Phenomenological approach to nuclei and nuclear matter

Kazuhiro Oyamatsu

Department of Library and Information Science*, Aichi Shukutoku University

* 1999-2008 Department of Media Theories and Production from April, Department of Human Informatics

1. Nuclear matter EOS, laboratory nuclei and neutron-star matter Iida(Kochi U.), Kohama(RIKEN), Koura(JAEA) since 2001

(2. a few words on Supernova Matter EOS Table)

NFQCD10, Jan. 26, 2010
くじらの会 at 入野

2008年10月29日（水）～31日（金）
# Approaches to obtain the EOS of (uniform) nuclear matter

<table>
<thead>
<tr>
<th>approach</th>
<th>starts from</th>
<th>ingredients</th>
<th>Theory/Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Phenomenological 1 (empirical)</strong></td>
<td>the parametrized EOS</td>
<td>nuclear mass, size, ...</td>
<td>Liquid-Drop Model&lt;br&gt;Droplet Model&lt;br&gt;Thomas-Fermi Theory&lt;br&gt; ......</td>
</tr>
<tr>
<td><strong>Phenomenological 2</strong></td>
<td>effective NN int.&lt;br&gt;(Hamiltonian, Lagrangean)</td>
<td>nuclear mass, size, ...</td>
<td>Skyrme HF&lt;br&gt;RMF&lt;br&gt;AMD&lt;br&gt; ......</td>
</tr>
<tr>
<td><strong>microscopic</strong></td>
<td>bare NN int.&lt;br&gt;(AV18, Bonn, Paris,...)</td>
<td>NN scattering, ...</td>
<td>Variational Calc.&lt;br&gt;DBHF&lt;br&gt; ......</td>
</tr>
</tbody>
</table>
Equation of state of nuclear matter
and nuclei in laboratories and in neutron-star crusts

Kazuhiro Oyamatsu (Aichi Shukutoku U.), Kei Iida (Kochi U.)

LARGEST UNCERTAINTY

L : density-derivative coefficient of symmetry energy

$y = -220 \text{ [MeV fm}^3\text{]}$

$y = -350 \text{ [MeV fm}^3\text{]}$

$y = -1800$

$K_0 \text{ (MeV)}$

$L \text{ (MeV)}$

$78\text{Ni}$ matter radius

Laboratory

uniform matter

pasta

spherical nuclei

proton clustering

fission instability

average nucleon density (fm$^{-3}$)

2010年1月27日水曜日
Which EOS parameter dominates macroscopic properties of neutron-rich nuclei in laboratory and in neutron-star crusts?

Energy per nucleon of nearly symmetric nuclear matter

\[ w(n, x) \approx w_0 + \frac{K_0}{18n_0^2}(n - n_0)^2 + (1 - 2x)^2 \left[ S_0 + \frac{L}{3n_0}(n - n_0) \right] \]

- \( n_0 \): nuclear density
- \( w_0 \): saturation energy
- \( K_0 \): incompressibility
- \( S_0 \): symmetry energy at \( n = n_0 \)
- \( L \): its density derivative coefficient

\[ L = 3n_0 \frac{dS(n)}{dn} \bigg|_{n=n_0} \]

\[ S_0 = S(n_0) \]
Strategy

We focus on macroscopic nuclear properties and adopt a macroscopic nuclear model.

1. From masses and radii of stable nuclei, we generate family of EOS and examine allowed regions of EOS parameter values.

2. We calculate neutron-rich nuclei in laboratories and identify key EOS parameter.

   *** mass and radius ***

3. We calculate nuclei in neutron-star crusts and identify key EOS parameter.

   *** proton number and ratio ***
   *** core-crust boundary density ***
   *** existence of pasta nuclei ***
Structure of neutron stars

Solid state of n-rich heavy nuclei

"pasta" phases

Transition from solid state of n-rich heavy nuclei to uniform nuclear matter

Pasta phases can appear in the bottom of inner crust

G. Watanabe
Existence of pasta nuclei depends on the EOS.

Dark domains mean nuclei (proton clusters).
At low densities in neutron-star crusts, we have nuclei which are more or less spherical.
In the core we have uniform matter. Pasta nuclei could exist in between.

Existence of pasta nuclei depends on the EOS.
Step 1

Generate all empirically allowed EOS's systematically

Adopted macroscopic model

Energy per cell (or Energy of a nucleus)

\[ W = \int_{cell} d\mathbf{r} \left[ \varepsilon_0(n_n, n_p) + m_n n_n + m_p n_p \right] + \int_{cell} d\mathbf{r} \ F_0 \left| \nabla n_i \right|^2 + \left( \text{electron kinetic energy} \right) + \left( \text{Coulomb} \right) \]

\( n_n, n_p \): local neutron (proton) density, \( n = n_n + n_p \): total density

\( \varepsilon_0(n_n, n_p) \): EOS of uniform nuclear matter (energy density)

\( F_0 \): surface energy parameter

Parametrization of the EOS (energy density)

\[ \varepsilon_0(n_n, n_p) = \frac{3}{5} \left( \frac{3}{2} \right)^{2/3} \left( \frac{\hbar^2}{2m_n} n_n^{5/3} + \frac{\hbar^2}{2m_p} n_p^{5/3} \right) + \left[ 1 - \left( 1 - 2Y_p \right)^2 \right] \nu_s(n) + \left( 1 - 2Y_p \right)^2 \nu_n(n) \]

Fermi kinetic energy density \hspace{1cm} potential energy density

\[ \nu_s(n) = a_1 n^2 + \frac{a_2 n^3}{1 + a_3 n} \hspace{1cm} \nu_n(n) = b_1 n^2 + \frac{b_2 n^3}{1 + b_3 n} \]

\( \star a_1 \sim b_2 \) and \( F_0 \): masses and radii of stable nuclei (\( b_3 = 1.59 \text{ fm}^3 \), a fit to FP EOS)

\( \star \) very flexible function form: \( a_3 \) can vary \( K_0 \) widely. (better than Skyrme)

The function can be fitted to SIII and TM1 EOS very well.
**Simplified Thomas-Fermi calculation**

energy minimization with respect to parameters of \(n_n(r)\) and \(n_p(r)\) (and lattice constant)

\[
n_i(r) = \begin{cases} 
(n_i^{in} - n_i^{out}) \left[ 1 - \left( \frac{r}{R_i} \right)^3 \right] + n_i^{out} & r < R_i \\
n_i^{out} & r < R_i
\end{cases}
\]

- \(R_n, R_p\): neutron (proton) radius parameter
- \(t_n, t_p\): neutron (proton) surface thickness parameter
- \(n_i^{in}\): central desnity
- \(n_i^{out}\): neutron gas density \((n_p^{out}=0)\)

A good function form

The \(n\) and \(p\) distributions are independent.

\(\Rightarrow\) neutron skin

The empirical information is limited: radius and thickness.

The gradient term in Euler Eq. is continuous.

The density is zero beyond the classical turning point.

The values of parameters \(a_1 \sim b_3\) (EOS) and \(F_0\) are determined to fit masses and radii of stable nuclei.

\(\Rightarrow\) about 200 sets of empirical EOS+\(F_0\)
EOS parameter values obtained from stable nuclei

\[ S_0 : \text{symmetry energy} \]

\[ L : \text{density symmetry coefficient} \]

\[ S_0(L) \text{ obtained from the fittings} \]

\[ S_0 = 27.752 + 0.075379 \times L \]

\[ K_0 \]
neutron matter uncertainties

empirical

microscopic

K. Hebeler, A. Schwenk, arXiv:0911.0483

2010年1月27日水曜日
Step 2

Calculate neutron-rich nuclei in labs with the 200 EOS's

The mass, radius and neutron skin are dependent on L but not on $K_0$. 
Step 3

Calculate nuclei in neutron star crusts with 200 EOS's
Proton number and fraction
Density region of pasta nuclei

Inner-crust nuclei: Proton number and fraction decrease with $L$

For large $L$, $S(n)$ at $n<n_0$ is small so that nuclei become more neutron-rich.
Estimate of density region of pasta nuclei


lower boundary

stability against fission of spherical nuclei

In the liquid drop model, (Coulomb self energy)=2*(surface energy)

==> (volume fraction of nucleus) = 1/8

upper boundary (core-crust boundary)

instability against forming proton clusters

\[ v(Q) = v_0 + 2(4\pi e^2 \beta)^{1/2} - \beta k_{TF}^2 > 0 \]

\[ Q^2 = \left( \frac{4\pi e^2}{\beta} \right)^{1/2} - k_{TF}^2 \]

\[
v(Q) = v_0 \frac{\partial \mu_p}{\partial n_p} - \frac{(\partial \mu_p/\partial n_n)^2}{\partial \mu_n/\partial n_n} = \left( \frac{\partial \mu_p}{\partial n_p} \right)_{\mu_n, \mu_c}
\]

\[ \beta = D_{pp} + 2D_{np}\zeta + D_{nn}\zeta^2, \quad \zeta = -\frac{\partial \mu_p/\partial n_n}{\partial \mu_n/\partial n_n}, \]

\[ k_{TF}^2 = \frac{4\pi e^2}{\partial \mu_e/\partial n_e} = \frac{4\alpha}{\pi} (3\pi^2 n_e)^{1/3}. \]
The upper bound (core-crust boundary density) is clearly dependent on L while the lower is almost constant.
Summary

• The values of $L$ and $K_0$ cannot be determined from masses and radii of stable nuclei.

• Radii and masses of unstable nuclei have sensitivity to $L$.

• Proton number and fraction of inner-crust nuclei is decrease with $L$.

• The core-crust boundary density of neutron star is dependent on $L$.

• The existence of the pasta phase is dominated by $L$. The pasta phase exists if $L < 100$ MeV.

• The present uncertainty in $L$ is too large.

• Systematic experimental study of nuclear mass and size of unstable nuclei in laboratories will help determine the $L$ value and the existence of pasta nuclei in neutron stars.
Efforts being done

• Estimate L value from global behavior of nuclear mass and size in nuclear chart with Prof. Iida (Kouchi U.), Drs. Kohama (RIKEN), Koura (JAEA), Abu-Ibrahim (Cairo U.)

• For nuclear size, we need to directly compare calculations and cross section measurements.
  • Kurotama (Black sphere) model
  • Glauber calculation of cross sections

• Preliminary result from nuclear mass L value is relatively large.
  (closer to EOS C than to EOS G)

to be studied during this workshop
Shen EOS table size

- \( \log_{10}(T(\text{MeV})) \) \(-1 \sim 2\) 31 grids
- \( \log_{10}(Y_p) \) \(-2 \sim -0.25\) 71 grids
- \( \log_{10}(\rho_B \text{ (g/cm}^3)) \) \(5 \sim 15.4\) 104 grids
- Total \( 31 \times 71 \times 104 = 228,904 \) data points
- MEGA data points
- Number of Nuclei in Nuclear Chart