Domain wall solution and variation of the fine structure constant in $F(R)$ gravity

Reference: K. Bamba, S. Nojiri and S. D. Odintsov,
Phys. Rev. D 85, 044012 (2012)
[arXiv:1107.2538 [hep-th]].

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Presenter : **Kazuharu Bamba** (*KMI, Nagoya University*)

Collaborators : **Shin'ichi Nojiri** (*KMI and Dep. of Phys., Nagoya University*)

Sergei D. Odintsov (*ICREA and CSIC-IEEC*)

I. Introduction

2011 Nobel Prize in Physics

- Recent observations of Supernova (SN) Ia confirmed that the current expansion of the universe is accelerating.

  [Perlmutter *et al*. [Supernova Cosmology Project Collaboration], Astrophys. J. 517, 565 (1999)]

  [Riess *et al*. [Supernova Search Team Collaboration], Astron. J. 116, 1009 (1998)]

  [Astier *et al*. [The SNLS Collaboration], Astron. Astrophys. 447, 31 (2006)]

- There are two approaches to explain the current cosmic acceleration. **[Copeland, Sami and Tsujikawa, Int. J. Mod. Phys. D 15, 1753 (2006)]**

  [Tsujikawa, arXiv:1004.1493 [astro-ph.CO]]

< Gravitational field equation >

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu}$$

Gravity **Matter**

$G_{\mu\nu}$: Einstein tensor

$T_{\mu\nu}$: Energy-momentum tensor

$\kappa^2 \equiv 8\pi / {M_{\mathrm{Pl}}}^2$

M_{Pl} : Planck mass

(1) **General relativistic approach** $\longrightarrow$ **Dark Energy**

(2) **Extension of gravitational theory**

(1) **General relativistic approach**

- **Cosmological constant**
- **Scalar fields**: **X matter**, **Quintessence**, **Phantom**, **K-essence**, **Tachyon**.
- **Fluid**: **Chaplygin gas**

$F(R)$: Arbitrary function of the Ricci scalar R

[Capozziello, Cardone, Carloni and Troisi, Int. J. Mod. Phys. D 12, 1969 (2003)]

(2) **Extension of gravitational theory**

- ***F*(*R*) gravity** **[Carroll, Duvvuri, Trodden and Turner, Phys. Rev. D 70, 043528 (2004)]**
- **Scalar-tensor theories** **[Nojiri and Odintsov, Phys. Rev. D 68, 123512 (2003)]**
- **Ghost condensates** **[Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405, 074 (2004)]**

$\mathcal{G}$: Gauss-Bonnet term

- **Higher-order curvature term** ▪ $f(\mathcal{G})$ **gravity**

T : torsion scalar

- **DGP braneworld scenario** **[Dvali, Gabadadze and Porrati, Phys. Lett B 485, 208 (2000)]**
- **Non-local gravity** **[Deser and Woodard, Phys. Rev. Lett. 99, 111301 (2007)]**
- ***f*(*T*) gravity** **[Bengochea and Ferraro, Phys. Rev. D 79, 124019 (2009)]** **[Linder, Phys. Rev. D 81, 127301 (2010) [Erratum-ibid. D 82, 109902 (2010)]]**
- **Galileon gravity** **[Nicolis, Rattazzi and Trincherini, Phys. Rev. D 79, 064036 (2009)]**

- We construct a domain wall solution in $F(R)$ gravity.
  - **Static domain wall solution in a scalar field theory.**
  - **Explicit $F(R)$ gravity model in which a static domain wall solution can be realized.**
- **We show that there could exist an effective (gravitational) domain wall in $F(R)$ gravity.**
- **It is demonstrated that a logarithmic non-minimal gravitational coupling of the electromagnetic theory in $F(R)$ gravity may produce time-variation of the fine structure constant which may increase with decrease of the curvature.**

II. Comparison of $F(R)$ gravity with a scalar field theory having a runaway type potential

$$g = \det(g_{\mu\nu})$$

< Action >

$$S = \int d^4x \sqrt{-g} \frac{F(R)}{2\kappa^2} + \int d^4x \mathcal{L}_{\mathrm{M}} \left(g_{\mu\nu}, \Psi_{\mathrm{M}}\right)$$

$g_{\mu\nu}$: Metric tensor

$\mathcal{L}_{\mathrm{M}}$: Matter Lagrangian

- We make a conformal transformation to the Einstein frame:

$$\boxed{\begin{aligned} &\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \\ &\Omega^2 \equiv F_{,R}\,, \quad F_{,R} \equiv \frac{dF(R)}{dR} \end{aligned}}$$

* A tilde represents quantities in the Einstein frame.

$$\phi \equiv \sqrt{\frac{3}{2}} \frac{1}{\kappa} \ln F_{,R}$$

$$S_{\mathrm{E}} = \int d^4x \sqrt{-\tilde{g}} \left(\frac{\tilde{R}}{2\kappa^2} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right) + \int d^4x \mathcal{L}_{\mathrm{M}} \left(\left(F_{,R}\right)^{-1} (\phi) \tilde{g}_{\mu\nu}, \Psi_{\mathrm{M}} \right)$$

$$V(\phi) = \frac{F_{,R} \tilde{R} - F}{2\kappa^2 \left(F_{,R}\right)^2}$$

$$R = e^{1/\sqrt{3}\kappa\phi} \left[\tilde{R} + \sqrt{3} \tilde{\Box} \left(\kappa\phi\right) - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \left(\kappa\phi\right) \partial_\nu \left(\kappa\phi\right) \right]$$

$$\tilde{\Box} \left(\kappa\phi\right) = \frac{1}{\sqrt{-\tilde{g}}} \partial_\mu \left[\sqrt{-\tilde{g}} \tilde{g}^{\mu\nu} \partial_\nu \left(\kappa\phi\right) \right]$$

$\Box \equiv g^{\mu\nu} \nabla_\mu \nabla_\nu$: Covariant d'Alembertian

∇_μ : Covariant derivative operator

- Action describing a runaway domain wall and a space-time varying fine structure constant α_{EM} :

$$S_{\mathrm{E}} = \int d^4x \sqrt{-\tilde{g}} \left(\frac{\tilde{R}}{2\kappa^2} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \underline{V(\phi)} \right) + \int d^4x \sqrt{-\tilde{g}} \left(-\frac{1}{4}\underline{B(\phi)}\tilde{g}^{\mu\alpha}\tilde{g}^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} \right) + S_{\mathrm{matter}}$$

$$V(\phi) = \frac{M^{2p+4}}{(\phi^2+\sigma^2)^p}$$

M : Mass scale

$p(>1)$, $\sigma(<\phi)$: Constants

$$B(\phi) = \mathrm{e}^{-\xi\kappa\phi}$$

ξ : Constant

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

: Electromagnetic field-strength tensor

A_μ : U(1) gauge field

Discrete symmetry $\phi \leftrightarrow -\phi$ can be broken dynamically.

$\longrightarrow$ **A domain wall can be formed.**

$$\boxed{\begin{array}{l} \alpha_{\mathrm{EM}}(\phi) = \alpha_{\mathrm{EM}}^{(0)}/B(\phi) \\ \alpha_{\mathrm{EM}}^{(0)} = e^2/(4\pi) \text{ : Bare fine structure constant} \\ e \text{ : Charge of the electron} \end{array}}$$

[Cho and Vilenkin, Phys. Rev. D 59, 021701 (1998)]

[Olive, Peloso and Uzan, Phys. Rev. D 83, 043509 (2011)]

[Chiba and Yamaguchi, JCAP 1103, 044 (2011)]

< Flat Friedmann-Lemaȋtre-Robertson-Walker (FLRW) space-time >

$$ds^2 = -dt^2 + a^2(t) \sum_{i=1,2,3} (dx^i)^2$$

$a(t)$: Scale factor

III. Reconstruction of a static domain wall solution in a scalar field theory

< Action > $S = \int d^D x \sqrt{-g} \left(\frac{R}{2\kappa^2} - \frac{\omega(\varphi)}{2} \partial_\mu \varphi \partial^\mu \varphi - \mathcal{V}(\varphi) \right)$

< $D = d+1$ dimensional warped metric >

$$\boxed{ds^2 = dy^2 + \mathrm{e}^{u(y)} \sum_{\mu,\nu=0}^{d-1} \hat{g}_{\mu\nu} dx^\mu dx^\nu}$$

We assume φ only depends on y .

$\hat{g}_{\mu\nu}$: Metric of the d-dimensional Einstein manifold: $\hat{R}_{\mu\nu} = \frac{d-1}{l^2} \hat{g}_{\mu\nu}$

- de Sitter space: $1/l^2 > 0$
- Flat space: $1/l^2 = 0$
- Anti-de Sitter space: $1/l^2 < 0$

< The Einstein equation >

(y, y) component: $-\frac{d(d-1)}{2l^2} \mathrm{e}^{-u} + \frac{d(d-1)}{8} (u')^2 = \frac{1}{2} \omega(\varphi) (\varphi')^2 - \mathcal{V}(\varphi)$

(μ, ν) component: $-\frac{(d-1)(d-2)}{2l^2} \mathrm{e}^{-u} + \frac{d-1}{2} u'' + \frac{d(d-1)}{8} (u')^2 = -\frac{1}{2} \omega(\varphi) (\varphi')^2 - \mathcal{V}(\varphi)$

- We may choose $\varphi = y$ and take $\kappa^2 = 1$.

$\ast$ The prime denotes the derivative with respect to y .

$$\Rightarrow \quad \omega(\varphi) = -\frac{d-1}{2} u'' - \frac{d-1}{l^2} \mathrm{e}^{-u}$$

$$\mathcal{V}(\varphi) = -\frac{d-1}{4} u'' - \frac{d(d-1)}{8} (u')^2 + \frac{(d-1)^2}{2l^2} \mathrm{e}^{-u}$$

$$\longrightarrow \quad \rho = \frac{\omega(\varphi)}{2}(\varphi')^2 + \mathcal{V}(\varphi) = -\frac{d-1}{2}u'' - \frac{d(d-1)}{8}(u')^2 + \frac{(d-1)(d-2)}{2l^2}\mathrm{e}^{-u}$$

- Example: $\boxed{u = u_0 \mathrm{e}^{-y^2/y_0^2}}$ $u_0,\ y_0$: Constants

$$\omega(\varphi) = -(d-1)\left(\frac{2\varphi^2}{y_0^4} - \frac{1}{y_0^2}\right)\mathrm{e}^{-\varphi^2/y_0^2} - \frac{(d-1)}{l^2}\mathrm{e}^{-u_0 \mathrm{e}^{-\varphi^2/y_0^2}}$$

$$\mathcal{V}(\varphi) = -\frac{d-1}{2}\left(\frac{2\varphi^2}{y_0^4} - \frac{1}{y_0^2}\right)\mathrm{e}^{-\varphi^2/y_0^2} + \frac{(d-1)^2}{l^2}\mathrm{e}^{-u_0 \mathrm{e}^{-\varphi^2/y_0^2}}$$

$$\longrightarrow \quad \rho(y) = -\frac{d-1}{2}\left(\frac{2y^2}{y_0^4} - \frac{1}{y_0^2}\right)\mathrm{e}^{-y^2/y_0^2} + \frac{(d-1)^2}{l^2}\mathrm{e}^{-u_0 \mathrm{e}^{-y^2/y_0^2}}$$

$\rho(y)$ is localized at $y \sim 0$ and makes a domain wall.

IV. Reconstruction of an explicit *F*(*R*) gravity model realizing a static domain wall solution

< Gravitational field equation >

$$T^{(\mathrm{M})}_{\alpha\beta} \equiv -\left(2/\sqrt{-g}\right)\left(\delta\mathcal{L}_{\mathrm{M}}/\delta g^{\alpha\beta}\right)$$

$$-\frac{1}{2}Fg_{\alpha\beta} + \left(R_{\alpha\beta} - \nabla_\alpha\nabla_\beta + g_{\alpha\beta}\Box\right)F_{,R} = \kappa^2 T^{(\mathrm{M})}_{\alpha\beta}$$

$T^{\alpha\ (\mathrm{M})}_{\ \beta} = \mathrm{diag}(-\rho_{\mathrm{M}}, P_{\mathrm{M}}, P_{\mathrm{M}}, P_{\mathrm{M}})$: Energy-momentum tensor of matter

$\rho_{\mathrm{M}},\ P_{\mathrm{M}}$: Energy density and pressure of matter

$$G_{\alpha\beta} = \kappa^2\left(T^{(\mathrm{M})}_{\alpha\beta} + T^{(\mathrm{D})}_{\alpha\beta}\right)$$

$$\kappa^2 T^{(\mathrm{D})}_{\alpha\beta} \equiv \frac{1}{2}(F-R)\,g_{\alpha\beta} + \left(1-F_{,R}\right)R_{\alpha\beta} + \left(\nabla_\alpha\nabla_\beta - g_{\alpha\beta}\Box\right)F_{,R}$$

$G_{\alpha\beta} \equiv R_{\alpha\beta} - (1/2)\,g_{\alpha\beta}R$: The Einstein tensor

$\kappa^2 T^{(\mathrm{D})}_{\alpha\beta}$: Contribution to the energy-momentum tensor from the deviation of *F*(*R*) gravity from general relativity

$D = d+1$ dimensional warped metric

$$ds^2 = dy^2 + \mathrm{e}^{u(y)}\sum_{\mu,\nu=0}^{d-1}\hat{g}_{\mu\nu}dx^\mu dx^\nu$$

$$g_{yy} = 1$$
$$g_{\mu\nu} = \mathrm{e}^{u}\hat{g}_{\mu\nu}$$

Gravitational field equation

(y, y) component: $\frac{d-1}{2}u'(F_{,R})' - \frac{d}{2}\left[u'' + \frac{1}{2}(u')^2\right]F_{,R} - \frac{1}{2}F = \kappa^2 T^{(\mathrm{M})}_{yy}$

(μ, ν) component:

$$d(F_{,R})'' + \frac{d(d-2)}{2}u'(F_{,R})' + \left\{-\frac{d}{2}\left[u'' + \frac{d}{2}(u')^2\right] + \frac{d(d-1)}{l^2}\mathrm{e}^{-u}\right\}F_{,R} - \frac{d}{2}F$$
$$= \kappa^2 \sum_{\mu,\nu=0}^{d-1} g^{\mu\nu} T^{(\mathrm{M})}_{\mu\nu}$$

$$(F_{,R})' \equiv dF_{,R}/dy\ , \qquad (F_{,R})'' \equiv d^2 F_{,R}/dy^2$$

- We derive an explicit form of $F(R)$ realizing a domain wall solution.

$$\boxed{u = u(y) \longrightarrow y = y(R) \longrightarrow u = u(y(R))}$$

$$\Uparrow$$

$$R = -\frac{d}{2}\left[2u'' + \frac{1+d}{2}(u')^2\right] + \frac{d(d-1)}{l^2}\mathrm{e}^{-u}$$

* We consider the case in which there is no matter.

$\longrightarrow$ $\boxed{(y,y)\text{component:}\quad \Xi_1(R)\frac{d^2F(R)}{dR^2}+\Xi_2(R)\frac{dF(R)}{dR}-F(R)=0}$

$$\Xi_1(R)\equiv(d-1)\,u'\frac{dR}{dy}=(d-1)\left(\frac{dR}{dy}\right)^2\frac{du(y(R))}{dR}$$

$$\Xi_2(R)\equiv(-d)\left[u''+\frac{1}{2}\left(u'\right)^2\right]=(-d)\left[\frac{d^2R}{dy^2}\frac{du(y(R))}{dR}+\left(\frac{dR}{dy}\right)^2\frac{d^2u(y(R))}{dR^2}+\frac{1}{2}\left(\frac{dR}{dy}\right)^2\left(\frac{du(y(R))}{dR}\right)^2\right]$$

- We solve the equation of the scalar curvature *R* in terms of *y*.

$\longrightarrow$ We define $Y\equiv y^2/y_0^2$. For $Y=y^2/y_0^2\ll 1$, we expand exponential terms and take the first leading terms in terms of *Y*.

$$\boxed{\frac{d^2F(R)}{dR^2}+\mathcal{C}\frac{dF(R)}{dR}+\mathcal{D}F(R)=0\ ,\quad \mathcal{C}\equiv\frac{\Xi_2^{(0)}}{\Xi_1^{(0)}}\ ,\quad \mathcal{D}\equiv-\frac{1}{\Xi_1^{(0)}}}$$

A general solution

$$F(R)=F_+\mathrm{e}^{\lambda_+R}+F_-\mathrm{e}^{\lambda_-R}$$

$$\lambda_\pm\equiv\frac{1}{2}\left(-\mathcal{C}\pm\sqrt{\mathcal{C}^2-4\mathcal{D}}\right)$$

Exponential model

$$\boxed{F(R)=F_+\mathrm{e}^{\lambda_+R}}$$

F_+ : Arbitrary constant

$$R=\frac{1}{\lambda_+}\left[\ln\left(\frac{1}{F_+\lambda_+}\right)+\sqrt{\frac{2}{3}}\kappa\phi\right]$$

$$V(\phi)=\frac{1}{2\kappa^2\lambda_+}\mathrm{e}^{-\sqrt{2/3}\kappa\phi}\left[\sqrt{\frac{2}{3}}\kappa\phi+\ln\left(\frac{1}{F_+\lambda_+}\right)-1\right]$$

V. Effective (gravitational) domain wall

< Reconstruction method >

$P(\psi),\ Q(\psi)$: Proper functions of the auxiliary scalar field ψ

$$P'(\psi) = dP(\psi)/d\psi$$

$$S_{F(R)} = \int d^4x\sqrt{-g}\left(\frac{F(R)}{2\kappa^2} + \mathcal{L}_{\mathrm{matter}}\right)$$

$$\Rightarrow \quad S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\underline{\left(P(\psi)R + Q(\psi)\right)} + \mathcal{L}_{\mathrm{matter}}\right]$$

The variation over ψ $\longrightarrow$ $0 = P'(\psi)R + Q'(\psi)$ $\longrightarrow$ $\psi = \psi(R)$

$$\Rightarrow \quad S = \int d^4x\sqrt{-g}\left(\frac{F(R)}{\underline{2\kappa^2}} + \mathcal{L}_{\mathrm{matter}}\right) \qquad \underline{F(R) \equiv P(\psi(R))R + Q(\psi(R))}$$

By the variation of the metric, we find

$$0 = \frac{1}{2}g_{\mu\nu}\left(P(\psi)R + Q(\psi)\right) - P(\psi)R_{\mu\nu} + \nabla_\mu\nabla_\nu P(\psi) - g_{\mu\nu}\Box P(\psi)$$

$D = d+1$ **dimensional warped metric**

$$\boxed{ds^2 = dy^2 + \mathrm{e}^{u(y)}\sum_{\mu,\nu=0}^{d-1}\hat{g}_{\mu\nu}dx^\mu dx^\nu}$$

* We have neglected the contribution from the matter.

(y,y) component: $$0=\frac{1}{2}\left\{P(\psi)\left[-du''-\frac{d(d+1)}{4}(u')^2+\frac{d(d-1)\mathrm{e}^{-u}}{l^2}\right]+Q(\psi)\right\} -P(\psi)\left[-\frac{d}{2}u''-\frac{d}{4}(u')^2\right]-\frac{d-1}{2}u'\psi' P'(\psi)$$

(i,j) component: $$0=\frac{1}{2}\mathrm{e}^u\left\{P(\psi)\left[-du''-\frac{d(d+1)}{4}(u')^2+\frac{d(d-1)\mathrm{e}^{-u}}{l^2}\right]+Q(\psi)\right\}$$
$$-P(\psi)\left\{\frac{d-1}{l^2}+\mathrm{e}^u\left[-\frac{1}{2}u''-\frac{d}{4}(u')^2\right]\right\}$$
$$+\frac{1}{2}\mathrm{e}^u u'\psi' P'(\psi)-\mathrm{e}^u\left[\psi''P'(\psi)+(\psi')^2P''(\psi)+\frac{d-1}{2}u'\psi'P'(\psi)\right]$$

$u'=du(y)/dy$

$u''=d^2u(y)/dy^2$

By choosing $\psi=y$, in case $1/l^2=0$, we obtain

$$0=P''(\psi)-\frac{u'(\psi)}{2}P'(\psi)+\frac{(d-1)u''(\psi)}{2}P(\psi)$$

$$Q(\psi)=\frac{d(d-1)\left(u'(\psi)\right)^2}{4}P(\psi)+(d-1)u'(\psi)P'(\psi)$$

$$u'(\psi)=-\frac{2}{d-1}P(\psi)^{\frac{1}{d-1}}\int d\psi P(\psi)^{-\frac{d}{d-1}}P''(\psi)$$
$$=-\frac{2}{d-1}\left[\frac{P'(\psi)}{P(\psi)}+\frac{d}{d-1}P(\psi)^{\frac{1}{d-1}}\int d\psi P(\psi)^{-\frac{2d-1}{d-1}}\left(P'(\psi)\right)^2\right]$$

$$P(\psi) = U(\psi)^{-2(d-1)} \quad \longrightarrow \quad u'(\psi) = \frac{4U'(\psi)}{U(\psi)} - \frac{8d}{U(\psi)^2}\int d\psi U'(\psi)^2$$

$$\boxed{U(\psi) = U_0\left(\psi^2 + \psi_0^2\right)^{\chi}}$$

$U_0,\ \psi_0,\ \chi$: Constants

$$u'(\psi) = \frac{2\chi\psi}{\psi^2+\psi_0^2} - \frac{32d\chi^2\psi^{4\chi-1}}{\left(\psi^2+\psi_0^2\right)^{2\chi}}\sum_{k=0}^{\infty}\frac{\Gamma(2\chi-1)}{(4\chi-1-2k)\,\Gamma(2\chi-1-k)\,k!}\left(\frac{\psi_0^2}{\psi^2}\right)^k$$

$$\Longrightarrow \quad u'(\psi) = \left(2\chi - \frac{32d\chi^2}{4\chi-1}\right)\frac{1}{\psi} + \left[-2\chi + \frac{64d\chi^3}{4\chi-1} - \frac{64d\chi^2(\chi-1)}{4\chi-3}\right]\frac{\psi_0}{\psi^2} + \mathcal{O}\left(\left(\frac{\psi_0^2}{\psi^2}\right)^2\right)$$

$\psi = y$

$$\Longrightarrow \quad u'(\psi) = \frac{1}{4(6d-1)}\frac{\psi_0}{\psi^2} + \mathcal{O}\left(\left(\frac{\psi_0^2}{\psi^2}\right)^2\right)$$

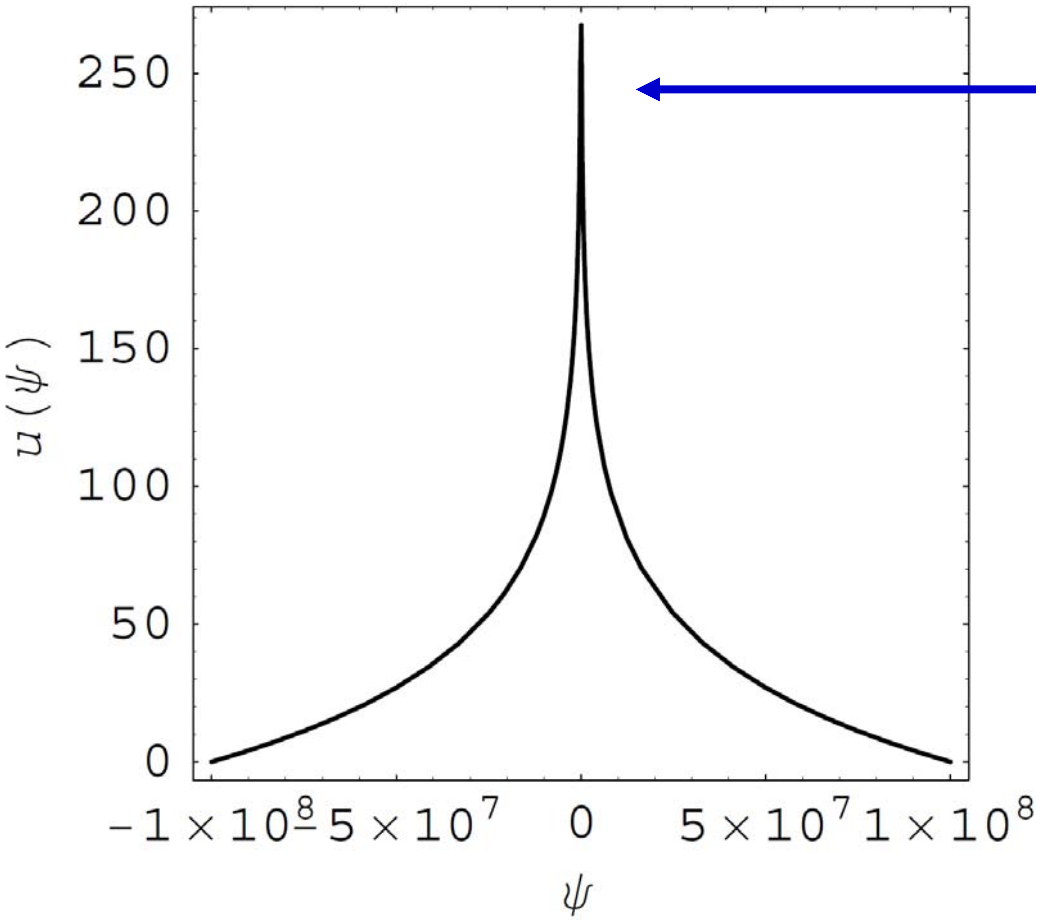
$$\chi = -\frac{1}{4(4d-1)}$$

$\longrightarrow$ By imposing the boundary condition that the universe becomes flat $(u \to 0)$ when $|y| = |\psi| \to \infty$, we find

$$u(\psi) = -\frac{1}{4(6d-1)}\frac{\psi_0}{\psi} + \mathcal{O}\left(\left(\frac{\psi_0}{\psi}\right)^3\right)$$

Since $u(\psi)$ behaves non-trivially when $\psi = y \sim 0$, we may regard that there could be an effective (gravitational) domain wall at $y=0$.

$$u(\psi) = 8\chi \int_{-\infty}^{\psi} d\psi \frac{\psi}{\psi^2+\psi_0^2} - 32d\chi^2 \int_{-\infty}^{\psi} d\psi \frac{1}{(\psi^2+\psi_0^2)^{2\chi}} \int_0^{\psi} d\tilde{\psi} \left(\tilde{\psi}^2+\psi_0^2\right)^{2(\chi-1)} \tilde{\psi}^2$$



$u(\psi)$ has a local maximum around $\psi = y \sim 0$.

There could exist an effective (gravitational) domain wall at $y=0$.

< Reconstruction of an explicit form of $F(R)$ >

$$0 = P'(\psi)R + Q'(\psi)$$

$$\longrightarrow \quad R = -\frac{Q'(\psi)}{P'(\psi)} = -(d-1)\left(\frac{d}{2}\frac{u'(\psi)u''(\psi)}{P'(\psi)} + u''(\psi) + u'(\psi)\frac{P''(\psi)}{P'(\psi)}\right)$$

- We derive an analytic relation $\psi = \psi(R)$. By substituting this relation into $0 = P'(\psi)R + Q'(\psi)$, we can obtain an explicit form of $F(R)$.
- We define $\bar{Y} \equiv \psi^2/\psi_0^2$. For $\bar{Y} = \psi^2/\psi_0^2 \ll 1$, we expand each quantities in terms of $\bar{Y}$ and take leading terms in terms of $\bar{Y}$.
- For $\chi = 1/2$, we acquire an analytic solution:

c_0 : Integration constant

$$u(\psi) = 2(1-2d)\ln\left(\psi^2 + \psi_0^2\right) + 4d\left(\arctan\left(\frac{\psi}{\sqrt{\psi_0^2}}\right)\right)^2 + c_0$$

$$\longrightarrow \quad P(\psi) = (U_0\psi_0)^{-2(d-1)}\left(1+\bar{Y}\right)^{-2(d-1)}$$

$\mathcal{F}_2$: Constant

$\Rightarrow$ **Power-law model**

$$\boxed{F(R) = R + \mathcal{F}_2 R^2}$$

VI. Non-minimal Maxwell-$F(R)$ gravity

< Action >

$$S = \int d^4x \sqrt{-g} \frac{F(R)}{2\kappa^2} + \int d^4x \sqrt{-g} \left(-\frac{1}{4} \underline{I(R)} g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu} F_{\alpha\beta} \right)$$

$$\boxed{I(R) = 1 + \ln\left(\frac{R}{R_0}\right)}$$

$$\boxed{\begin{gathered} \alpha_{\rm EM}(R) = \frac{\alpha_{\rm EM}^{(0)}}{I(R)} \\ \alpha_{\rm EM}^{(0)} = \alpha_{\rm EM}(R_0) \end{gathered}}$$

R_0 : Current curvature

$\alpha_{\rm EM}^{(0)}$: Bare fine structure constant

- It has been found that such a logarithmic-type non-minimal gravitational coupling appears in the effective renormalization-group improved Lagrangian for an SU(2) gauge theory in matter sector for a de Sitter background.

[Elizalde, Odintsov and Romeo, Phys. Rev. D <u>54</u>, 4152 (1996)]

< Observations >

- Keck/HIRES (High Resolution Echelle Spectrometer) quasi-stellar object (QSO) absorption spectra over the redshift range $0.2 < z_{\text{abs}} < 3.7$:

$$\frac{\alpha_{\text{EM}} - \alpha_{\text{EM}}^{(0)}}{\alpha_{\text{EM}}^{(0)}} = \underline{(-0.543 \pm 0.116) \times 10^{-5}} \leftarrow \textbf{Time variation of } \alpha_{\text{EM}}$$

(4.7σ significance level)

[Murphy, Webb and Flambaum, Mon. Not. Roy. Astron. Soc. 345, 609 (2003)]

- Combined dataset from the Keck telescope and the ESO Very Large Telescope (VLT)

$$\frac{\alpha_{\text{EM}} - \alpha_{\text{EM}}^{(0)}}{\alpha_{\text{EM}}^{(0)}} = \underline{(1.10 \pm 0.25) \times 10^{-6} r \cos\Theta \,\text{Glyr}^{-1}} \leftarrow \textbf{Spatial variation of } \alpha_{\text{EM}}$$

(4.2σ significance level)

$r(z) \equiv ct(z)$: Look-back time at redshift z

c : Speed of light

Θ : Angle on the sky between sightline and best-fit dipole position

[Webb, King, Murphy, Flambaum, Carswell and Bainbridge, Phys. Rev. Lett. 107, 191101 (2011)]

< Theoretical estimation (in the Jordan frame) >

$$\boxed{\frac{\alpha_{\rm EM}(R(z=0.21)) - \alpha_{\rm EM}^{(0)}}{\alpha_{\rm EM}^{(0)}} = \underline{-0.364}}$$

$$R/R_0 \approx (1+z)^3$$

$$R(z=0.21)/R_0 \approx 1.77$$

$$H_0 = 2.1h \times 10^{-42}\mathrm{GeV}$$

: Current value of H

$$h = 0.7$$

[Freedman *et al*. [HST Collaboration], Astrophys. J. <u>553</u>, 47 (2001)]

- Naive model of a logarithmic non-minimal gravitational coupling of the electromagnetic field could not satisfy the constraints on the time variation of the fine structure constant from quasar absorption lines and therefore it would be ruled out.

< Relation to a coupling between the electromagnetic field and a scalar field in the Einstein frame >

$$S_{\mathrm{E}} = \int d^4x \sqrt{-\tilde{g}} \left(\frac{\tilde{R}}{2\kappa^2} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) \right) + \int d^4x \sqrt{-\tilde{g}} \left(-\frac{1}{4} \underline{J(\phi)} \tilde{g}^{\mu\alpha}\tilde{g}^{\nu\beta} F_{\mu\nu}F_{\alpha\beta} \right)$$

$$\boxed{\begin{aligned} J(\phi) \equiv{}& \mathrm{e}^{-2/\sqrt{3}\kappa\phi} \left(I(\tilde{R}) - \frac{dI(\tilde{R})}{d\tilde{R}}\tilde{R} \right) \\ &+ \mathrm{e}^{-1/\sqrt{3}\kappa\phi} \frac{dI(\tilde{R})}{d\tilde{R}} \left[\tilde{R} + \sqrt{3}\tilde{\Box}(\kappa\phi) - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu(\kappa\phi)\partial_\nu(\kappa\phi) \right] \end{aligned}}$$

- If the scalar curvature can be expressed by ϕ, J can be described as a function of ϕ. $\longrightarrow$ $J(\phi) = B(\phi)$

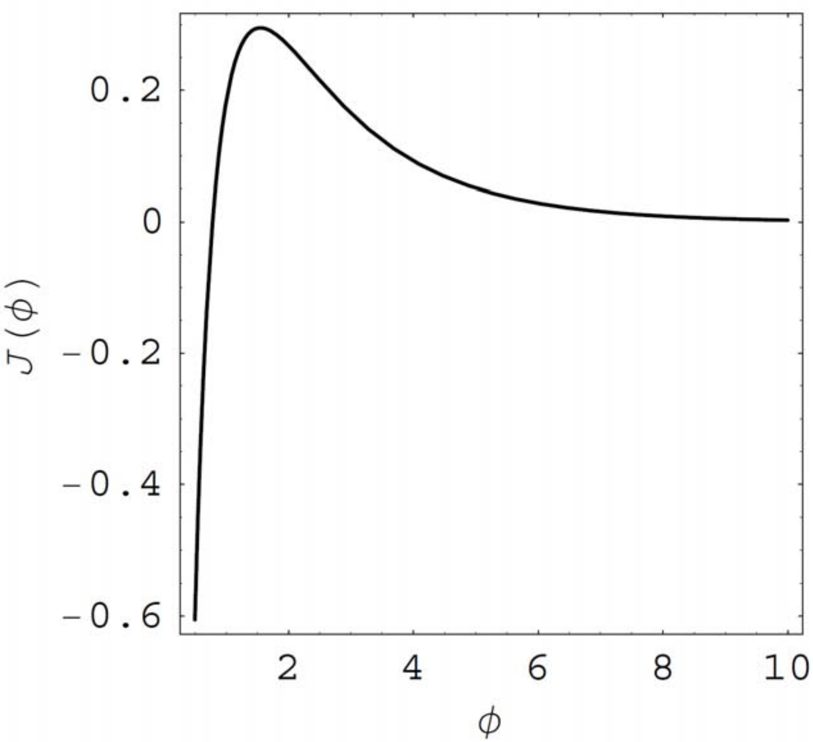
- We can obtain the relation between a non-minimal gravitational coupling of the electromagnetic field in the Jordan frame and a coupling of the electromagnetic field to a scalar field in the Einstein frame.

< Case for an exponential model >

$$F(R) = F_+ \mathrm{e}^{\lambda_+ R}$$

$$\alpha_{\mathrm{EM}}(\phi) = \frac{\alpha_{\mathrm{EM}}^{(0)}}{J(\phi)}$$

$$J(\phi) = \mathrm{e}^{-2/\sqrt{3}\kappa\phi} \ln\left(\frac{R}{R_0}\right) + \mathrm{e}^{-1/\sqrt{3}\kappa\phi}\left[1 - 3\sqrt{3}\left(\frac{\phi}{y_0}\right)\mathrm{e}^{-(\phi/y_0)^2}\frac{u_0}{y_0}\frac{\kappa}{R}\left(\frac{d\phi}{dy}\right) - \frac{1}{2}\frac{\kappa^2}{R}\left(\frac{d\phi}{dy}\right)^2\right]$$



$$R_0 = (1/\lambda_+)\left\{\ln\left[1/\left(F_+\lambda_+\right)\right] + \sqrt{2/3}\kappa\phi_{\mathrm{p}}\right\}$$

$$\phi(z = 0.21) = \left(R(z = 0.21)/R_0\right)\phi_{\mathrm{p}}$$

$$R = \sqrt{2/3}\phi$$

$$F_+ = 1,\ \lambda_+ = 1,\ u_0 = 1,\ y_0 = \phi_0 = 10,$$

$$\phi_{\mathrm{p}} = 1$$

$$\frac{\alpha_{\mathrm{EM}}(\phi(z = 0.21)) - \alpha_{\mathrm{EM}}^{(0)}}{\alpha_{\mathrm{EM}}^{(0)}} = \frac{1}{J(\phi(z = 0.21))} - 1 = \underline{1.38}$$

< Case for a power-law model >

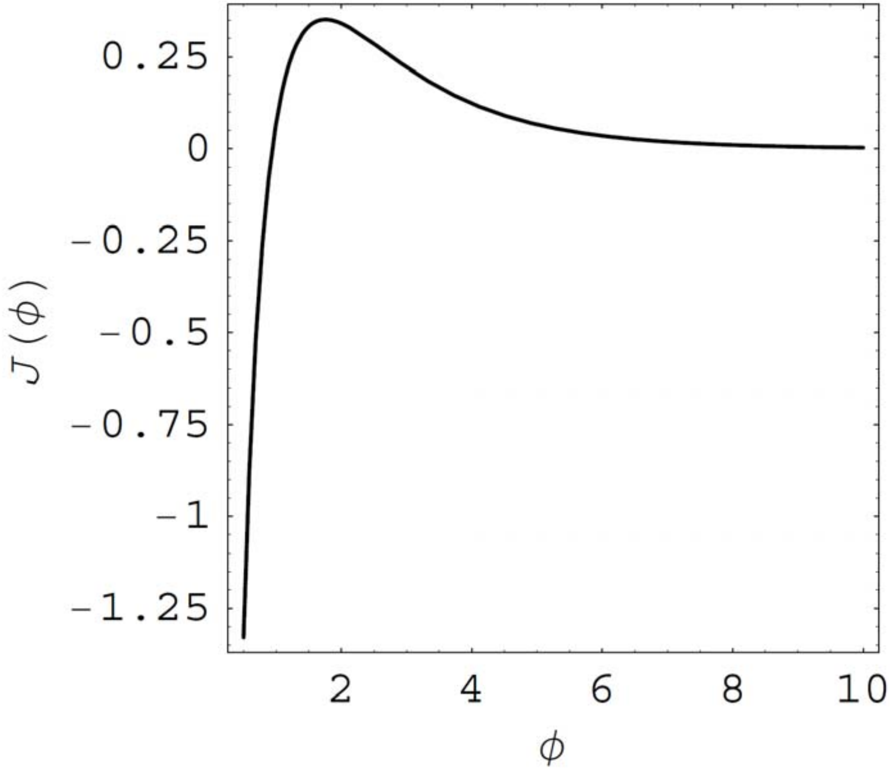
$$F(R) = R + \mathcal{F}_2 R^2$$

$$R = \frac{1}{2\mathcal{F}_2}\left(\mathrm{e}^{\sqrt{2/3}\kappa\phi} - 1\right)$$

$$R_0 = \left(\mathrm{e}^{\sqrt{2/3}\kappa\phi_\mathrm{p}} - 1\right) / (2\mathcal{F}_2)$$

$$\phi(z = 0.21) = 1.44$$

$$J(\phi(z = 0.21)) = 0.632$$



$$R = \left(\mathrm{e}^{\sqrt{2/3}\phi} - 1\right)/2$$

$$J(\phi(z = 0.21)) = 0.420$$

$$\phi(z = 0.21) =$$

$$\sqrt{3/2}\ln\left[(R(z = 0.21)/R_0)\left(\mathrm{e}^{\sqrt{2/3}\phi_\mathrm{p}} - 1\right) + 1\right]$$

$$\mathcal{F}_2 = 1,\ u_0 = 1,\ y_0 = \phi_0 = 10,$$

$$\phi_\mathrm{p} = 1$$

$$\frac{\alpha_\mathrm{EM}(\phi(z = 0.21)) - \alpha_\mathrm{EM}^{(0)}}{\alpha_\mathrm{EM}^{(0)}} = \frac{1}{J(\phi(z = 0.21))} - 1 = \underline{0.583}$$

VII. Summary

- We have studied a domain wall solution in $F(R)$ gravity.
  - Static domain wall solution in a scalar field theory.
  - Explicit $F(R)$ gravity model in which a static domain wall solution can be realized.
- We have shown that there could exist an effective (gravitational) domain wall in the framework of $F(R)$ gravity.
- It has been demonstrated that a logarithmic non-minimal gravitational coupling of the electromagnetic theory in $F(R)$ gravity may produce time-variation of the fine structure constant which may increase with decrease of the curvature.

< Cosmological consequences of the coupling of the electromagnetic field to not only a scalar field but also the scalar curvature >

$$S_{\mathrm{E}} = \int d^4x \sqrt{-\tilde{g}} \left(\frac{\tilde{R}}{2\kappa^2} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) \right) + \int d^4x \sqrt{-\tilde{g}} \left(-\frac{1}{4}\Upsilon(\phi, \tilde{R})\tilde{g}^{\mu\alpha}\tilde{g}^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} \right)$$

$$+ S_{\mathrm{matter}}$$

$\Upsilon(\phi, \tilde{R})$: Arbitrary function of ϕ and R

- A domain wall can be used to account for the spatial variation through a scalar field coupled to electromagnetism, whereas the non-minimal gravitational coupling of the electromagnetic field to the scalar curvature can explain the time variation of the fine structure constant. Thus, there exist more choices of the scalar field potential which can make a domain wall.

- The conformal invariance of the electromagnetic field can be broken and therefore large-scale magnetic fields can be generated.

$$S=\int d^4x\sqrt{-g}\left(\frac{F(R)}{2\kappa^2}-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V(\phi)\right)+\int d^4x\sqrt{-g}\left(-\frac{1}{4}\underline{\Upsilon(\phi,R)}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}\right)$$
$$+S_{\mathrm{matter}}$$

- Power-law inflation can occur due to the non-minimal gravitational coupling of the electromagnetic field as well as the deviation of *F*(*R*) gravity from general relativity and the late-time accelerated expansion of the universe can also be realized through the modified part of *F*(*R*) gravity in a unified model action.

- In the scalar-tensor sector of the theory, the domain wall may be created due to combined effect of scalar potential and modified gravity. Then, combined effect of scalar and curvature in the non-minimal electromagnetic sector gives us wider possibility for realizing the time-variation of the fine structure constant in accordance with observational data.

Backup slides

$$\longrightarrow \quad \rho = \frac{\omega(\varphi)}{2}(\varphi')^2 + \mathcal{V}(\varphi) = -\frac{d-1}{2}u'' - \frac{d(d-1)}{8}(u')^2 + \frac{(d-1)(d-2)}{2l^2}\mathrm{e}^{-u}$$

- We assume Z_2 symmetry of the metric, which is the invariance under the transformation $y \to -y$.

$\longrightarrow$ There must be a region where $\omega(\varphi)$ becomes negative and therefore φ becomes a ghost. ρ often becomes negative.

- Example: $\boxed{u = u_0 \mathrm{e}^{-y^2/y_0^2}}$ $\quad u_0,\ y_0$: Constants

$$\Rightarrow \quad \omega(\varphi) = -(d-1)\left(\frac{2\varphi^2}{y_0^4} - \frac{1}{y_0^2}\right)\mathrm{e}^{-\varphi^2/y_0^2} - \frac{(d-1)}{l^2}\mathrm{e}^{-u_0\mathrm{e}^{-\varphi^2/y_0^2}}$$

$$\mathcal{V}(\varphi) = -\frac{d-1}{2}\left(\frac{2\varphi^2}{y_0^4} - \frac{1}{y_0^2}\right)\mathrm{e}^{-\varphi^2/y_0^2} + \frac{(d-1)^2}{l^2}\mathrm{e}^{-u_0\mathrm{e}^{-\varphi^2/y_0^2}}$$

$$\longrightarrow \quad \rho(y) = -\frac{d-1}{2}\left(\frac{2y^2}{y_0^4} - \frac{1}{y_0^2}\right)\mathrm{e}^{-y^2/y_0^2} + \frac{(d-1)^2}{l^2}\mathrm{e}^{-u_0\mathrm{e}^{-y^2/y_0^2}}$$

$\rho(y)$ is localized at $y \sim 0$ and makes a domain wall.

Gravitational field equation

(y,y) component: $$\frac{d-1}{2}u'\left(F_{,R}\right)' - \frac{d}{2}\left[u''+\frac{1}{2}\left(u'\right)^2\right]F_{,R} - \frac{1}{2}F = \kappa^2 T^{(\mathrm{M})}_{yy}$$

(μ,ν) component: $$\left(F_{,R}\right)' \equiv dF_{,R}/dy\,, \qquad \left(F_{,R}\right)'' \equiv d^2F_{,R}/dy^2$$

$$d\left(F_{,R}\right)'' + \frac{d(d-2)}{2}u'\left(F_{,R}\right)' + \left\{-\frac{d}{2}\left[u''+\frac{d}{2}\left(u'\right)^2\right] + \frac{d(d-1)}{l^2}\mathrm{e}^{-u}\right\}F_{,R} - \frac{d}{2}F$$

$$= \kappa^2 \sum_{\mu,\nu=0}^{d-1} g^{\mu\nu}T^{(\mathrm{M})}_{\mu\nu}$$

$$R = -\frac{d}{2}\left[2u''+\frac{1+d}{2}\left(u'\right)^2\right] + \frac{d(d-1)}{l^2}\mathrm{e}^{-u}$$

$$-\frac{d}{2}\left[u''+\frac{1}{2}\left(u'\right)^2\right] - \frac{R}{2} = \kappa^2\left(T^{(\mathrm{M})}_{yy} + T^{(\mathrm{D})}_{yy}\right)$$

$$-\frac{d}{2}\left[u''+\frac{d}{2}\left(u'\right)^2\right] + \frac{d(d-1)}{l^2}\mathrm{e}^{-u} - \frac{d}{2}R = \kappa^2\left(\sum_{\mu,\nu=0}^{d-1} g^{\mu\nu}T^{(\mathrm{M})}_{\mu\nu} + \sum_{\mu,\nu=0}^{d-1} g^{\mu\nu}T^{(\mathrm{D})}_{\mu\nu}\right)$$

$$\kappa^2 T^{(\mathrm{D})}_{yy} \equiv -\frac{d-1}{2}u'\left(F_{,R}\right)' + \frac{d}{2}\left[u''+\frac{1}{2}\left(u'\right)^2\right]\left(F_{,R}-1\right) + \frac{1}{2}\left(F-R\right)$$

$$\kappa^2 \sum_{\mu,\nu=0}^{d-1} g^{\mu\nu}T^{(\mathrm{D})}_{\mu\nu} \equiv -d\left(F_{,R}\right)'' - \frac{d(d-2)}{2}u'\left(F_{,R}\right)' + \left\{\frac{d}{2}\left[u''+\frac{d}{2}\left(u'\right)^2\right] - \frac{d(d-1)}{l^2}\mathrm{e}^{-u}\right\}\left(F_{,R}-1\right) + \frac{d}{2}\left(F-R\right)$$

- We derive an explicit form of $F(R)$ realizing a domain wall solution.

$$u = u(y) \longrightarrow y = y(R) \longrightarrow u = u(y(R))$$

* We consider the case in which there is no matter.

$$\boxed{(y, y)\,\text{component:} \quad \Xi_1(R)\frac{d^2F(R)}{dR^2} + \Xi_2(R)\frac{dF(R)}{dR} - F(R) = 0}$$

$$\Xi_1(R) \equiv (d-1)\,u'\frac{dR}{dy} = (d-1)\left(\frac{dR}{dy}\right)^2 \frac{du(y(R))}{dR}$$

$$\Xi_2(R) \equiv (-d)\left[u'' + \frac{1}{2}(u')^2\right] = (-d)\left[\frac{d^2R}{dy^2}\frac{du(y(R))}{dR} + \left(\frac{dR}{dy}\right)^2 \frac{d^2u(y(R))}{dR^2} + \frac{1}{2}\left(\frac{dR}{dy}\right)^2\left(\frac{du(y(R))}{dR}\right)^2\right]$$

- We solve the equation of the scalar curvature R in terms of y.

$\longrightarrow$ We define $Y \equiv y^2/y_0^2$. For $Y = y^2/y_0^2 \ll 1$, we expand exponential terms and take the first leading terms in terms of Y.

$$\Rightarrow \quad Y \equiv \frac{y^2}{y_0^2} \approx \frac{R - \gamma_1}{\gamma_2}$$

$$\gamma_1 \equiv 2d\frac{u_0}{y_0^2} + \frac{d\,(d-1)}{l^2}$$

$$\gamma_2 \equiv -d\frac{u_0}{y_0^2}\left[6 + (1+d)\,u_0\right] + \frac{d\,(d-1)}{l^2}u_0$$

$$\frac{dR}{dy} \approx \zeta_1 + \zeta_2 \frac{y^2}{y_0^2} \qquad \zeta_1 \equiv d\frac{u_0}{y_0^3}\left(1 + \frac{1+d}{2}u_0\right) + \frac{d\,(d-1)}{l^2}\frac{u_0}{y_0}$$

$$\zeta_2 \equiv -d\frac{u_0}{y_0^3}\left[1 + (1+d)\,u_0\right] - \frac{d\,(d-1)}{l^2}\frac{u_0\,(u_0+1)}{y_0}$$

$$\frac{d^2R}{dy^2} \approx \eta_1 + \eta_2 \frac{y^2}{y_0^2} \qquad \eta_1 \equiv -d\frac{u_0}{y_0^4}\left[1 + (1+d)\,u_0\right] - \frac{d\,(d-1)}{l^2}\frac{u_0\,(1-u_0)\,\mathrm{e}^{-u_0}}{y_0^2}$$

$$\eta_2 \equiv d\frac{u_0}{y_0^4}\left[1 + 2\,(1+d)\,u_0\right] + \frac{d\,(d-1)}{l^2}\frac{u_0\,(u_0^2 - 3u_0 + 1)}{y_0^2}$$

$$\Rightarrow \boxed{\Xi_i(R) = \Xi_i^{(0)} + \Xi_i^{(1)}Y = \Xi_i^{(0)} - \frac{\gamma_1}{\gamma_2}\Xi_i^{(1)} + \Xi_i^{(1)}R}$$

$$\Xi_1^{(0)} \equiv (d-1)\left(-\frac{u_0}{\gamma_2}\zeta_1^2\right) \qquad \Xi_1^{(1)} \equiv (d-1)\,\frac{u_0}{\gamma_2}\zeta_1\left(\zeta_1 - 2\zeta_2\right)$$

$$\Xi_2^{(0)} \equiv (-d)\left[-\frac{u_0}{\gamma_2}\eta_1 + \frac{u_0}{\gamma_2^2}\zeta_1^2\left(1 + \frac{u_0}{2}\right)\right]$$

$$\Xi_2^{(1)} \equiv (-d)\left\{\frac{u_0}{\gamma_2}\left(\eta_1 - \eta_2\right) - \zeta_1\frac{u_0}{\gamma_2^2}\left[\zeta_1\left(1+u_0\right) - 2\zeta_2\left(1 + \frac{u_0}{2}\right)\right]\right\}$$

- For $Y = y^2/y_0^2 \ll 1$, when $\Xi_i^{(1)}/\Xi_i^{(0)} \lesssim \mathcal{O}(1)$, we can consider $\Xi_i \approx \Xi_i^{(0)} (= \text{constant})$.

$$\Rightarrow \boxed{\frac{d^2 F(R)}{dR^2} + \mathcal{C}\frac{dF(R)}{dR} + \mathcal{D}F(R) = 0 \ , \quad \mathcal{C} \equiv \frac{\Xi_2^{(0)}}{\Xi_1^{(0)}} \ , \quad \mathcal{D} \equiv -\frac{1}{\Xi_1^{(0)}}}$$

A general solution

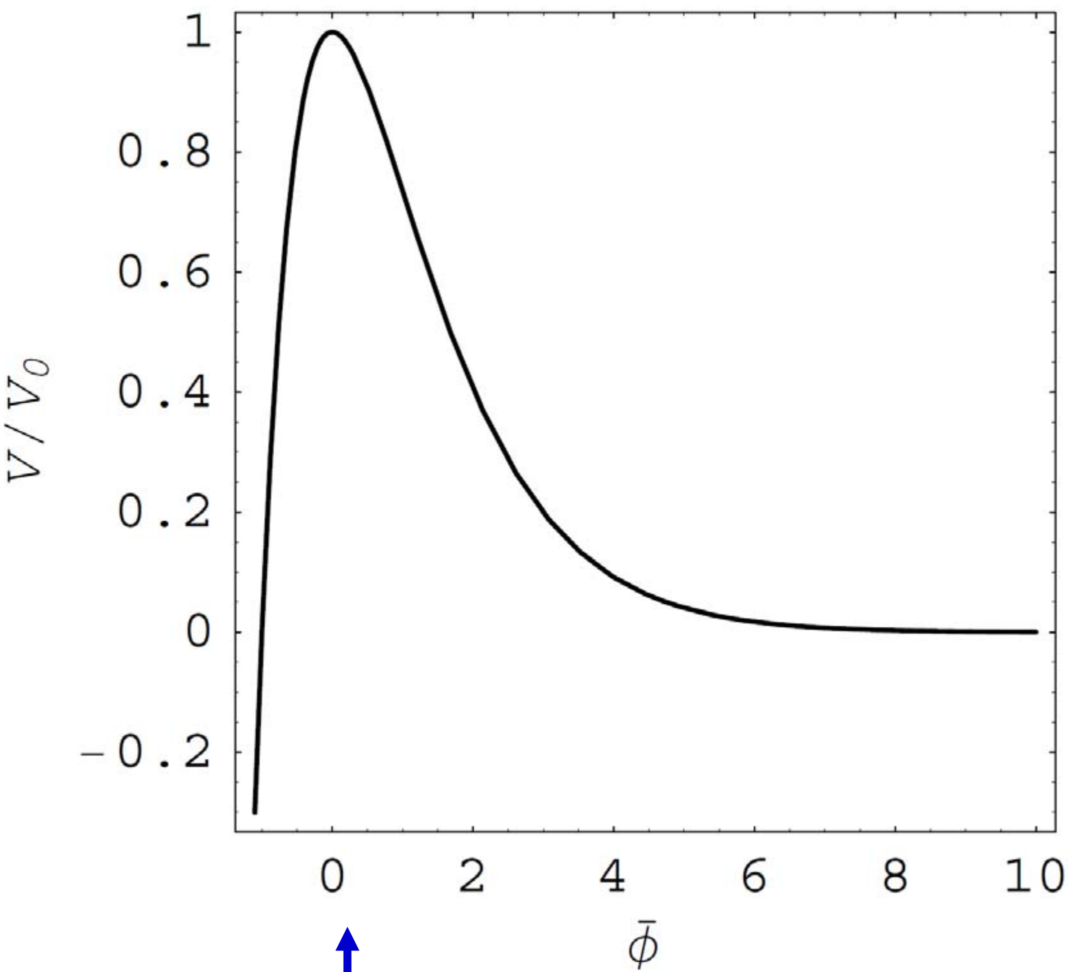
$$\to \quad F(R) = F_+ \mathrm{e}^{\lambda_+ R} + F_- \mathrm{e}^{\lambda_- R}$$

$$\lambda_\pm \equiv \frac{1}{2}\left(-\mathcal{C} \pm \sqrt{\mathcal{C}^2 - 4\mathcal{D}}\right)$$

Exponential model

$$\Rightarrow \boxed{F(R) = F_+ \mathrm{e}^{\lambda_+ R}} \qquad R = \frac{1}{\lambda_+}\left[\ln\left(\frac{1}{F_+\lambda_+}\right) + \sqrt{\frac{2}{3}}\kappa\phi\right]$$

$$V(\phi) = \frac{1}{2\kappa^2\lambda_+}\mathrm{e}^{-\sqrt{2/3}\kappa\phi}\left[\sqrt{\frac{2}{3}}\kappa\phi + \ln\left(\frac{1}{F_+\lambda_+}\right) - 1\right]$$



$$\boxed{V(\bar{\phi}) = V_0 \mathrm{e}^{-\bar{\phi}} \left(\bar{\phi} + \bar{\phi}_0\right)}$$

$$\bar{\phi} \equiv \sqrt{2/3}\kappa\phi$$

$$\bar{\phi}_0 \equiv \ln\left[1/\left(F_+\lambda_+\right)\right] - 1$$

$$V_0 \equiv 1/\left(2\kappa^2\lambda_+\right)$$

For $\bar{\phi}_0 = 1 \ (F_+\lambda_+ = 1/\mathrm{e}^2)$

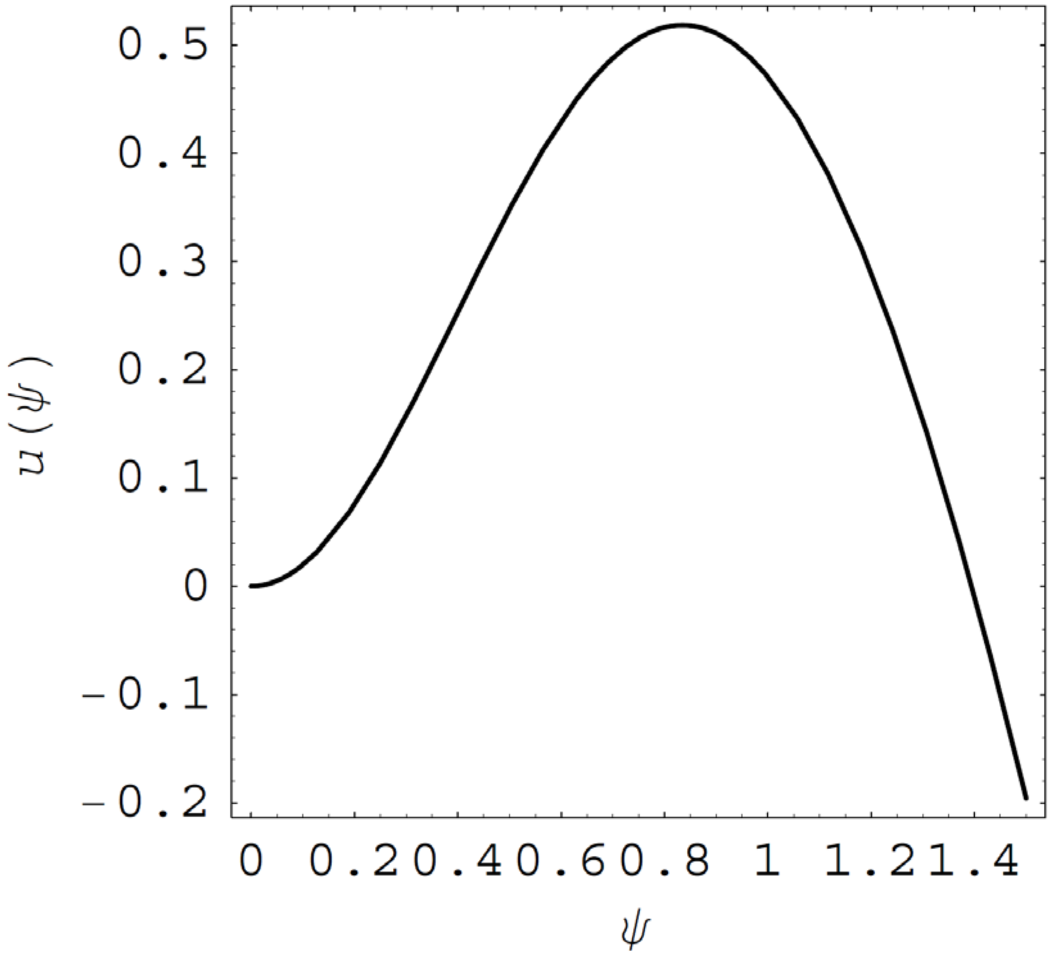
The potential energy is localized at $\bar{\phi} \sim 0$.

For $\chi = 1/2$, we acquire an analytic solution:

$$u(\psi) = 2(1-2d)\ln\left(\psi^2+\psi_0^2\right) + 4d\left(\arctan\left(\frac{\psi}{\sqrt{\psi_0^2}}\right)\right)^2 + c_0$$

$\psi \sim 0.8$

$\psi_0 = 1, \ c_0 = 0$



In the range of $|\psi| \lesssim 1.4$, the distribution of the energy density is localized.

This configuration could be regarded as an effective (gravitational) domain wall.

< Reconstruction of an explicit form of $F(R)$ >

$$0 = P'(\psi)R + Q'(\psi)$$

$$R = -\frac{Q'(\psi)}{P'(\psi)} = -(d-1)\left(\frac{d}{2}\frac{u'(\psi)u''(\psi)}{P'(\psi)} + u''(\psi) + u'(\psi)\frac{P''(\psi)}{P'(\psi)}\right)$$

$$\bar{Y} \equiv \psi^2/\psi_0^2 \qquad \bar{Y} = \psi^2/\psi_0^2 \ll 1 \qquad \psi = \psi(R)$$

For $\chi = 1/2$, $\quad P(\psi) = (U_0\psi_0)^{-2(d-1)}\left(1+\bar{Y}\right)^{-2(d-1)}$

$$R = \mathcal{R}_0 + \mathcal{R}_1\bar{Y} \qquad \mathcal{R}_0 \equiv \frac{4(d-1)}{\psi_0^2}\left[\frac{d}{(d-1)}\frac{1}{(U_0\psi_0)^{-2(d-1)}} - 2\right]$$

$$\mathcal{R}_1 \equiv -\frac{4(d-1)}{\psi_0^2}\left[\frac{d}{(d-1)}\frac{1}{(U_0\psi_0)^{-2(d-1)}}\left(4+\frac{5d}{3}\right) - \left(4+\frac{14d}{3}\right)\right]$$

$$Q = \mathcal{Q}_1\bar{Y} + \mathcal{Q}_2\bar{Y}^2 \qquad \mathcal{Q}_1 \equiv \frac{4(d-1)}{\psi_0^2}\left[d - 2(d-1)(U_0\psi_0)^{-2(d-1)}\right]$$

$$\mathcal{Q}_2 \equiv \frac{8(d-1)}{\psi_0^2}\left[-d\left(1+\frac{2d}{3}\right) + (U_0\psi_0)^{-2(d-1)}(d-1)\left(1+\frac{5d}{3}\right)\right]$$

$$\bar{Y} = \bar{Y}_0 + \bar{Y}_1 R \,, \qquad \bar{Y}_0 \equiv -\frac{\mathcal{R}_0}{\mathcal{R}_1} \,, \qquad \bar{Y}_1 \equiv \frac{1}{\mathcal{R}_1}$$

$$P(\psi) \approx (U_0\psi_0)^{-2(d-1)} \left\{1 - (d-1)\,\bar{Y} + [d\,(d-1)\,/2]\,\bar{Y}^2\right\}$$

$$F(R) = \mathcal{F}_0 + \mathcal{F}_1 R + \mathcal{F}_2 R^2$$

$$\mathcal{F}_0 \equiv \mathcal{Q}_1 \bar{Y}_0 + \mathcal{Q}_2 \bar{Y}_0^2$$

$$\mathcal{F}_1 \equiv (U_0\psi_0)^{-2(d-1)} \left[1 - (d-1)\,\bar{Y}_0 + \frac{d\,(d-1)}{2}\bar{Y}_0^2\right] + \mathcal{Q}_1 \bar{Y}_1 + 2\mathcal{Q}_2 \bar{Y}_0 \bar{Y}_1$$

$$\mathcal{F}_2 \equiv (U_0\psi_0)^{-2(d-1)} (d-1)\,\bar{Y}_1 \left(-1 + d\bar{Y}_0\right) + \mathcal{Q}_2 \bar{Y}_1^2$$

Power-law model

$$\Rightarrow \boxed{F(R) = R + \mathcal{F}_2 R^2}$$

$$R \sim \mathcal{O}(1)$$

$$\mathcal{R}_0 \sim \mathcal{O}(1)$$

$$\bar{Y} = \psi^2/\psi_0^2 \ll 1$$

- In Sec. IV, we first suppose the existence of a static domain wall solution in $F(R)$ gravity. Then, through the comparison of gravitational field equations in $F(R)$ gravity with those in a scalar field theory in general relativity, we reconstruct an explicit form of $F(R)$.

- In Sec. V, by using the reconstruction method of $F(R)$ gravity, we directly show that the distribution of the energy density could be localized and hence such a configuration could be regarded as an effective (gravitational) domain wall. Here, a domain wall solution obtained in Sec. V is realized by a pure gravitational effect.

- An effective (gravitational) domain wall in Sec. V is realized by a pure gravitational effect. On the other hand, a static domain wall solution is made by a scalar field. In Sec. IV, the deviation of $F(R)$ gravity from general relativity contributes to the energy-momentum tensor geometrically, and eventually it plays an equivalent role of matter, such as a scalar field in Sec. III.

VI. Non-minimal Maxwell-*F*(*R*) gravity

< Action >

$$S = \int d^4x \sqrt{-g} \frac{F(R)}{2\kappa^2} + \int d^4x \sqrt{-g} \left(-\frac{1}{4} \underline{I(R)} g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu} F_{\alpha\beta} \right)$$

$$\boxed{I(R) = 1 + \ln\left(\frac{R}{R_0}\right)}$$

$$\boxed{\begin{aligned} \alpha_{\mathrm{EM}}(R) &= \frac{\alpha_{\mathrm{EM}}^{(0)}}{I(R)} \\ \alpha_{\mathrm{EM}}^{(0)} &= \alpha_{\mathrm{EM}}(R_0) \end{aligned}}$$

R_0 : Current curvature

$\alpha_{\mathrm{EM}}^{(0)}$: Bare fine structure constant

- It has been found that such a logarithmic-type non-minimal gravitational coupling appears in the effective renormalization-group improved Lagrangian for an SU(2) gauge theory in matter sector for a de Sitter background.

[Elizalde, Odintsov and Romeo, Phys. Rev. D <u>54</u>, 4152 (1996)]

This comes from the running gauge coupling constant with the asymptotic freedom in a non-Abelian gauge theory, which approaches zero in very high energy regime.

< Theoretical estimation (in the Jordan frame) >

$$\frac{\alpha_{\mathrm{EM}}(R(z=0.21)) - \alpha_{\mathrm{EM}}^{(0)}}{\alpha_{\mathrm{EM}}^{(0)}} = \underline{-0.364}$$

$$R/R_0 \approx (1+z)^3$$

$$R(z=0.21)/R_0 \approx 1.77$$

$$H_0 = 2.1h \times 10^{-42}\mathrm{GeV}$$

: Current value of H

$$h = 0.7$$

[Freedman *et al*. [HST Collaboration], Astrophys. J. 553, 47 (2001)]

- Naive model of a logarithmic non-minimal gravitational coupling of the electromagnetic field could not satisfy the constraints on the time variation of the fine structure constant from quasar absorption lines and therefore it would be ruled out.

- It is not possible to estimate the spatial variation of α_{EM} and only the time-variation of it could be estimated.

V. Summary

- **We have studied a domain wall solution in $F(R)$ gravity.**
    - **Static domain wall solution in a scalar field theory.**
    - **Explicit $F(R)$ gravity model in which a static domain wall solution can be realized.**
- **We have shown that there could exist an effective (gravitational) domain wall in the framework of $F(R)$ gravity.**
- It has been demonstrated that a logarithmic non-minimal gravitational coupling of the electromagnetic theory in $F(R)$ gravity may produce time-variation of the fine structure constant which may increase with decrease of the curvature.
- We have presented cosmological consequences of the coupling of the electromagnetic field to a scalar field as well as the scalar curvature.

< Flat Friedmann-Lemaître-Robertson-Walker (FLRW) space-time >

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) d\boldsymbol{x}^2$$

$a(t)$: Scale factor

< Equation for $a(t)$ with a perfect fluid >

$$T_{\mu\nu} = \mathrm{diag}(\rho, P, P, P)$$

$$\frac{\ddot{a}}{a} = -\frac{\kappa^2}{6}\underline{(1+3w)\,\rho}$$

ρ : Energy density

P : Pressure

$\dot{} = \partial/\partial t$

$$\boxed{w \equiv \frac{P}{\rho}}$$: **Equation of state (EoS)**

$\ddot{a} > 0$: **Accelerated expansion**

$$\Longrightarrow \boxed{w < -\frac{1}{3}}$$: **Condition for accelerated expansion**

Cf. Cosmological constant $\Longrightarrow w = -1$

< $f(R)$ gravity >

$$S = \int d^4x \sqrt{-g}\, \frac{f(R)}{2\kappa^2}$$

$f(R)$ gravity

$f(R) = R$: General Relativity

[Nojiri and Odintsov, Phys. Rept. 505, 59 (2011) [arXiv:1011.0544 [gr-qc]]; Int. J. Geom. Meth. Mod. Phys. 4, 115 (2007) [arXiv:hep-th/0601213]]

[Capozziello and Francaviglia, Gen. Rel. Grav. 40, 357 (2008)]

[Sotiriou and Faraoni, Rev. Mod. Phys. 82, 451 (2010)]

[De Felice and Tsujikawa, Living Rev. Rel. 13, 3 (2010)]

< Gravitational field equation >

$f'(R) = df(R)/dR$

$$f'(R)R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f(R) + g_{\mu\nu}\Box f'(R) - \nabla_\mu\nabla_\nu f'(R) = 0$$

$\Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$: Covariant d'Alembertian

∇_μ : Covariant derivative operator

- In the flat FLRW background, gravitational field equations read

$$H^2 = \frac{\kappa^2}{3}\rho_{\rm eff}\,, \quad \dot{H} = -\frac{\kappa^2}{2}\left(\rho_{\rm eff} + p_{\rm eff}\right)$$

$\rho_{\rm eff}$, $p_{\rm eff}$: Effective energy density and pressure from the term $f(R) - R$

$$\rho_{\rm eff} = \frac{1}{\kappa^2 f'(R)}\left[\frac{1}{2}\left(-f(R) + Rf'(R)\right) - 3H\dot{R}f''(R)\right]$$

$$p_{\rm eff} = \frac{1}{\kappa^2 f'(R)}\left[\frac{1}{2}\left(f(R) - Rf'(R)\right) + \left(2H\dot{R} + \ddot{R}\right)f''(R) + \dot{R}^2 f'''(R)\right]$$

$$\longrightarrow \boxed{w_{\rm eff} = \frac{p_{\rm eff}}{\rho_{\rm eff}} = \frac{\left(f(R) - Rf'(R)\right)/2 + \left(2H\dot{R} + \ddot{R}\right)f''(R) + \dot{R}^2 f'''(R)}{\left(-f(R) + Rf'(R)\right)/2 - 3H\dot{R}f''(R)}}$$

$H = \dot{a}/a$: Hubble parameter

- Example : $f(R) \propto R^n \ (n \neq 1)$

$$\longrightarrow a \propto t^q, \quad q = \frac{-2n^2+3n-1}{n-2}$$

$$w_{\rm eff} = -\frac{6n^2-7n-1}{6n^2-9n+3}$$

[Capozziello, Carloni and Troisi, Recent Res. Dev. Astron. Astrophys. <u>1</u>, 625 (2003)]

If $q > 1$, accelerated expansion can be realized.

(For $n = 3/2$ or $n = -1$, $q = 2$ and $w_{\rm eff} = -2/3$.)

- In the flat FLRW background, gravitational field equations read

$$H^2 = \frac{\kappa^2}{3}\rho_{\rm eff}\,, \quad \dot{H} = -\frac{\kappa^2}{2}\left(\rho_{\rm eff} + p_{\rm eff}\right)$$

$\rho_{\rm eff}$, $p_{\rm eff}$: Effective energy density and pressure from the term $f(R) - R$

$$\rho_{\rm eff} = \frac{1}{\kappa^2 f'(R)}\left[\frac{1}{2}\left(-f(R) + Rf'(R)\right) - 3H\dot{R}f''(R)\right]$$

$$p_{\rm eff} = \frac{1}{\kappa^2 f'(R)}\left[\frac{1}{2}\left(f(R) - Rf'(R)\right) + \left(2H\dot{R} + \ddot{R}\right)f''(R) + \dot{R}^2 f'''(R)\right]$$

$$\rightarrow \boxed{w_{\rm eff} = \frac{p_{\rm eff}}{\rho_{\rm eff}} = \frac{\left(f(R) - Rf'(R)\right)/2 + \left(2H\dot{R} + \ddot{R}\right)f''(R) + \dot{R}^2 f'''(R)}{\left(-f(R) + Rf'(R)\right)/2 - 3H\dot{R}f''(R)}}$$

- Example: $f(R) = R \underline{- \frac{\mu^{2(n+1)}}{R^n}}$ **[Carroll, Duvvuri, Trodden and Turner, Phys. Rev. D 70, 043528 (2004)]**

μ : Mass scale, n : Constant

Second term become important as R decreases.

$\Rightarrow$ $\underline{a \propto t^q}$, $\quad q = \frac{(2n+1)(n+1)}{n+2}$

$$w_{\rm eff} = -1 + \frac{2(n+2)}{3(2n+1)(n+1)}$$

(For $n = 1$, $\underline{q = 2 \text{ and } w_{\rm eff} = -2/3}$.)

If $q > 1$, accelerated expansion can be realized.