YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

インフレーション宇宙論

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Big Bang Cosmology

The universe starts with a fireball.

- Friedmann Universe
  ～Hubble's law
- Nucleosynthesis
  Baryon/Photon～$10^{-9\sim-10}$
- Cosmic Microwave
  Background

Progress of Theoretical Physics, Vol. 5, No. 2, March～April, 1950.

Proton-Neutron Concentration Ratio in the Expanding Universe at the Stages preceding the Formation of the Elements.

Chushiro HAYASHI.

Department of Physics, Naniwa University.

(Received January 12, 1950)

§ 1. Introduction.

In the theory of the origin of the elements by Gamow, Alpher, and colaborators[1)], primordial matter (ylem) of the universe, which afterwards has been cooled down owing to the expansion of the universe and has formed the elements through nuclear reactions such as radiative capture and beta-decays, is assumed to consist solely of neutrons. At early stages, however, of high temperatures ($kT \gtrsim mc^2$, m being the electron mass) in the expanding universe before the formation of the elements, induced beta-processes caused by energetic electrons, positrons, neutrinos and antineutrinos, in addition to the natural decay of neutrons, such as

$$n+e^+ \rightleftarrows p+a\nu, \quad (1a)$$

$$n+\nu \rightleftarrows p+e^-, \quad (1b)$$

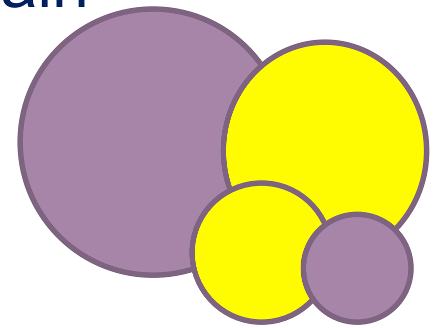
$$n \rightleftarrows p+e^-+a\nu, \quad (1c)$$

Beginning of Particle Cosmology

Various motivations for inflation @1980

Large domain of baryon/anti-baryon domain
Superhorizon scale correlation
Monopole problem

K. Sato



Horizon problem
Flatness problem

A. Guth

Inflation solves all problems just by assuming an early phase of exponential expansion of the universe.

Initial singularity avoidance

A. Starobinsky

$$L_{grav} = M_{pl}^2 \left(R + R^2 / 6M^2 \right)$$

+ Initial density perturbation

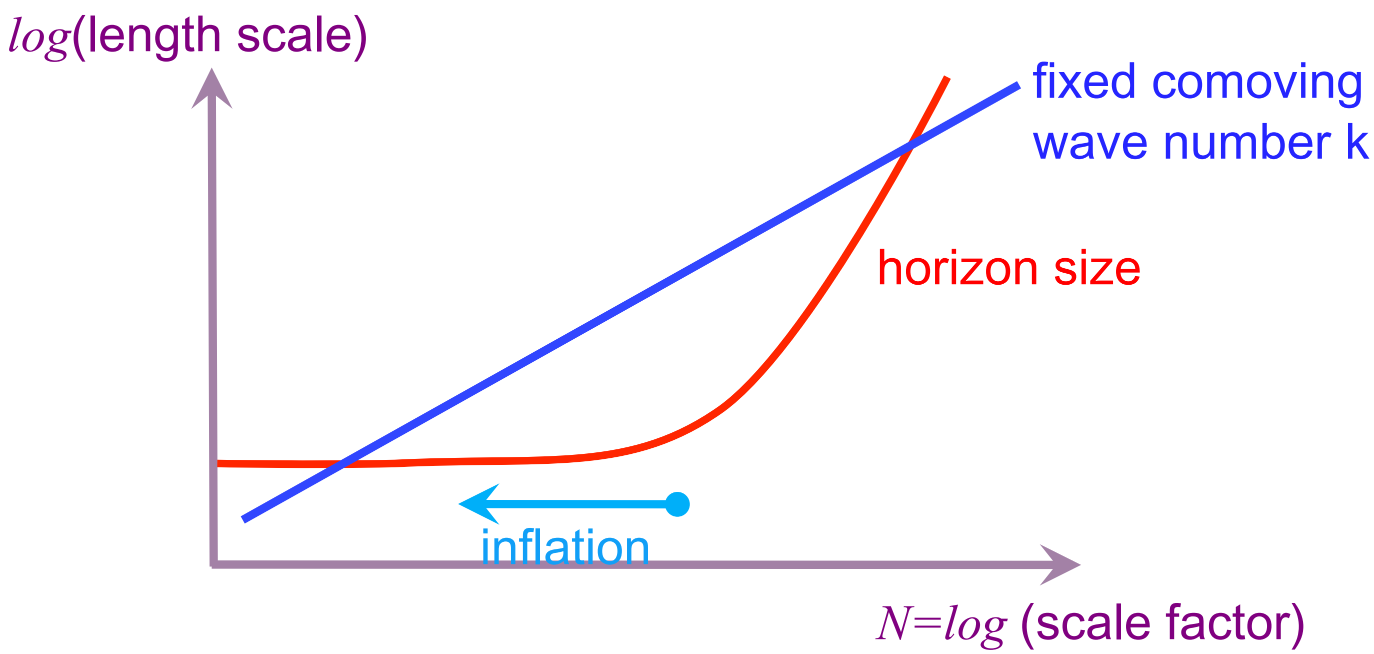
Evolution of scales

Horizon scale: H^{-1}

Distance that light can travel for the cosmic expansion timescale

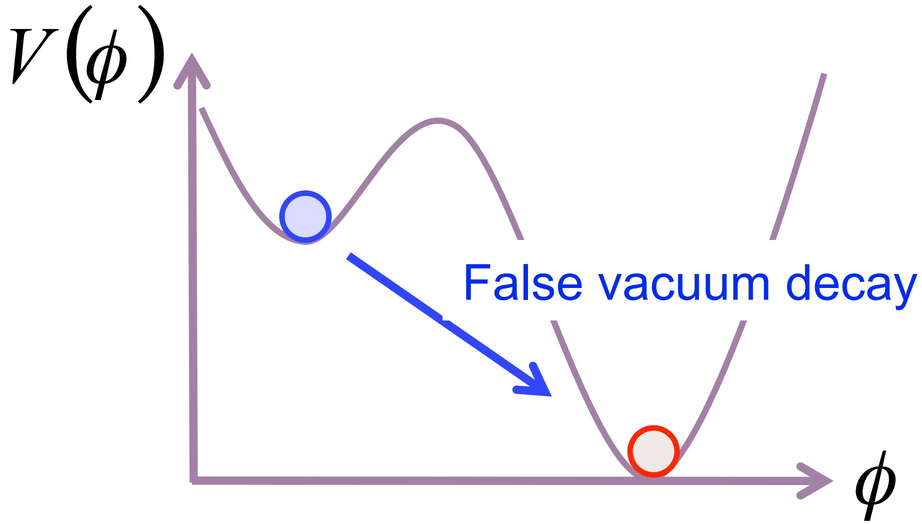
Comoving wavenumber: $\mathbf{k} = -i \dfrac{\partial}{\partial \mathbf{x}}$

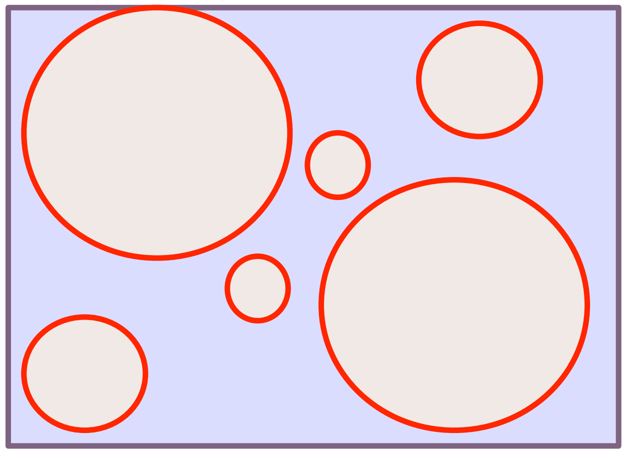
In linear perturbation, different $\mathbf{k}$ modes evolve independently.



Various inflation models

False vacuum inflation (Old inflation)





Small nucleation rate for inflation

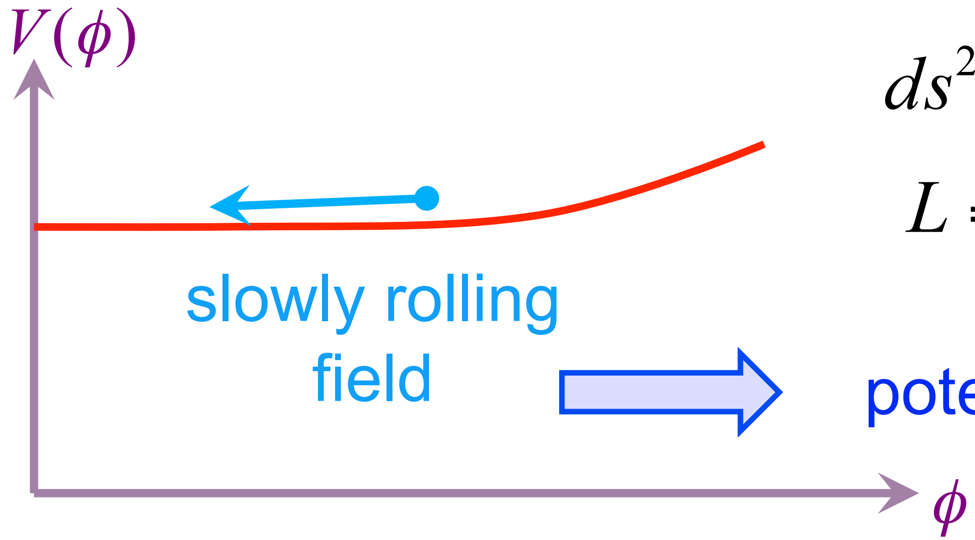
⇨ Some region continues to inflate

⇨ Difficult to terminate inflation / too large fluctuation

A synchronized clock to control the transition is necessary

Standard slow roll inflation

New inflation, Chaotic inflation   A. Linde (1982,3)



$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$$

$$L = -\frac{1}{2}(\nabla\phi)^2 - V(\phi)$$

potential term dominant

Expansion rate: $$H^2 = \frac{8\pi G}{3}\rho \approx \frac{8\pi G}{3}V$$

$$H \equiv \frac{\dot{a}}{a} \quad \Rightarrow \quad a \propto e^{Ht}$$

exponential
expansion

ϕ plays the role of synchronized clock

Generation of density perturbation

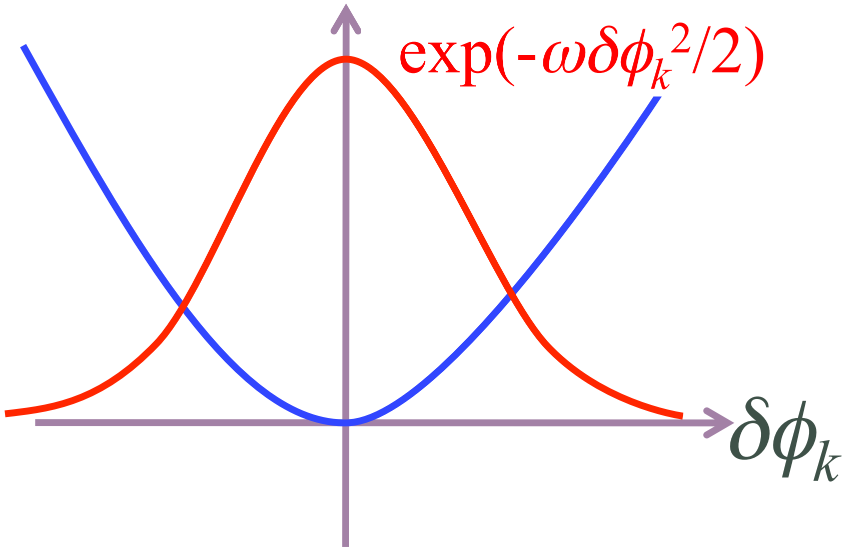
Mukhanov, Viatcheslav(1981)
Hawking(1982)
Starobinsky(1982)
Guth, Pi (1982)
Bardeen(1982)
Kodama-Sasaki
PTP supplement (1984)

Quantum fluctuation of inflaton ϕ during inflation:

$$\Box\delta\phi + V''\delta\phi = 0 \Rightarrow \left[\partial_t^2 + 3H\partial_t + \frac{k^2}{a^2}\right]\delta\phi_k = 0$$

$\frac{k^2}{a^2} \equiv \omega^2$

Time-dependent harmonic oscillator



1) $\omega \gg H$: sub-horizon

$\omega\searrow$ ⇒ wider wave function

$\frac{\dot{\omega}}{\omega^2} << 1$:adiabatic evolution

2) $\omega \ll H$: super-horizon

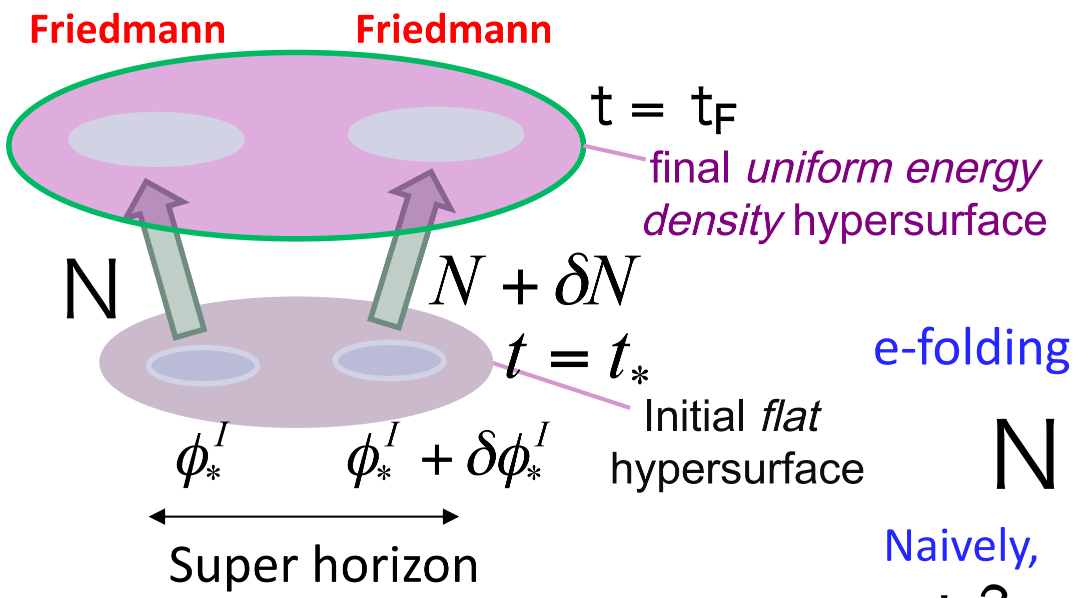
freeze out at $\omega \lesssim H$

$(k/a)^3\,\delta\phi_k^2 \sim H^2$

Super-horizon dynamics – δN formalism-

- Super-horizon dynamics is locally described by the FRW universe.

Starobinsky (1985)
Salopek & Bond (1990)
Sasaki & Stewart(1996)
Sasaki & TT(1998),
Lyth et al.(2005)



e-folding number

$$N(t_F; t_*) \equiv \int_{t_*}^{t_F} H\,dt$$

Naively,

$$ds^2 = -dt^2 + a^2 e^{2\zeta}\gamma_{ij}\,dx^i dx^j$$

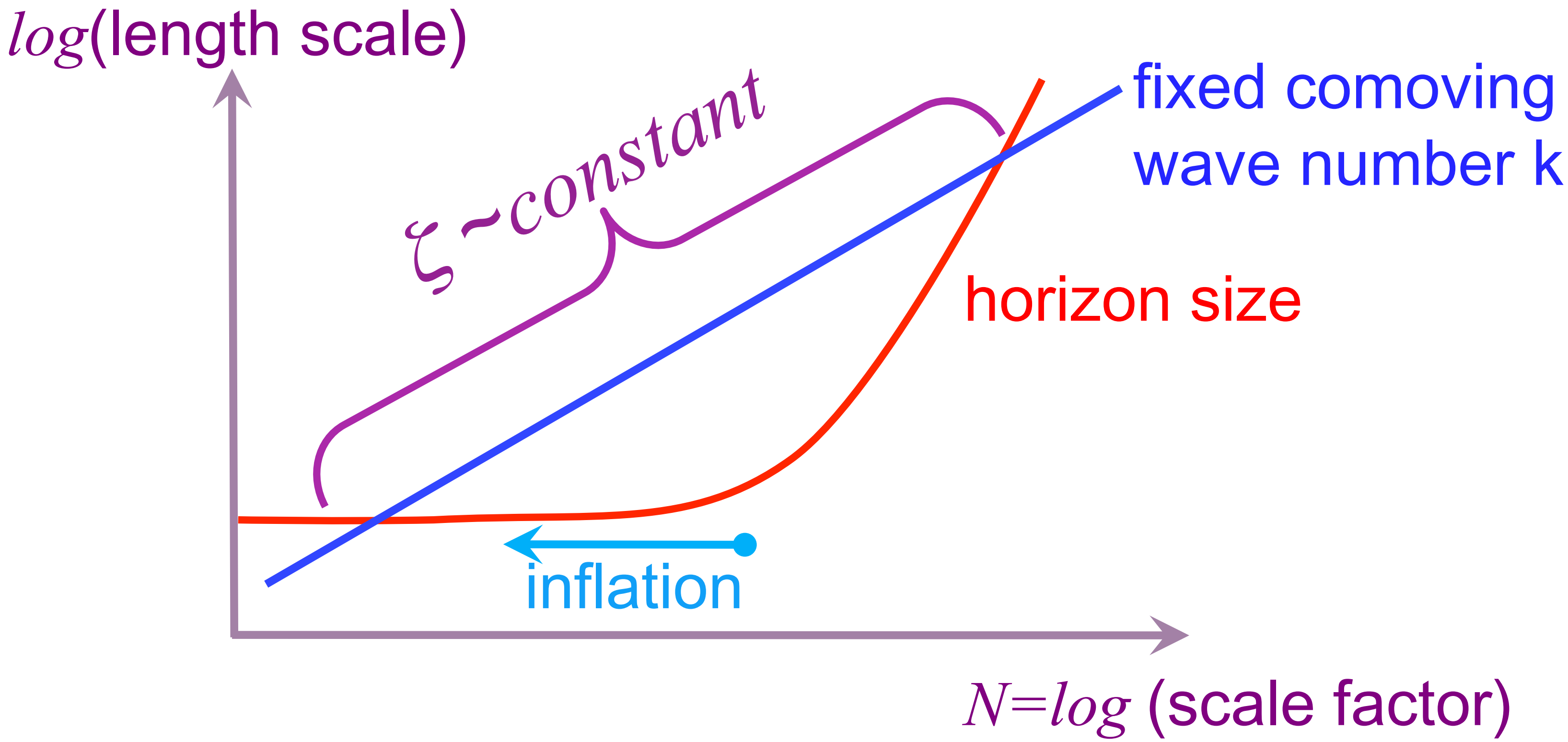
$$e^{2(N+\delta N)}$$

$$\zeta(t_F; x) \simeq \delta N(t_F; \phi^I(t_*; x))$$

ζ is conserved for single field inflation on super horizon scale.

$$\zeta \approx H\delta t = H\frac{\delta\phi}{\dot\phi} \approx \frac{H^2}{\dot\phi}$$

Single field inflation



Tensor perturbations

$$L_{grav} = \frac{M_{pl}^2}{2} R \approx -\frac{M_{pl}^2}{4} \nabla_\rho h_{\mu\nu} \nabla^\rho h^{\mu\nu} + \cdots$$

canonically normalized gravitational wave perturbation

$$\left(\frac{\delta T}{T}\right)_{tensor} \approx \delta h_{\mu\nu} \approx \frac{\delta\psi_{\mu\nu}}{M_{pl}} \approx \frac{H}{M_{pl}}$$

Direct probe of the energy scale of inflation.

Formulas for slow roll inflation

Slow roll parameters

$$\varepsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \qquad \eta = \frac{V''}{V}$$

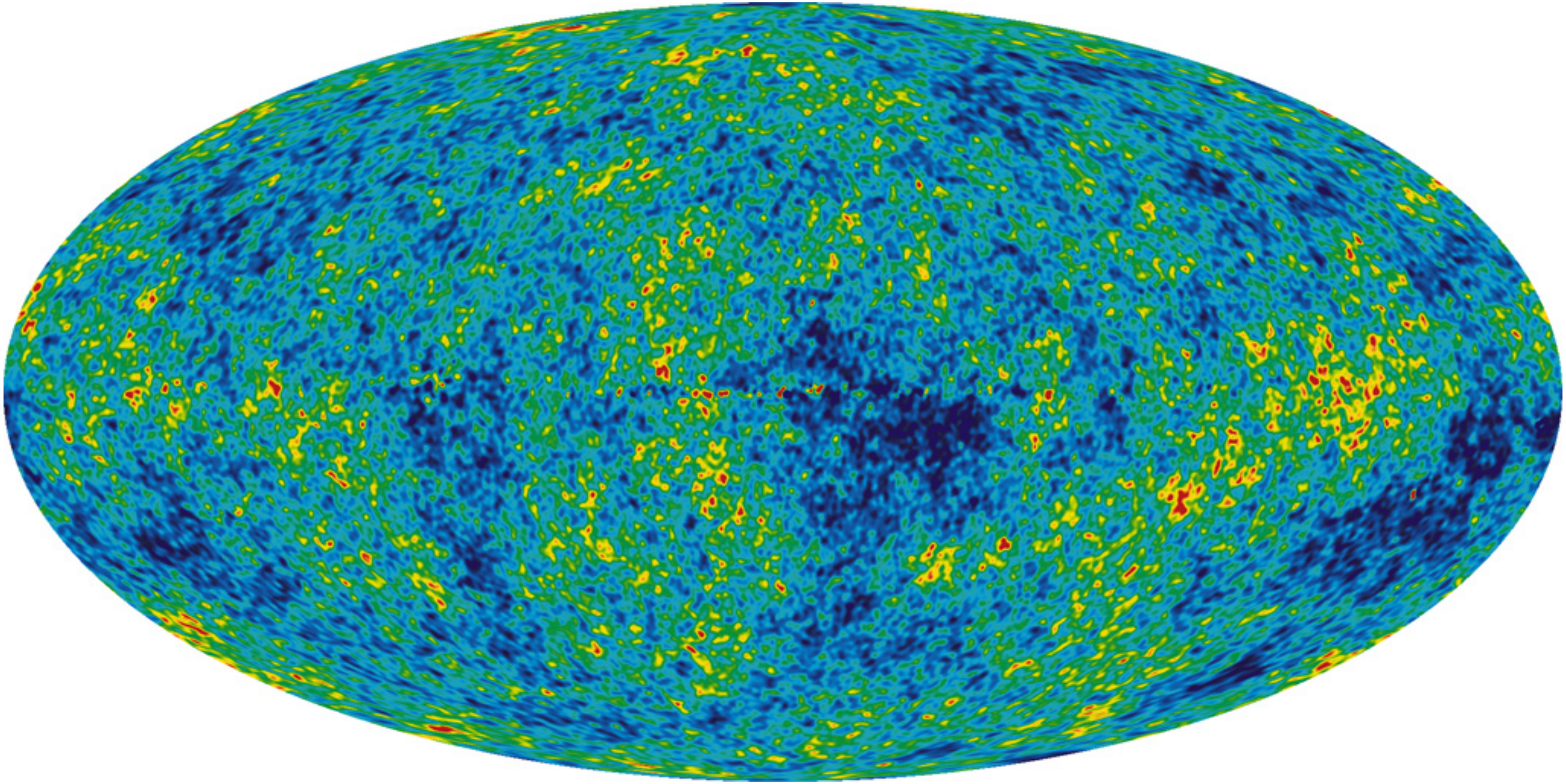
$$\Delta_\zeta^2 = \frac{V}{24\pi^2 \varepsilon}$$:squared amplitude of curvature perturbation

$$n_s - 1 = -6\varepsilon + 2\eta$$:tilt of the spectrum $\Delta_\zeta \propto k^{n_s - 1}$

$$r \equiv \frac{\Delta_h^2}{\Delta_\zeta^2} = 16\varepsilon$$:tensor-to-scalar ratio

$$n_t = -2\varepsilon$$:tilde of tensor perturbation

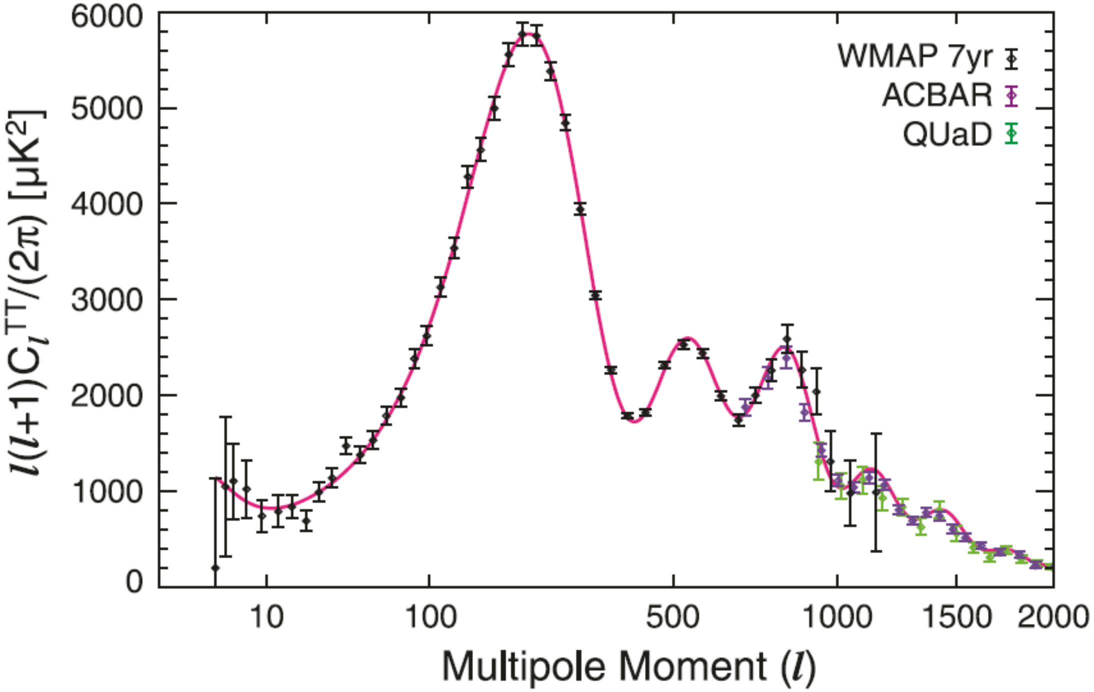
CMB map by WMAP satellite

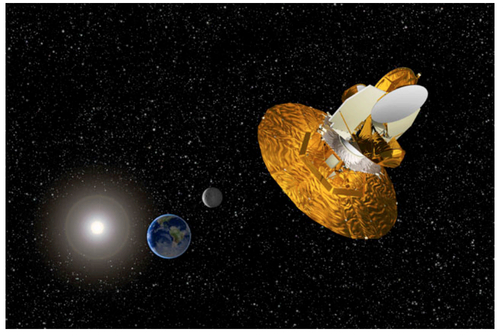


Amplitude of fluctuation is about 10^{-5}.

Success of inflation

WMAP7yr+ACBAR+QUaD





WMAP

$$-0.0133 < \Omega_K < 0.0084$$

$$\Omega_\Lambda = 0.725 \pm 0.016$$

$$\Delta_\zeta^2 = 2.42 \times 10^{-9}$$

$$n_s = 0.968 \pm 0.012$$

$$r < 0.24$$

Further steps from observations

1) Tensor perturbations:

r < 0.24 : WMAP

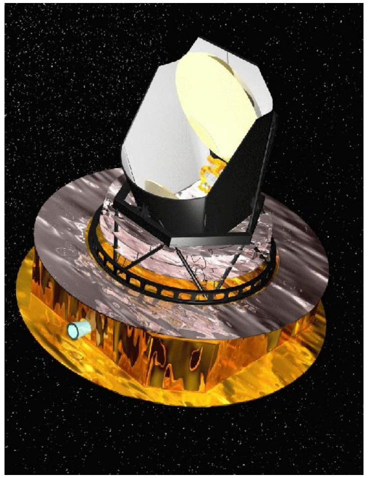
r < 0.05 : Planck

r < 0.01 : QUIET, PolarBeaR, BICEP2, SPTpol, Spider

r < 0.001 : LiteBIRD, EPIC, B-Pol

2) Non-Gaussianity ⟺ effects of non-linear dynamics during and after inflation

$$\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \zeta_{-\mathbf{k}-\mathbf{k}'} \rangle \neq 0$$



WMAP 7yr

$$-10 < f_{NL}^{local} < 74 \, (95\% CL)$$

$$-214 < f_{NL}^{equil} < 266 \, (95\% CL)$$

$$-410 < f_{NL}^{orthog} < 6 \, (95\% CL)$$

Planck (launched 14 May 2009)

Large tensor from inflation

Large tensor perturbation requires large field inflation

$$r = 16\varepsilon = 8\left(\frac{V'}{V}\right)^2 \approx \frac{8\Delta\phi^2}{N^2}$$: Lyth bound $$\frac{d\phi}{dN} = \frac{\dot{\phi}}{H} \approx -\frac{V'}{3H^2} \approx -\frac{V'}{V}$$

SUGRA:

Scalar field potential

$$V = e^K \left[K_{\Phi\overline{\Phi}}^{-1}|D_\Phi W|^2 - 3|W|^2\right]$$

$$D_\Phi W = \frac{\partial W}{\partial \Phi} + \frac{\partial K}{\partial \Phi} W$$

$K_{\Phi\overline{\Phi}}\partial_\mu \Phi \partial^\mu \overline{\Phi}$:kinetic term

Canonical choice of Kähler potential is $K = \Phi\overline{\Phi}$, for which $K_{\Phi\overline{\Phi}} = 1$.

- Exponential growth of potential for $\phi > 1$.
- η -problem: $m^2 = O(H^2)$

A solution is to choose $K = -\frac{1}{2}\left(\Phi - \overline{\Phi}\right)^2 = \Phi\overline{\Phi} - \frac{1}{2}\left(\Phi^2 + \overline{\Phi}^2\right)$

Kawasaki, Yamaguchi and Yanagida (2000)

Realizing Large field inflation

String:

Moduli/brane in internal space:
$\Delta\phi >> M_{pl}$ ⇨ long internal space.

If the volume of internal space is large, it is also disfavored:

$$M_{pl}^2 \approx M_{10D}^8 Vol_6$$

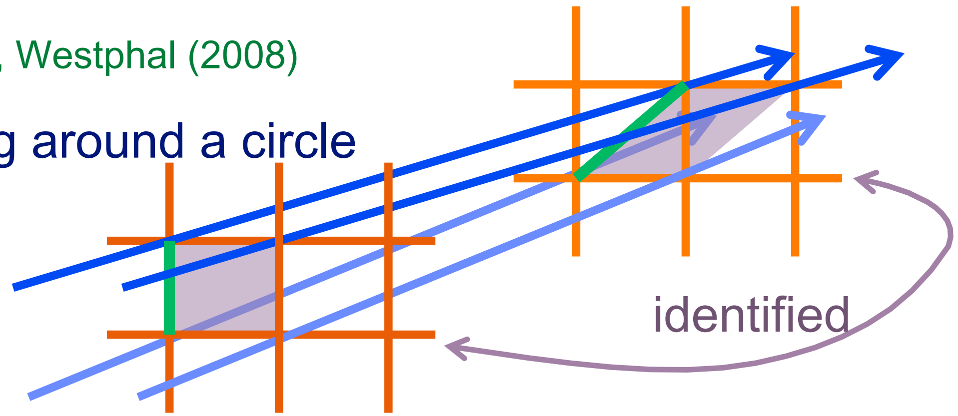
⇕ difficult to be compatible

Small ε and η ⇨ small backreaction to the whole internal space
⇨ strong stabilization

• Monodromy

Silverstein, Westphal (2008)

Roughly speaking, wrapping around a circle

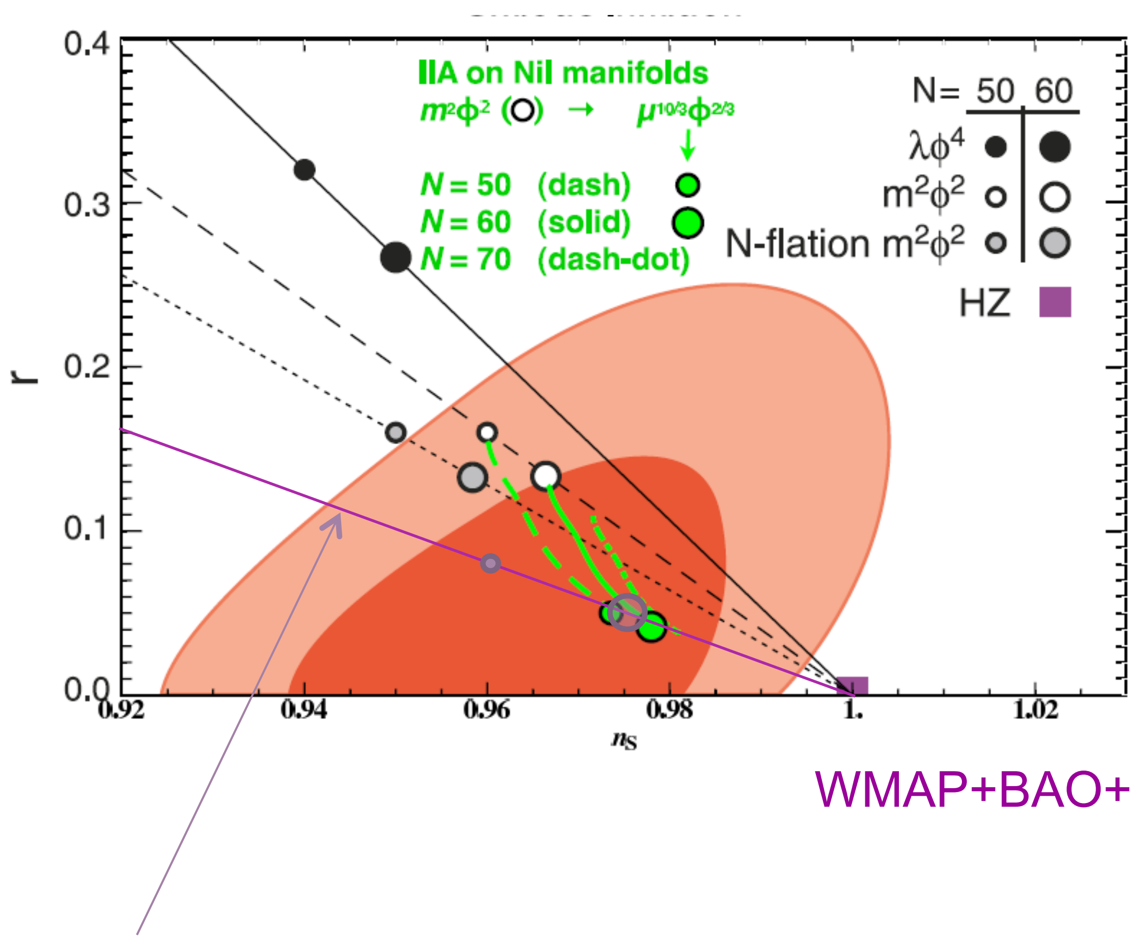


•N-flation

Dimopoulos, Kachru, McGreevy, Wacker (2005)

N scalar fields ⇨ lager H

⇨ slower rolling ⇨ smaller ε and η



WMAP+BAO+H_0 (not latest)

- Starobinsky inflation

$$L_{grav} = M_{pl}^2\left(R + R^2/6M^2\right) \Longleftrightarrow$$

Einstein gravity + single scalar

$$V = \frac{3M^2}{4}\left(1 - e^{-\sqrt{2/3}\phi}\right)^2$$

Non-Gaussianity

- In the standard slow roll inflation, non-Gaussianity is extremely suppressed.

  ⇨ Non-Gaussianity requires non-standard inflation models.

- Non-linear dynamics gives non-linear mapping

$$\zeta_G \to \zeta = \zeta(\zeta_G)$$ Komatsu and Spergel (2001)

$$\zeta(\mathbf{x}) = \underline{\zeta_G(\mathbf{x})} + \frac{3}{5}\underline{f_{NL}}\zeta_G^2(\mathbf{x})$$

Gaussian variable   Non-linear parameter

$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle \equiv \delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\,\underline{B_\zeta(k_1,k_2,k_3)}$$

bispectrum

$$B_\zeta(k_1,k_2,k_3) = \frac{6}{5}\frac{f_{NL}}{(2\pi)^{3/2}}\left[P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_3)P_\zeta(k_1)\right]$$

In general, mapping is non-local.

Local interaction only ~ Super horizon dynamics

$$\zeta(t_c) \approx \delta N = N^*_a \varphi^a_* + \frac{1}{2} N^*_{ab} \varphi^a_* \varphi^b_* + \cdots$$

$$N_a(t) \equiv \left. \frac{\partial N(t_c, \phi)}{\partial \phi^a} \right|_{\phi^a = \phi^a(t)}$$

"*" indicates a time just after initial horizon crossing

$$N_{ab}(t) \equiv \left. \frac{\partial^2 N(t_c, \phi)}{\partial \phi^a \partial \phi^b} \right|_{\phi^a = \phi^a(t)}$$

$$\langle \zeta(\mathbf{x}_1) \zeta(\mathbf{x}_2) \zeta(\mathbf{x}_3) \rangle = N^*_a N^*_b N^*_c \underline{\langle \varphi^a_*(\mathbf{x}_1) \varphi^b_*(\mathbf{x}_2) \varphi^c_*(\mathbf{x}_3) \rangle}$$

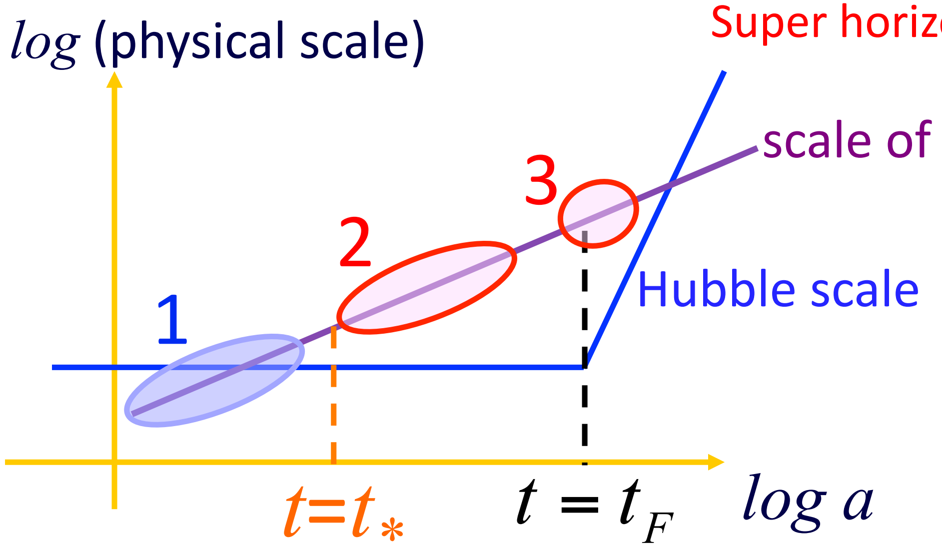
Early generation of non-Gaussianity

➔ suppressed by slow-roll parameters. (Seery & Lidsey (2005))

Exception is fast roll inflation.

$$+ \frac{1}{2} N^*_a N^*_b N^*_{cd} \left[\langle \varphi^a_*(\mathbf{x}_1) \varphi^b_*(\mathbf{x}_2) \varphi^c_*(\mathbf{x}_3) \varphi^d_*(\mathbf{x}_3) \rangle + \text{perm} \right]$$

Super horizon part of non-Gaussianity



Non Gaussianity is produced

1) before horizon crossing
2) during super horizon evolution
3) at the end of or after inflation

2) or 3) are local ⇨

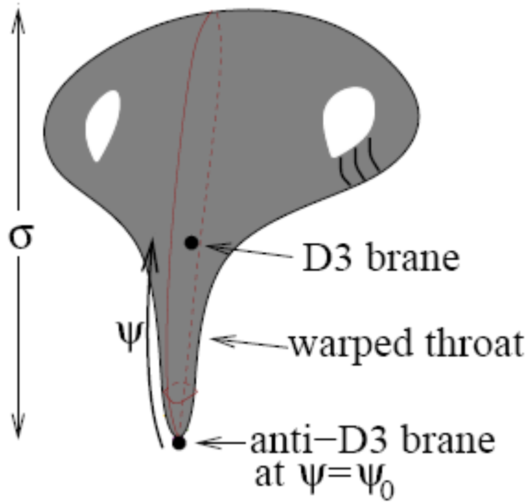
$$\frac{6}{5} f_{NL} \approx \frac{N^a_* N^b_* N^*_{ab}}{\left(N^c_* N^*_c\right)^2}$$

Non-local Non-Gaussianity from non-canonical kinetic term

- Typical example is DBI inflation — Alishhiha, Silverstein, Tong (2008)

Moving D3-brane in a higher-dimensional background ⇔ (AdS/CFT) Strong coupling large N CFT

$$ds^2 = h^{-1/2}(y^K) g_{\mu\nu} dx^\mu dx^\nu + h^{1/2}(y^K) G_{IJ}(y^K) dy^I dy^J$$



$$\Rightarrow \quad L_{eff} = -\frac{1}{f}\sqrt{-\det\left(g_{\mu\nu} + f\, G_{IJ}\partial_\mu \phi^I \partial_\nu \phi^J\right)} - V(\phi)$$

$$f = \frac{h}{T_3} \qquad \phi^I = \sqrt{T_3}\,\delta y^I$$

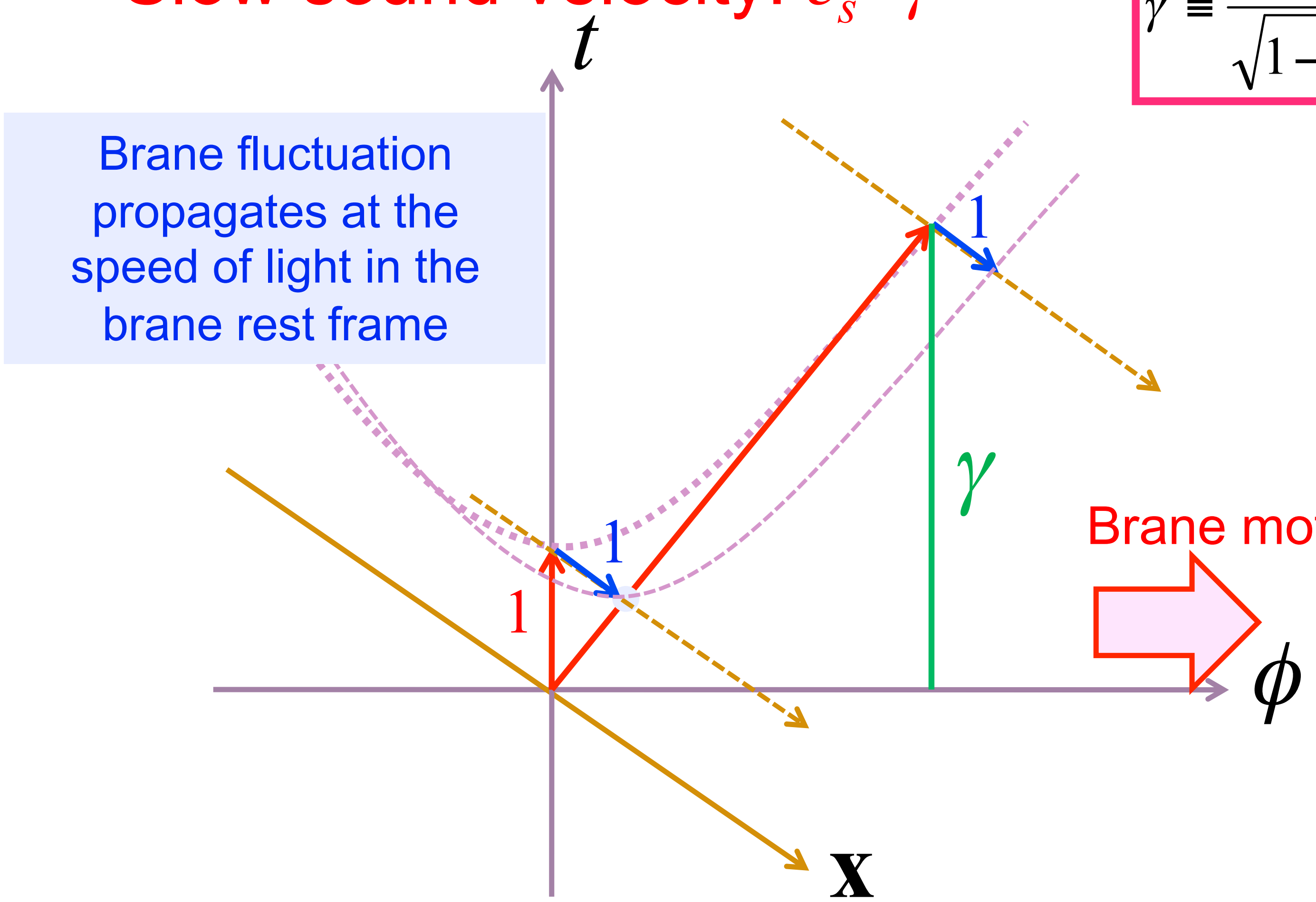
Speed limit:

$$\sqrt{-\det(\cdots)} \xrightarrow{\text{spatially homogeneous}} \sqrt{-g}\sqrt{\left(1 - f\dot{\phi}^2\right)}$$

Even if V' is large, $|\dot{\phi}| < f^{-1/2}$ ⇒ smaller ε and η

Slow sound velocity: $c_s = \gamma^{-1}$

$$\gamma \equiv \frac{1}{\sqrt{1 - f\dot{\phi}^2}}$$



$$\Delta_\zeta^2 = \frac{H^2}{8\pi^2 \varepsilon\, c_s}$$

$$f_{NL}^{equil} = -\frac{35}{108}\frac{1}{c_s^2}$$

enhanced

$$r = 16\varepsilon\, c_s$$

suppressed

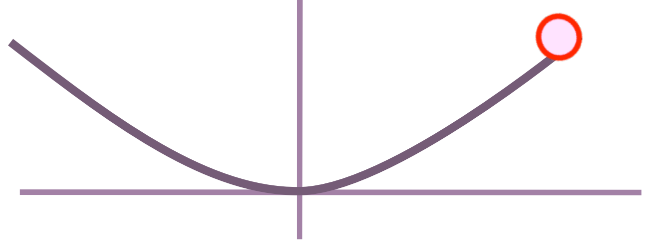
Non-Gaussianity produced at the end of or after inflation

Curvaton	(Lyth & Wands (2002))
Modulated reheating	(Dvali, Gruzinov & Zaldarriaga (2004))
Modulated waterfall	(Bernardeau, Kofman and Uzan (2004), Lyth (2004))

Ex.) Curvaton

$$\zeta_\sigma \approx \frac{\delta\rho_\sigma}{\rho_{tot}} = r\left(2\frac{\delta\sigma}{\sigma} + \left(\frac{\delta\sigma}{\sigma}\right)^2\right) \qquad r \equiv \frac{\rho_\sigma}{\rho_{tot}}$$

σ starts to roll after inflation.



$$\rho_\sigma \approx \frac{m^2}{2}(\sigma + \delta\sigma)^2$$

Suppose ζ_σ is the dominant component of fluctuation.

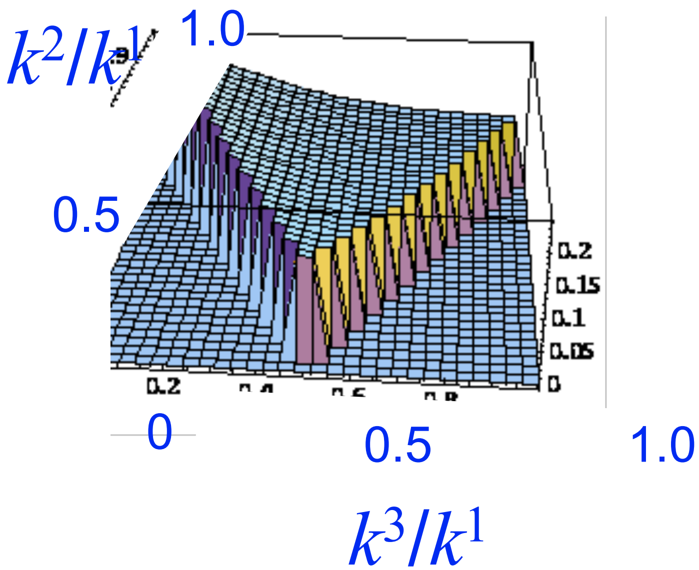
Amplitude is observationally fixed.

$$P_\zeta \approx \left(r\frac{\delta\sigma}{\sigma}\right)^2 = 10^{-9}$$

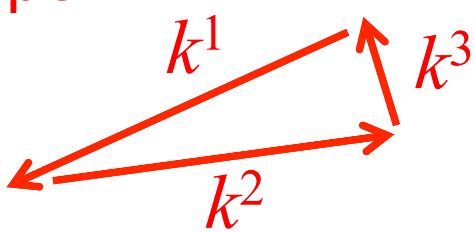
$$f_{NL} = \frac{1}{r}$$

can be as large as 10^5.

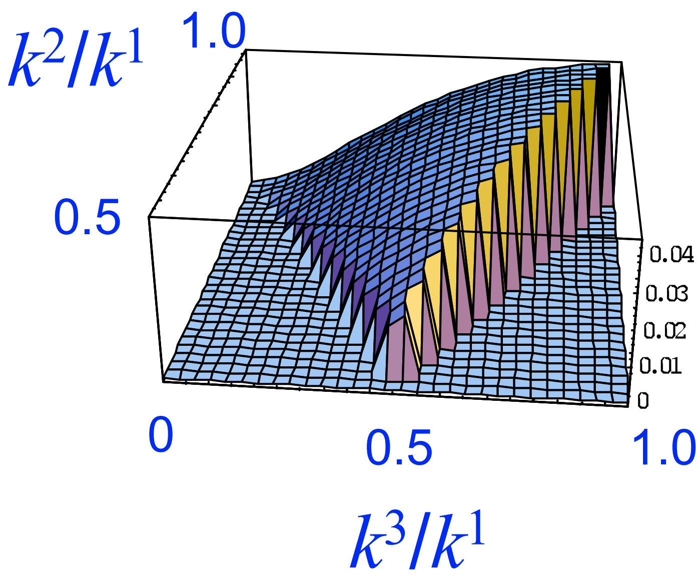
Curvaton bi-spectrum



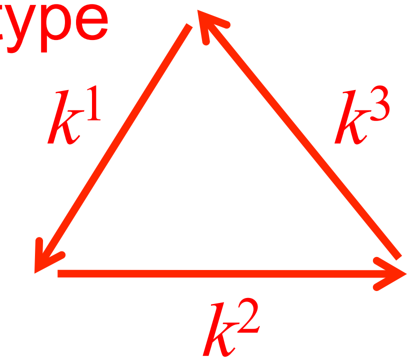
local-type



DBI bi-spectrum



equilateral-type



Summary

- Tensor perturbations and non-Gaussianities in CMB will be the key for the next step.
- Observations of the next generation will reduce the error by factor 1/10 or more.
- Various inflation models make different prediction about tensor amplitude and amplitude/shapes of non-Gaussianities.
- Once they are detected, it becomes a powerful tool to distinguish different models of inflation.