
<thead>
<tr>
<th>Determination</th>
<th>Parton Distribution Functions (PDFs), Fragmentation Functions (FFs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nulceonic PDFs</td>
<td>****</td>
</tr>
<tr>
<td>Determination</td>
<td>Uncertainty</td>
</tr>
</tbody>
</table>

**Uncertainty ranges of determined fragmentation functions were not estimated, although there are such studies in nucleonic and nuclear PDFs.**

**The large differences indicate that the determined FFs have much ambiguities.**
Determination of Fragmentation Functions

Ref. M. Hirai, SK, T.-H. Nagai, K. Sudoh

Code for calculating the fragmentation functions is available at http://research.kek.jp/people/kumanos/ffs.html.
New aspects in our analysis (compared with Kretzer, KKP, AKK)

• Determination of fragmentation functions (FFs) and their uncertainties in LO and NLO.

• Discuss NLO improvement in comparison with LO by considering the uncertainties. (Namely, roles of NLO terms in the determination of FFs)

• Comparison with other parametrizations

• Avoid assumptions on parameters as much as we can, Avoid contradiction to the momentum sum rule

• SLD (2004) data are included.
Initial functions for pion

Note: constituent-quark composition $\pi^+ = u\bar{d}$, $\pi^- = \bar{u}d$

$$D_u^{\pi^+}(z, Q_0^2) = N_u^{\pi^+} z^{\alpha_u^{\pi^+}} (1 - z)^{\beta_u^{\pi^+}} = D_{\bar{d}}^{\pi^+}(z, Q_0^2)$$

$$D_{\bar{u}}^{\pi^+}(z, Q_0^2) = N_{\bar{u}}^{\pi^+} z^{\alpha_{\bar{u}}^{\pi^+}} (1 - z)^{\beta_{\bar{u}}^{\pi^+}} = D_d^{\pi^+}(z, Q_0^2) = D_s^{\pi^+}(z, Q_0^2) = D_{\bar{s}}^{\pi^+}(z, Q_0^2)$$

$$D_c^{\pi^+}(z, m_c^2) = N_c^{\pi^+} z^{\alpha_c^{\pi^+}} (1 - z)^{\beta_c^{\pi^+}} = D_{\bar{c}}^{\pi^+}(z, m_c^2)$$

$$D_b^{\pi^+}(z, m_b^2) = N_b^{\pi^+} z^{\alpha_b^{\pi^+}} (1 - z)^{\beta_b^{\pi^+}} = D_{\bar{b}}^{\pi^+}(z, m_b^2)$$

$$D_g^{\pi^+}(z, Q_0^2) = N_g^{\pi^+} z^{\alpha_g^{\pi^+}} (1 - z)^{\beta_g^{\pi^+}}$$

Constraint: 2\textsuperscript{nd} moment should be finite and less than 1

$$N = \frac{M}{B(\alpha + 2, \beta + 1)}, \quad M \equiv \int_0^1 zD(z) \, dz \quad (2\text{nd} \text{ moment}), \quad B(\alpha + 2, \beta + 1) = \text{beta function}$$

$$0 < M_i^h < 1 \quad \text{because of the sum rule} \sum_h M_i^h = 1$$
Experimental data for pion

Total number of data: 264

<table>
<thead>
<tr>
<th>Collaboration</th>
<th>Lab</th>
<th>$\sqrt{s}$ (GeV)</th>
<th># of data</th>
</tr>
</thead>
<tbody>
<tr>
<td>TASSO</td>
<td>DESY SLAC</td>
<td>12,14,22,30,34,4</td>
<td>29</td>
</tr>
<tr>
<td>TPC</td>
<td>SLAC</td>
<td>4</td>
<td>18</td>
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<tr>
<td>HRS</td>
<td>KEK SLAC</td>
<td>29</td>
<td>2</td>
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<tr>
<td>TOPAZ</td>
<td>KEK SLAC</td>
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<td>4</td>
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<tr>
<td>SLD</td>
<td></td>
<td>58</td>
<td>29</td>
</tr>
<tr>
<td>SLD [light quark]</td>
<td></td>
<td>91.2</td>
<td>29</td>
</tr>
<tr>
<td>SLD [c quark]</td>
<td></td>
<td>91.2</td>
<td>29</td>
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<tr>
<td>SLD [b quark]</td>
<td></td>
<td>91.2</td>
<td>29</td>
</tr>
<tr>
<td>ALEPH</td>
<td>CERN</td>
<td>91.2</td>
<td>22</td>
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<tr>
<td>OPAL</td>
<td>CERN</td>
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<tr>
<td>DELPHI</td>
<td>CERN</td>
<td>91.2</td>
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<tr>
<td>DELPHI [light quark]</td>
<td></td>
<td>91.2</td>
<td>17</td>
</tr>
<tr>
<td>DELPHI [b quark]</td>
<td></td>
<td>91.2</td>
<td>17</td>
</tr>
</tbody>
</table>

![Graph: Data distribution](image)
Analysis

Initial scale: \( Q_0^2 = 1 \text{ GeV}^2 \)

Scale parameter: \( \Lambda_{QCD}^{n_f=4} = 0.220 \) (LO), 0.323 (NLO)
\( \alpha_s \) varies with \( n_f \)

Heavy-quark masses: \( m_c = 1.43 \text{ GeV}, \ m_b = 4.3 \text{ GeV} \)

Results for the pion \( \chi^2 / \text{d.o.f.} = 1.81 \) (LO), 1.73 (NLO)

Uncertainty estimation: Hessian method

\[
\Delta \chi^2 \equiv \chi^2(\hat{a} + \delta a) - \chi^2(\hat{a}) = \sum_{i,j} H_{ij} \delta a_i \delta a_j, \\
H_{ij} = \frac{\partial^2 \chi^2(\hat{a})}{\partial a_i \partial a_j}
\]

\[
[\delta D(z)]^2 = \Delta \chi^2 \sum_{i,j} \frac{\partial D(z, \hat{a})}{\partial a_i} H_{ij}^{-1} \frac{\partial D(z, \hat{a})}{\partial a_j}
\]
Comparison with pion data

\[ F^{\pi^\pm}(z, Q^2) = \frac{1}{\sigma_{tot}} \frac{d\sigma(e^+ e^- \rightarrow \pi^\pm X)}{dz} \]

Our fit is successful to reproduce the pion data.

The DELPHI data deviate from our fit at large \( z \).

Rational difference between data and theory

\[ \frac{F^{\pi^\pm}(z, Q^2)_{\text{data}} - F^{\pi^\pm}(z, Q^2)_{\text{theory}}}{F^{\pi^\pm}(z, Q^2)_{\text{theory}}} \]
Comparison with pion data: \( \frac{(\text{data-theory})}{\text{theory}} \)
Determined fragmentation functions for pion

- Gluon and light-quark fragmentation functions have large uncertainties.

- Uncertainty bands become smaller in NLO in comparison with LO. → The data are sensitive to NLO effects.

- The NLO improvement is clear especially in gluon and disfavored functions.

- Heavy-quark functions are relatively well determined.
Comparison with other parametrizations in pion

(KKP) Kniehl, Kramer, Pötter
(AKK) Albino, Kniehl, Kramer
(HKNS) Hirai, Kumano, Nagai, Sudoh
(DSS) De Florian, Sassot, Stratmann

- Gluon and light-quark disfavored fragmentation functions have large differences, but they are within the uncertainty bands.

→ The functions of KKP, Kretzer, AKK, DSS, and HKNS are consistent with each other.

\[ \hat{s} = x_a x_b s \sim (0.1)^2 (200 \text{ GeV})^2 \] for RHIC

\[ \sqrt{\hat{s}} = 0.1 \cdot 200 = 20 \text{ GeV} \]

\[ z \sim \frac{p_T}{\sqrt{\hat{s}} / 2} = \frac{p_T}{10} \sim 0.5 \] (relatively large \( z \))
Pion production at RHIC: $\bar{p} + \bar{p} \rightarrow \pi^0 + X$

Subprocesses

$gg \rightarrow q(g)X, \quad qg \rightarrow q(g)X, \quad qq \rightarrow qX,$
$q\bar{q} \rightarrow q(g,q')X, \quad qq' \rightarrow qX, \quad q\bar{q}' \rightarrow qX$

$g + g \rightarrow q(g) + X$ processes are dominant at small $p_T$
$q + g \rightarrow q(g) + X$ at medium $p_T$

Gluon polarization $\Delta g$ at small $p_T$

$\rightarrow$ Gluon fragmentation function plays a major role
Comparison with other parametrizations in kaon and proton

**kaon**

- **gluon** $Q^2 = 2\text{ GeV}^2$
- **d quark** $Q^2 = 2\text{ GeV}^2$
- **u quark** $Q^2 = 2\text{ GeV}^2$
- **s quark** $Q^2 = 2\text{ GeV}^2$
- **c quark** $Q^2 = 10\text{ GeV}^2$
- **b quark** $Q^2 = 100\text{ GeV}^2$

**proton**

- **gluon** $Q^2 = 2\text{ GeV}^2$
- **s quark** $Q^2 = 2\text{ GeV}^2$
- **u quark** $Q^2 = 2\text{ GeV}^2$
- **d quark** $Q^2 = 2\text{ GeV}^2$
- **c quark** $Q^2 = 10\text{ GeV}^2$
- **b quark** $Q^2 = 100\text{ GeV}^2$
Comments on “low-energy” experiments, Belle & BaBar

Gluon fragmentation function is very important for hadron production at small $p_T$ at RHIC (heavy ion, spin) and LHC, and it is “not determined” as shown in this analysis.
→ Need to determine it accurately.
→ Gluon function is a NLO effect with the coefficient function and in $Q^2$ evolution.

We have precise data such as the SLD ones at $Q=Mz$, so that accurate small-$Q^2$ data are needed for probing the $Q^2$ evolution, namely the gluon fragmentation functions. (Belle, BaBar)
Expected segmentation functions by Belle

Expected Belle data by R. Seidl

Scaling violation = Determination of gluon fragmentation function

Current data
Summary I

Determination of the optimum fragmentation functions for π, K, p in LO and NLO by a global analysis of e^+e^−→ h+X data.

• It was the first time that uncertainties of the fragmentation functions are estimated.

• Gluon and disfavored light-quark functions have large uncertainties.
  → The uncertainties could be important for discussing physics in
  \[ \bar{p} + \bar{p} \rightarrow \pi^0 + X, \quad A + A' \rightarrow h + X \text{ (RHIC, LHC), HERMES, COMPASS, JLab, ...} \]
  → Need accurate data at low energies (Belle and BaBar).

• For the pion and kaon, the uncertainties are reduced in NLO in comparison with LO.
  For the proton, such improvement is not obvious.

• Heavy-quark functions are well determined.

• Code for calculating the fragmentation functions is available at http://research.kek.jp/people/kumanos/ffs.html.
Fragmentation Functions
for Exotic-Hadron Search

$f_0(980)$ as an example

(1) Introduction to exotic hadrons
   • Recent discoveries
   • Exotic hadrons at $M \sim 1$ GeV, especially $f_0(980)$
   • FFs in heavy-ion collisions

(2) Criteria for determining quark configurations by fragmentation functions
   • Functional forms, Second moments

(3) Analysis of $e^+ + e^- \rightarrow f_0 + X$ data for determining fragmentation functions for $f_0(980)$
   • Analysis method, Results, Discussions

(4) Summary
Recent progress in exotic hadrons

\[ q\bar{q} \quad \text{Meson} \]
\[ q^3 \quad \text{Baryon} \]
\[ q^2\bar{q}^2 \quad \text{Tetraquark} \]
\[ q^4\bar{q} \quad \text{Tetraquark} \]
\[ q^6 \quad \text{Dibaryon} \]
\[ \ldots \]
\[ q^{10}\bar{q} \quad \text{e.g. Strange tribaryon} \]
\[ \ldots \]
\[ gg \quad \text{Glueball} \]

(Japanese ?) Exotics

- **\( \Theta^+(1540) ? \): LEPS**
  - Pentaquark?

- **\( S^0(3115), S^+(3140) \): KEK-PS**
  - Strange tribaryons?

- **\( X (3872), Y(3940) \): Belle**
  - Tetraquark, DD molecule

- **\( D_{sJ}(2317), D_{sJ}(2460) \): BaBar, CLEO, Belle**
  - Tetraquark, DK molecule

- **\( Z(4430) \): Belle**
  - Tetraquark, …

| \( uudd\bar{s} ? \) |
| \( K^- pnn \) |
| \( K^- ppn ? \) |
| \( c\bar{c} \) |
| \( D^0(c\bar{u})\bar{D}^0(\bar{c}u) \) |
| \( D^+(cd)\bar{D}^-(\bar{c}\bar{d}) \) ? |
| \( c\bar{s} \) |
| \( D^0(c\bar{u})K^+(u\bar{s}) \) |
| \( D^+(cd)K^0(d\bar{s}) \) ? |
| \( c\bar{c}ud\bar{d}, D \text{ molecule?} \) |

Note: \( Z(4430) \neq q\bar{q} ? \)
Scalar mesons $J^P=0^+$ at $M\sim 1$ GeV

**Naïve quark-model**

<p>| | | |</p>
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>$a_1(1230)$</td>
<td>1.0 GeV</td>
<td>$a_0(980),,,,f_0(980)$</td>
</tr>
<tr>
<td>$\rho(770)$</td>
<td>0.5 GeV</td>
<td>$f_0(600) = \sigma$</td>
</tr>
</tbody>
</table>

$\sigma = f_0(600) = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$

$f_0(980) = s\bar{s} \rightarrow$ denote $f_0$ in this talk

$a_0(980) = u\bar{d}, \, \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), \, d\bar{u}$

Naive model: $m(\sigma) \sim m(a_0) < m(f_0)$

**Strong-decay issue:** The experimental widths

$\Gamma(f_0, a_0) = 40 - 100$ MeV are too small to be predicted by a typical quark model.

\[\uparrow \text{ contradiction}\]

**Experiment:** $m(\sigma) < m(a_0) \sim m(f_0)$

These issues could be resolved if $f_0$ is a tetraquark $(qq\bar{q}\bar{q})$ or a $K\bar{K}$ molecule, namely an "exotic" hadron.

Determination of $f_0(980)$ structure by electromagnetic decays


Radiative decay: $\phi \rightarrow S \gamma$  $S=f_0(980), a_0(980)$  $J^P = 1^- \rightarrow 0^+$  $E1$ transition  Electric dipole:  $e \vec{r}$ (distance!)

$q\bar{q}$ model:  $\Gamma = \text{small}$  

$K\bar{K}$ molecule  or $qq\bar{q}\bar{q}$:  $\Gamma = \text{large}$

Experimental results of VEPP-2M and DAΦNE suggest that $f_0$ is a tetraquark state (or a $K\bar{K}$ molecule?).

CMD-2 (1999):  $B(\phi \rightarrow f_0\gamma) = (1.93 \pm 0.46 \pm 0.50) \times 10^{-4}$
SND (2000):  $(3.5 \pm 0.3^{+1.3}_{-0.5}) \times 10^{-4}$
KLOE (2002):  $(4.47 \pm 0.21_{\text{stat+syst}}) \times 10^{-4}$

For recent discussions,
N. N. Achasov and A. V. Kiselev, PRD 73 (2006) 054029;
D74 (2006) 059902(E); D76 (2007) 077501;

See also Belle (2007)  $\Gamma(f_0 \rightarrow \gamma\gamma) = 0.205^{+0.095}_{-0.083}(\text{stat})^{+0.147}_{-0.117}(\text{syst})$ keV
Criteria for determining internal structure by fragmentation functions

(Naïve estimates)

Criteria for determining $f_0$ structure by its fragmentation functions

Possible configurations of $f_0(980)$

1. **ordinary $u,d$-meson**
   \[
   \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d})
   \]

2. **strange meson**, $s\bar{s}$

3. **tetraquark ($K\bar{K}$)**,
   \[
   \frac{1}{\sqrt{2}} (u\bar{s}s + d\bar{s}s)
   \]

4. **glueball**, $gg$

Contradicts with experimental widths

- $\Gamma_{\text{theo}}(f_0 \rightarrow \pi\pi) = 500 - 1000$ MeV
  $\Rightarrow \Gamma_{\text{exp}} = 40 - 100$ MeV
- $\Gamma_{\text{theo}}(f_0 \rightarrow \gamma\gamma) = 1.3 - 1.8$ keV
  $\Rightarrow \Gamma_{\text{exp}} = 0.205$ keV

Contradicts with lattice-QCD estimate

- $m_{\text{lattice}}(f_0) = 1600$ MeV
  $\Rightarrow m_{\text{exp}} = 980$ MeV

Discuss 2nd moments and functional forms (peak positions) of the fragmentation functions for $f_0$ by assuming the above configurations, (1), (2), (3), and (4).
$s\bar{s}$ picture for $f_0(980)$

**$u$ (disfavored)**

- O($g^2$)
- + one O($g^3$) term of gluon radiation from the antiquark

**$s$ (favored)**

- O($g^2$)
- + one O($g^3$) term of gluon radiation from the antiquark

**$g$**

- O($g^2$)
- + one O($g^3$) term of gluon radiation from the quark

- + two O($g^3$) terms of gluon radiation from the quark or antiquark

2nd moment: $M(u) < M(s) \leq M(g)$

Peak of function: $z_{\text{max}}(u) < z_{\text{max}}(s) \approx z_{\text{max}}(g)$
\( f_0 = (u\bar{u}s\bar{s} + d\bar{d}s\bar{s}) / \sqrt{2} \)

\( f_0 = \left[ K^+ (u\bar{s})K^- (\bar{u}s) + K^0 (d\bar{s})\bar{K}^0 (\bar{d}s) \right] / \sqrt{2} \)

\( n\bar{n}s\bar{s} \) picture for \( f_0(980) \)

\( K\bar{K} \) picture for \( f_0(980) \)

\( u, s \) (favored)

\( O (g^4) \)

2nd moment: \( M(u) = M(s) \lesssim M(g) \)

Peak of function: \( z_{\text{max}}(u) = z_{\text{max}}(s) \approx z_{\text{max}}(g) \)
Judgment

<table>
<thead>
<tr>
<th>Type</th>
<th>Configuration</th>
<th>2nd Moment</th>
<th>Peak z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonstrange $q\bar{q}$</td>
<td>$(u\bar{u} + d\bar{d}) / \sqrt{2}$</td>
<td>$M(s) &lt; M(u) &lt; M(g)$</td>
<td>$z_{\text{max}}(s) &lt; z_{\text{max}}(u) \approx z_{\text{max}}(g)$</td>
</tr>
<tr>
<td>Strange $q\bar{q}$</td>
<td>$s\bar{s}$</td>
<td>$M(u) &lt; M(s) \lesssim M(g)$</td>
<td>$z_{\text{max}}(u) &lt; z_{\text{max}}(s) \approx z_{\text{max}}(g)$</td>
</tr>
<tr>
<td>Tetraquark</td>
<td>$(u\bar{u}s\bar{s} + d\bar{d}s\bar{s}) / \sqrt{2}$</td>
<td>$M(u) = M(s) \lesssim M(g)$</td>
<td>$z_{\text{max}}(u) = z_{\text{max}}(s) \approx z_{\text{max}}(g)$</td>
</tr>
<tr>
<td>$K\bar{K}$ Molecule</td>
<td>$(K^+K^- + K^0\bar{K}^0) / \sqrt{2}$</td>
<td>$M(u) = M(s) \lesssim M(g)$</td>
<td>$z_{\text{max}}(u) = z_{\text{max}}(s) \approx z_{\text{max}}(g)$</td>
</tr>
<tr>
<td>Glueball</td>
<td>$g\bar{g}$</td>
<td>$M(u) = M(s) &lt; M(g)$</td>
<td>$z_{\text{max}}(u) = z_{\text{max}}(s) &lt; z_{\text{max}}(g)$</td>
</tr>
</tbody>
</table>

Since there is no difference between $D_{u}^{f_0}$ and $D_{d}^{f_0}$ in the models, they are assumed to be equal. On the other hand, $D_{s}^{f_0}$ and $D_{g}^{f_0}$ are generally different from them, so that they should be used for finding the internal structure. Therefore, simple and "model-independent" initial functions are

$$D_{u}^{f_0}(z, Q^2_0) = D_{d}^{f_0}(z, Q^2_0) = D_{d}^{f_0}(z, Q^2_0) = D_{d}^{f_0}(z, Q^2_0), \quad D_{s}^{f_0}(z, Q^2_0) = D_{s}^{f_0}(z, Q^2_0),$$

$$D_{g}^{f_0}(z, Q^2_0), \quad D_{c}^{f_0}(z, m^2_c) = D_{c}^{f_0}(z, m^2_c), \quad D_{b}^{f_0}(z, m^2_b) = D_{b}^{f_0}(z, m^2_b).$$
2nd moments of favored • and disfavored • fragmentation functions

Actual HKNS07 analysis results (M. Hirai et al., PRD75 (2007) 094009) for the 2nd moments: \( M \equiv \int_0^1 zD(z)dz \)

<table>
<thead>
<tr>
<th>2nd moment</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_{u}^{\pi^+} )</td>
<td>0.401 ± 0.052</td>
</tr>
<tr>
<td>( D_{u}^{\pi^0} )</td>
<td>0.094 ± 0.029</td>
</tr>
<tr>
<td>( D_{c}^{\pi^+} )</td>
<td>0.178 ± 0.018</td>
</tr>
<tr>
<td>( D_{b}^{\pi^+} )</td>
<td>0.236 ± 0.009</td>
</tr>
<tr>
<td>( D_{g}^{\pi^+} )</td>
<td>0.238 ± 0.029</td>
</tr>
<tr>
<td>( D_{u}^{K^+} )</td>
<td>0.0740 ± 0.0268</td>
</tr>
<tr>
<td>( D_{s}^{K^+} )</td>
<td>0.0878 ± 0.0506</td>
</tr>
<tr>
<td>( D_{c}^{K^+} )</td>
<td>0.0255 ± 0.0173</td>
</tr>
<tr>
<td>( D_{b}^{K^+} )</td>
<td>0.0583 ± 0.0052</td>
</tr>
<tr>
<td>( D_{g}^{K^+} )</td>
<td>0.0522 ± 0.0024</td>
</tr>
<tr>
<td>( D_{b}^{K^+} )</td>
<td>0.0705 ± 0.0099</td>
</tr>
</tbody>
</table>

There is a tendency that 2nd moments are larger for the favored functions. → It suggests that the 2nd moments could be used for exotic hadron determination (quark / gluon configuration in hadrons).
Global analysis for fragmentation functions of $f_0(980)$
Fragmentation functions for $f_0(980)$

$e^+ \gamma, Z \rightarrow h = f_0(980)$

$e^- q \rightarrow h$

$F^h(z, Q^2) = \sum_i \int_1^0 \frac{dy}{y} C_i \left( \frac{z}{y}, Q^2 \right) D^h_i(y, Q^2)$

Initial functions

$D^{f_0}_u(z, Q^2_0) = D^{f_0}_d(z, Q^2_0) = N^{f_0}_u z^{\alpha^{f_0}_u} (1 - z)^{\beta^{f_0}_u}$

$D^{f_0}_s(z, Q^2_0) = N^{f_0}_s z^{\alpha^{f_0}_s} (1 - z)^{\beta^{f_0}_s}$

$D^{f_0}_g(z, Q^2_0) = N^{f_0}_g z^{\alpha^{f_0}_g} (1 - z)^{\beta^{f_0}_g}$

$D^{f_0}_c(z, m^2_c) = N^{f_0}_c z^{\alpha^{f_0}_c} (1 - z)^{\beta^{f_0}_c}$

$D^{f_0}_b(z, m^2_b) = N^{f_0}_b z^{\alpha^{f_0}_b} (1 - z)^{\beta^{f_0}_b}$

$z \equiv \frac{E_h}{\sqrt{s} / 2} = \frac{2E_h}{Q} = \frac{E_h}{E_q}$, \hspace{1em} s = Q^2$

$F^h(z, Q^2) = \frac{1}{\sigma_{tot}} d\sigma(e^+ e^- \rightarrow hX) dz$

$\sigma_{tot} = \text{total hadronic cross section}$

- $D^{f_0}_q(z, Q^2_0) = D^{f_0}_{\bar{q}}(z, Q^2_0)$

- $Q_0 = 1 \text{ GeV}$
  \hspace{1em} $m_c = 1.43 \text{ GeV}$
  \hspace{1em} $m_b = 4.3 \text{ GeV}$

$N = M \frac{\Gamma(\alpha + \beta + 3)}{\Gamma(\alpha + 2)\Gamma(\beta + 1)}$, \hspace{1em} $M \equiv \int_0^1 zD(z)dz$
One could foresee the difficulty in getting reliable FFs for $f_0$ at this stage.

### Experimental data for $f_0$

**Total number of data:** only

<table>
<thead>
<tr>
<th>Exp. collaboration</th>
<th>$\sqrt{s}$ (GeV)</th>
<th># of data</th>
</tr>
</thead>
<tbody>
<tr>
<td>HRS</td>
<td>29</td>
<td>4</td>
</tr>
<tr>
<td>OPAL</td>
<td>91.2</td>
<td>8</td>
</tr>
<tr>
<td>DELPHI</td>
<td>91.2</td>
<td>11</td>
</tr>
</tbody>
</table>

**Pion**

**Total number of data:** 264

<table>
<thead>
<tr>
<th>Exp. collaboration</th>
<th>$\sqrt{s}$ (GeV)</th>
<th># of data</th>
</tr>
</thead>
<tbody>
<tr>
<td>TASSO</td>
<td>12,14,22,30,34,44</td>
<td>29</td>
</tr>
<tr>
<td>TCP</td>
<td>29</td>
<td>18</td>
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<tr>
<td>HRS</td>
<td>29</td>
<td>2</td>
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<tr>
<td>TOPAZ</td>
<td>58</td>
<td>4</td>
</tr>
<tr>
<td>SLD</td>
<td>91.2</td>
<td>29</td>
</tr>
<tr>
<td>SLD [light quark]</td>
<td></td>
<td>29</td>
</tr>
<tr>
<td>SLD [c quark]</td>
<td></td>
<td>29</td>
</tr>
<tr>
<td>SLD [b quark]</td>
<td></td>
<td>29</td>
</tr>
<tr>
<td>ALEPH</td>
<td>91.2</td>
<td>22</td>
</tr>
<tr>
<td>OPAL</td>
<td>91.2</td>
<td>22</td>
</tr>
<tr>
<td>DELPHI</td>
<td>91.2</td>
<td>17</td>
</tr>
<tr>
<td>DELPHI [light quark]</td>
<td></td>
<td>17</td>
</tr>
<tr>
<td>DELPHI [b quark]</td>
<td></td>
<td>17</td>
</tr>
</tbody>
</table>
Results on the fragmentation functions

- Functional forms
  
  (1) $D_{u}^{f_{0}}(z)$, $D_{s}^{f_{0}}(z)$ have peaks at large $z$
  
  (2) $z_{u}^{\text{max}} \sim z_{s}^{\text{max}}$

(1) and (2) indicate tetraquark structure

$$f_{0} \sim \frac{1}{\sqrt{2}} (u\bar{u}s\bar{s} + d\bar{d}s\bar{s})$$

- 2nd moments: $\frac{M_{u}}{M_{s}} = 0.43$

This relation indicates $s\bar{s}$-like structure (or admixture)

$$f_{0} \sim s\bar{s}$$

$\Rightarrow$ Why do we get the conflicting results?

$\rightarrow$ Uncertainties of the FFs should be taken into account (next page).
Large uncertainties

2nd moments

\[ M_u = 0.0012 \pm 0.0107 \]
\[ M_s = 0.0027 \pm 0.0183 \]
\[ M_g = 0.0090 \pm 0.0046 \]

\[ \rightarrow \frac{M_u}{M_s} = 0.43 \pm 6.73 \]

The uncertainties are order-of-magnitude larger than the distributions and their moments themselves.

At this stage, the determined FFs are not accurate enough to discuss internal structure of \( f_0(980) \).

\[ \rightarrow \ ] \text{Accurate data are awaited not only for } f_0(980) \text{ but also for other exotic and “ordinary” hadrons.} \]
Exotic hadrons could be found by studying fragmentation functions. As an example, the $f_0(980)$ meson was investigated.

1. We proposed to use 2nd moments and functional forms as criteria for finding quark configuration.

2. Global analysis of $e^+e^-\rightarrow f_0 + X$ data
   - The results may indicate $s\bar{s}$ or $qq\bar{q}\bar{q}$ structure. However, ...
   - Large uncertainties in the determined FFs
     - The obtained FFs are not accurate enough to discuss the quark configuration of $f_0(980)$.

3. Accurate experimental data are important
   - Small-$Q^2$ data as well as large-$Q^2$ ($M_z^2$) ones
   - $c$- and $b$-quark tagging