Perturbations in Lee-Wick Bouncing Universe

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- PRD 82, 025013 (2010)
- JCAP, 1111:043 (2011)
with O-Kab Kwon (Sungkyunkwan Univ)

Outline

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    - B. Lee-Wick Bouncing Universe
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    - A. Series Expansion about bouncing point
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4. Tensor Perturbation
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HD: Higher-Derivative Field Theory → LW: Lee-Wick Form

:- In String Theory, infinite number of field derivative is accompanied
e.g.) tachyon from open string field theory, p-adic string theory, etc.

:- Quantum mechanical system (Pais-Uhlenbeck, 1950)

:- N-th order HD Lag → N Scalar Fields (ordinary fields + Lee-Wick partners)
→ LW-partner is ghost: but safe b/c decays early to ordinary particles
('69 Lee-Wick)

HD Lagrangian

HD parameters

$$\mathcal{L}_{\text{HD}}^{(N)} = \frac{1}{2}\sum_{n=1}^{N}(-1)^{n+1} a_n \phi \Box^n \phi - \frac{1}{2} m^2 \phi^2 + \mathcal{L}_{\text{int}}(\phi)$$

Transformations

Lee-Wick Lagrangian

LW parameters

$$\mathcal{L}_{\text{LW}}^{(N)} = \frac{1}{2}\sum_{n=1}^{N} \kappa_n \psi_n (\Box - \mu_n) \psi_n + \mathcal{L}_{\text{int}}(\psi),$$

$$\mu_1 < \mu_2 < \mu_3 < \ldots < \mu_N .$$

sign

LW Field

: assume no degeneracy

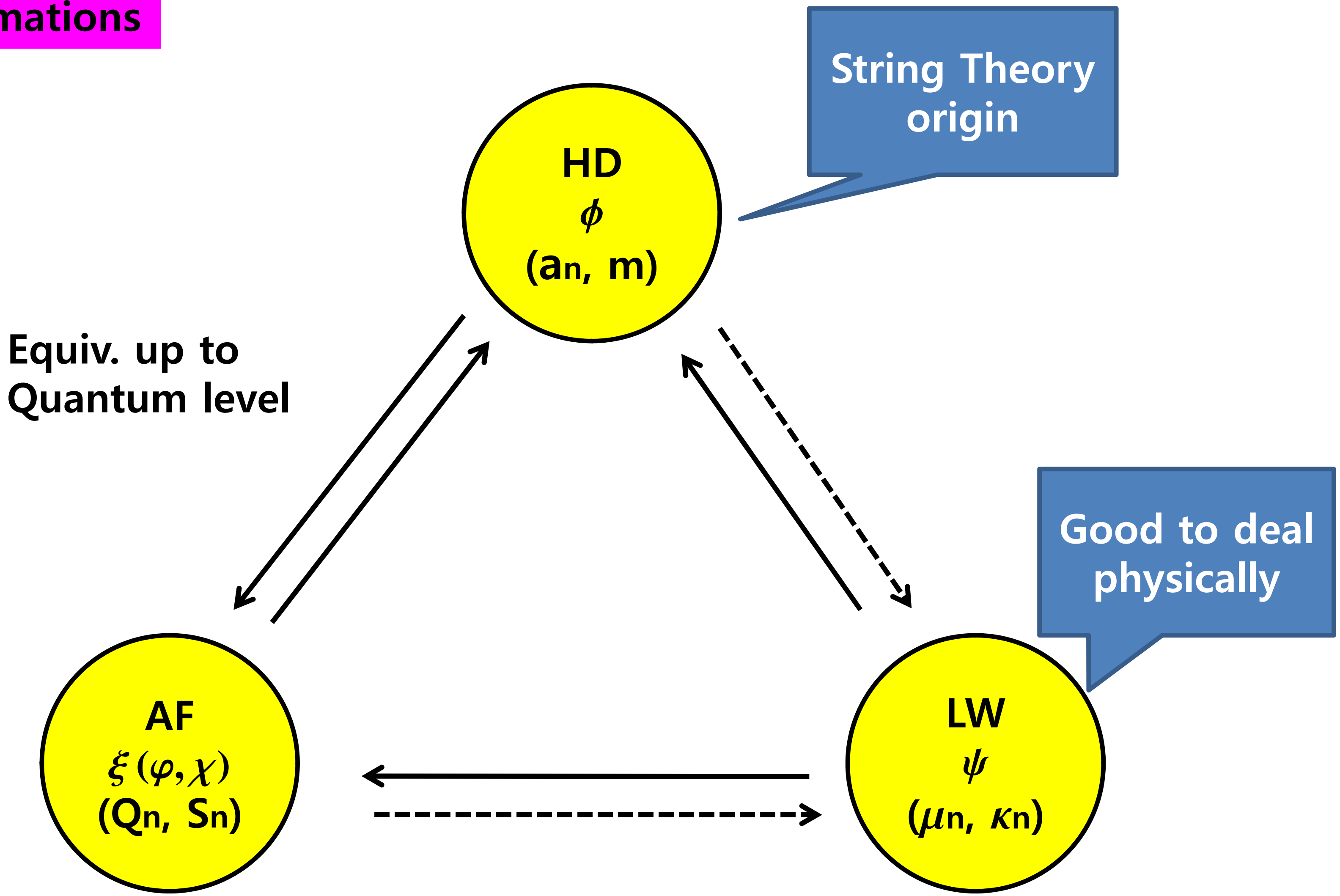
κ_n = +1, -1, +1, -1, ……. : alters its sign

Ghosts : Lee-Wick partners

→ If mass is larger than the ordinary field, these decay early into other particles and may cause NO macroscopic physical problem

Generalized Lee-Wick Formalism

Transformations



[For details, see I.C. and O. Kwon, PRD 82, 025013 (2010)]

Lee-Wick Bouncing Universe

Lee-Wick Model

: consider only N=2 in this work

$$S_{\rm LW} = \int d^4\sqrt{-g}\Big[\sum_{n=1}^{N}(-1)^n\big(\frac{1}{2}\partial_\mu\varphi_n\partial^\mu\varphi_n + \frac{1}{2}m_n^2\varphi_n^2\big)\Big].$$

$$T^0_{\ 0} = \sum_{n=1}^{N}(-1)^{n+1}\Big[-\frac{1}{2}\dot{\varphi}_n^2 - \frac{1}{2}m_n^2\varphi_n^2\Big],$$

$$T^i_{\ i} = \sum_{n=1}^{N}(-1)^{n+1}\Big[\frac{1}{2}\dot{\varphi}_n^2 - \frac{1}{2}m_n^2\varphi_n^2\Big],$$

- Background metric:

$$ds^2 = -dt^2 + a(t)^2 dx^i dx^i,$$

- Background evolution in Lee-Wick matter:

$$H^2 = \frac{8\pi G}{3}\sum_{n=1}^{N}(-1)^{n+1}\Big(\frac{1}{2}\dot{\varphi}_n^2 + \frac{1}{2}m_n^2\varphi_n^2\Big),$$

$$\dot{H} = -4\pi G\sum_{n=1}^{N}(-1)^{n+1}\dot{\varphi}_n^2,$$

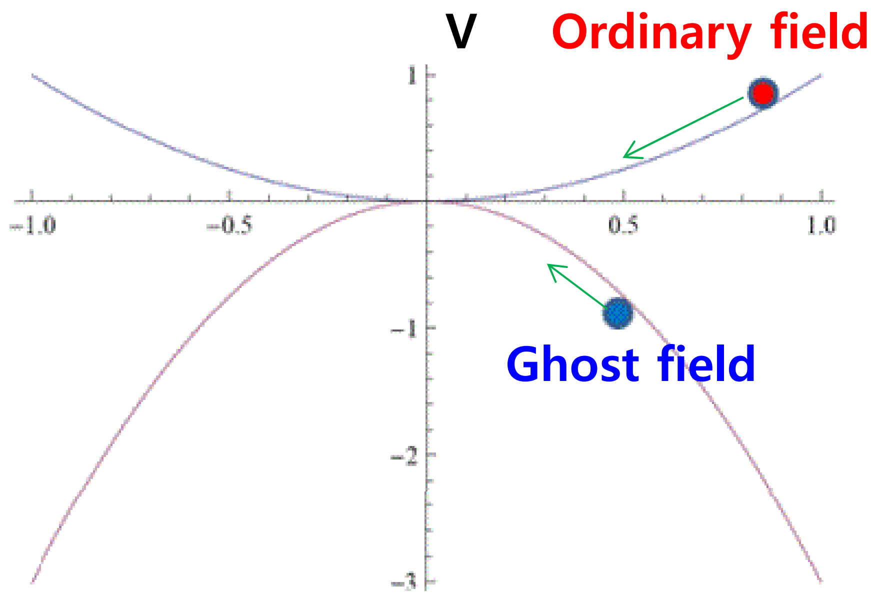
$$\ddot{\varphi}_n + 3H\dot{\varphi}_n + m_n^2\varphi_n = 0.$$

Why Bouncing?

Contracting U → Expanding U

↑

GHOST



By adjusting conditions at t=0,
one can make H=0 ;

$$H^2 = \frac{8\pi G}{3}\sum_{n=1}^{N}(-1)^{n+1}\underbrace{\left(\frac{1}{2}\dot{\varphi}_n^2 + \frac{1}{2}m_n^2\varphi_n^2\right)}_{\text{adjust}} \equiv 0 \qquad \Rightarrow \ \dot{a} = 0 \ : \ \text{Bouncing !!}$$

If we restrict further,

$$\dot{H} = -4\pi G\sum_{n=1}^{N}(-1)^{n+1}\underbrace{\dot{\varphi}_n^2}_{=\,0} \equiv 0$$

➔ Symmetric about t=0

Symmetric Bouncing Conditions:

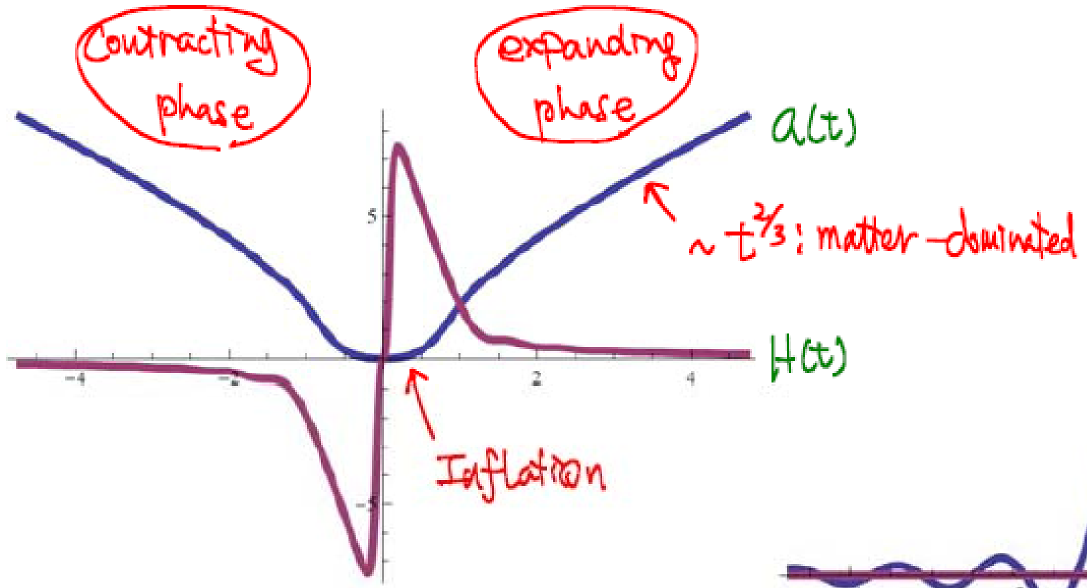
	$\dot{\varphi}_1(0)$	$\dot{\varphi}_2(0)$	$\varphi_1(0)$	$\varphi_2(0)$	$\dot{a}(0)=0$	$\ddot{a}(0)$	$\dddot{a}(0)$	$\ddddot{a}(0)$
Case 1	$=0$	$=0$	$\neq 0$	$\neq 0$	$m_1^2\varphi_1^2 = m_2^2\varphi_2^2$	$=0$	$=0$	>0 for $m_1^2 < m_2^2$
Case 2	$\neq 0$	$\neq 0$	$=0$	$=0$	$\dot{\varphi}_1^2 = \dot{\varphi}_2^2$	$=0$	$=0$	$=0$
Case 3	$\neq 0$	$=0$	$=0$	$\neq 0$	$\dot{\varphi}_1^2 = m_2^2\varphi_2^2$	<0	$=0$	$>,=,<$
Case 4	$=0$	$\neq 0$	$\neq 0$	$=0$	$\dot{\varphi}_2^2 = m_1^2\varphi_1^2$	>0	$=0$	<0

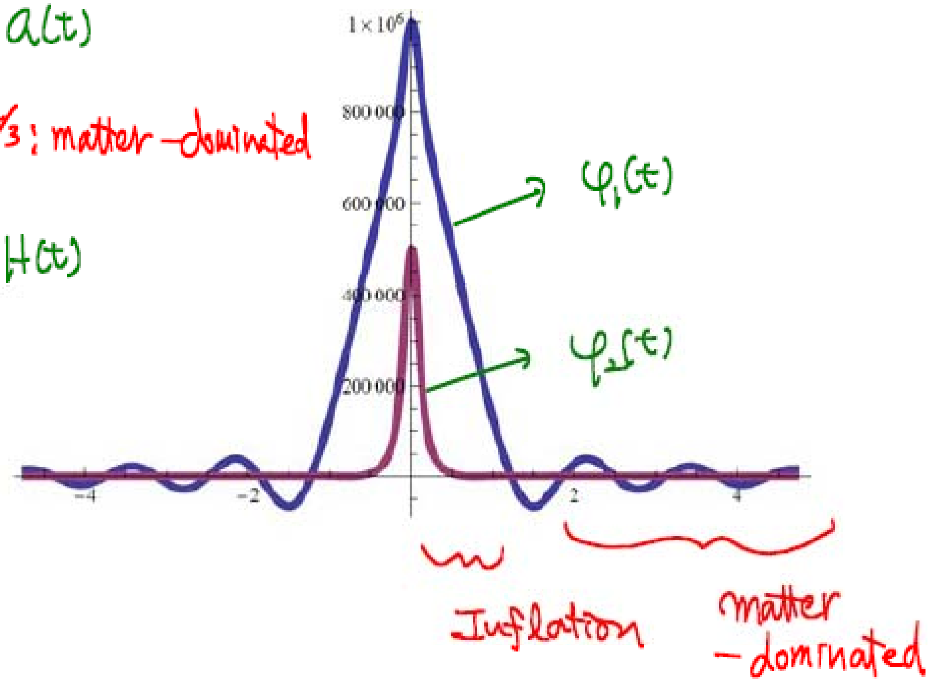
For this "Symmetric Bouncing Universe",
in order to solve Field Equations numerically
the only necessary Initial Condition is

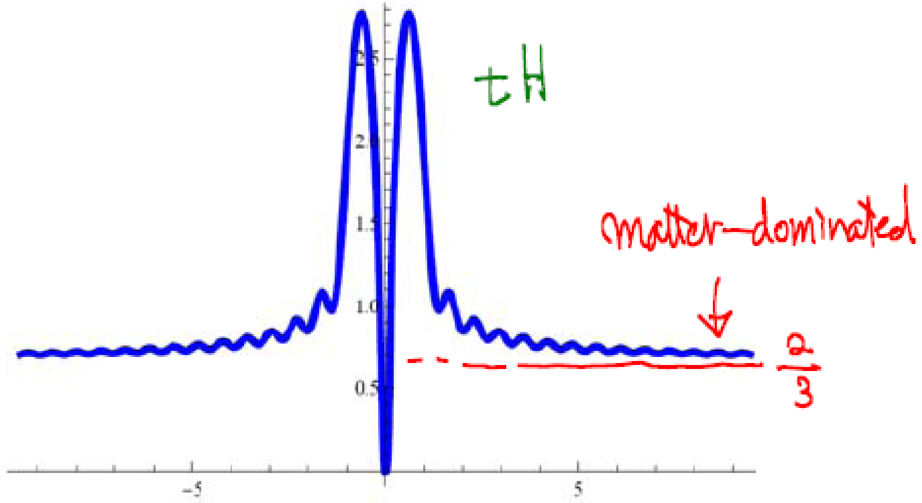
$$\varphi_1(t=0) \quad \text{or} \quad \varphi_2(t=0)$$

Solutions of Field Equations

$$m_2 = 2m_1 = 10^{-5} m_p, \quad \varphi_1(0) = m_p$$







We shall consider SYMMETRIC case about t=0

Asymptotic Background Solutions

: φ_1 is dominant

: φ_2 is important mainly during bouncing

Approximate Field Equations

$$H^2 \approx \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\varphi}_1^2 + \frac{1}{2}m_1^2\varphi_1^2\right)$$

$$\dot{H} \approx -4\pi G\dot{\varphi}_1^2$$

$$\ddot{\varphi}_1 + 3H\dot{\varphi}_1 + m_1^2\varphi_1 \approx 0$$

Approximate Asymptotic Solutions

$$\varphi_1(t) \approx \frac{\cos(m_1 t + \alpha_1)}{\sqrt{3\pi G}\, m_1 t}$$

$$H(t) \approx \frac{2}{3t} + \frac{\sin(2m_1 t + 2\alpha_1)}{3m_1 t^2}$$

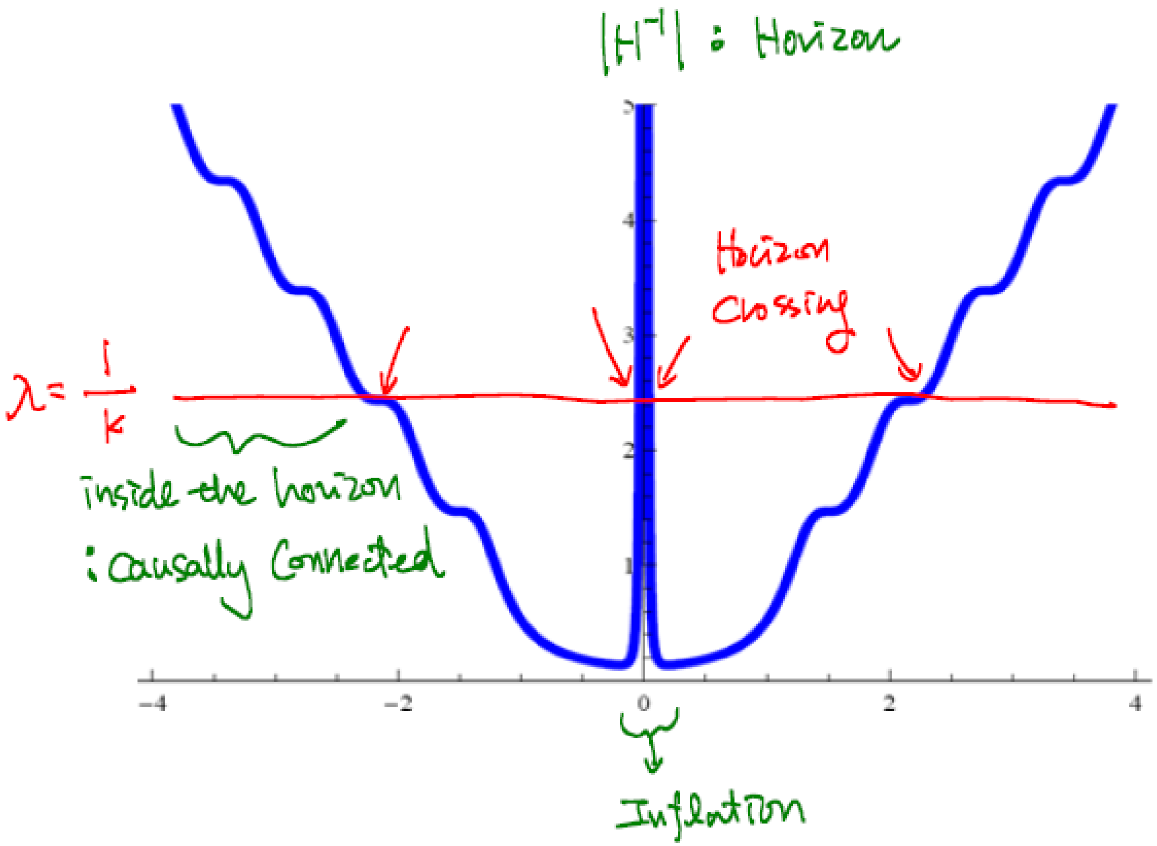
Bouncing Universe

:- 60 e-folding is NOT necessary

i) Horizon Problem: Solved during the contracting phase

ii) Flatness Problem: Ω_k deviates from 0 during expanding phase, but it approaches 0 during contracting phase exactly at the same rate.

:- Remaining Condition: SHOULD produce proper Density Perturbation



Scalar Perturbation in N=2 Lee-Wick Model

Lee-Wick Bouncing Model

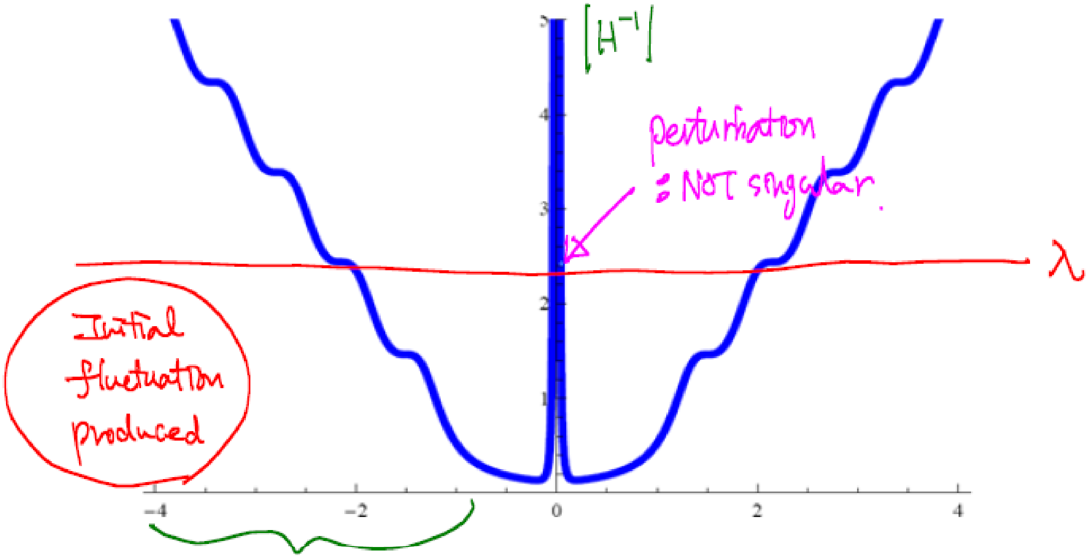
:- perturbation is **NON-singular** ('09 Cai, Qui, Brandenberger, Zhang.)

:- **Singular** in other models such as "Ekpyrotic Bouncing Universe"

Initial Perturbation

:- produced in the **contracting phase**

:- survives during bouncing, and provides "**scale-invariant spectrum**" in the expanding phase



φ : Oscillating $\Rightarrow$ massive particle

$\Rightarrow$ Bunch-Davis vacuum.

Scalar Perturbation

- Perturbed FRW spacetime (linear scalar perturbations):

$$ds^2 = -(1+2A)dt^2 + 2a\partial_i B dx^i dt + a^2 \left[(1-2\psi)\delta_{ij} + 2\partial_i\partial_j E\right] dx^i dx^i.$$

- Matter field equations: $\nabla_\mu T^\mu_{\ 0} = 0$

$$\delta\ddot{\varphi}_n + 3H\delta\dot{\varphi}_n + \frac{k^2}{a^2}\delta\varphi_n + m_n^2\delta\varphi_n = -2m_n^2\varphi_n A + \dot{\varphi}_n\left[\dot{A} + 3\dot{\psi} + \frac{k^2}{a^2}\left(a^2\dot{E} - aB\right)\right]$$

Sasaki-Mukhanov Variable Q: gauge invariant quantity $x^\mu \longrightarrow x^\mu + \xi^\mu$

$$Q_n \equiv \delta\varphi_n + \frac{\dot{\varphi}_n}{H}\psi.$$

Spatially flat gauge

$$\psi = 0, \qquad E = 0.$$

Then,
Field Eq. & Others :
Expressed in Qn & Background Fields

Field Equation:

$$\ddot{Q}_n + 3H\dot{Q}_n + \frac{k^2}{a^2}Q_n + m_n^2 Q_n - \frac{8\pi G}{a^3}\sum_{l=1}^{N}(-1)^{l+1}\frac{d}{dt}\left(\frac{a^3}{H}\dot{\varphi}_n\dot{\varphi}_l\right)Q_l = 0.$$

: Solved when "Background" is known !!

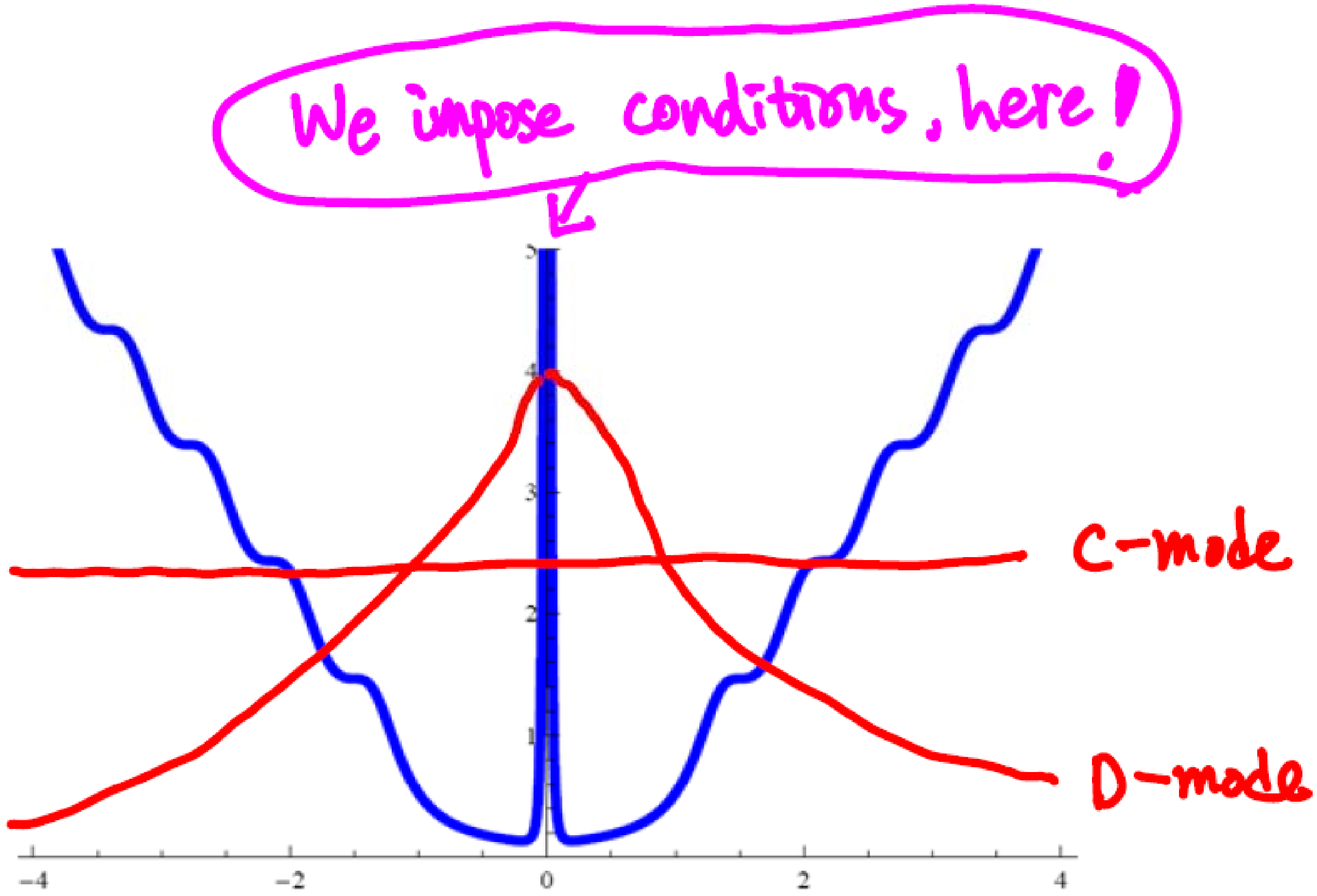
Comoving Curvature R:

$$\mathcal{R} = \psi - \frac{H}{\rho + p}\delta q = H \times \left[\frac{\sum_{n=1}^{N}(-1)^{n+1}\dot{\varphi}_n Q_n}{\sum_{m=1}^{N}(-1)^{m+1}\dot{\varphi}_m^2}\right]$$

Power Spectrum:

$$\mathcal{P}_{\mathcal{R}} = \frac{k^3}{2\pi^2}|\mathcal{R}|^2.$$

Our Policy



Series Expansion about bouncing point

Consider the "Bouncing Point (t=0)", apply "Regularity Condition", and "Solve Q-equations".
(rather than considering initial perturbations during contracting phase)

Background evolution of "a" and "φ_n" are already solved and fixed.

Need to get initial behavior of Q_n at t=0.

Need series expansion of Background Fields:

$$\boxed{\begin{aligned} \varphi_n(t) &= p_{n0} + p_{n1}t + p_{n2}t^2 + p_{n3}t^3 + \cdots \\ H(t) &= h_1 t + h_2 t^2 + h_3 t^3 + \cdots \,. \end{aligned}}$$

$$\begin{aligned} p_{n2} &= -\frac{m_n^2}{2} p_{n0}, \\ p_{n3} &= 0, \\ p_{n4} &= \frac{m_n^4}{24} p_{n0}, \\ p_{n5} &= 0, \\ p_{n6} &= -\frac{1}{30}\left(6h_3 p_{n2} + m_n^2 p_{n4}\right), \\ &\cdots, \end{aligned}$$

$$\begin{aligned} h_1 &= 0, \\ h_2 &= 0, \\ h_3 &= -\frac{4\pi G}{3}\sum_{n=1}^{N}(-1)^{n+1} m_n^4 p_{n0}^2, \\ h_4 &= 0, \\ h_5 &= \frac{4\pi G}{15}\sum_{n=1}^{N}(-1)^{n+1} m_n^6 p_{n0}^2, \\ &\cdots, \end{aligned}$$

Series expansion for Q_n,

$$\boxed{Q_n(t) = t^s(q_{n0} + q_{n1}t^1 + q_{n2}t^2 + q_{n3}t^3 + q_{n4}t^4 + \cdots),}$$

Series forms of φ_n, H and Q_n → Q-equation

$$\boxed{\ddot{Q}_n + 3H\dot{Q}_n + \frac{k^2}{a^2}Q_n + m_n^2 Q_n - \frac{8\pi G}{a^3}\sum_{l=1}^{N}(-1)^{l+1}\frac{d}{dt}\left(\frac{a^3}{H}\dot{\varphi}_n\dot{\varphi}_l\right)Q_l = 0.}$$

Then, s is determined

→ admits 2 linearly independent solutions (even & odd)

$$Q_n^{\rm even}(t) = t^{-2}(q_{n0} + q_{n2}t^2 + q_{n4}t^4 + \cdots),$$
$$Q_n^{\rm odd}(t) = q_{n1}t^1 + q_{n3}t^3 + q_{n5}t^5 + \cdots ,$$

Singular at t=0, but gives finite "R"

Even- & Odd-Mode Perturbations

From Q-equation,

→ Relation b/w coefficients & parameters are determined

To solve Q-equation numerically

(i) even case

$$(1) \quad q_{20} = \frac{m_2}{m_1} q_{10}$$

$$(2) \quad q_{22} = \frac{m_1}{m_2} q_{12} - \frac{(m_1^2 - m_2^2)(5k^2 + m_1^2 + m_2^2)}{30 m_1 m_2} q_{10}$$

(ii) odd case

$$(1) \quad q_{21} = \frac{m_1}{m_2} q_{11}$$

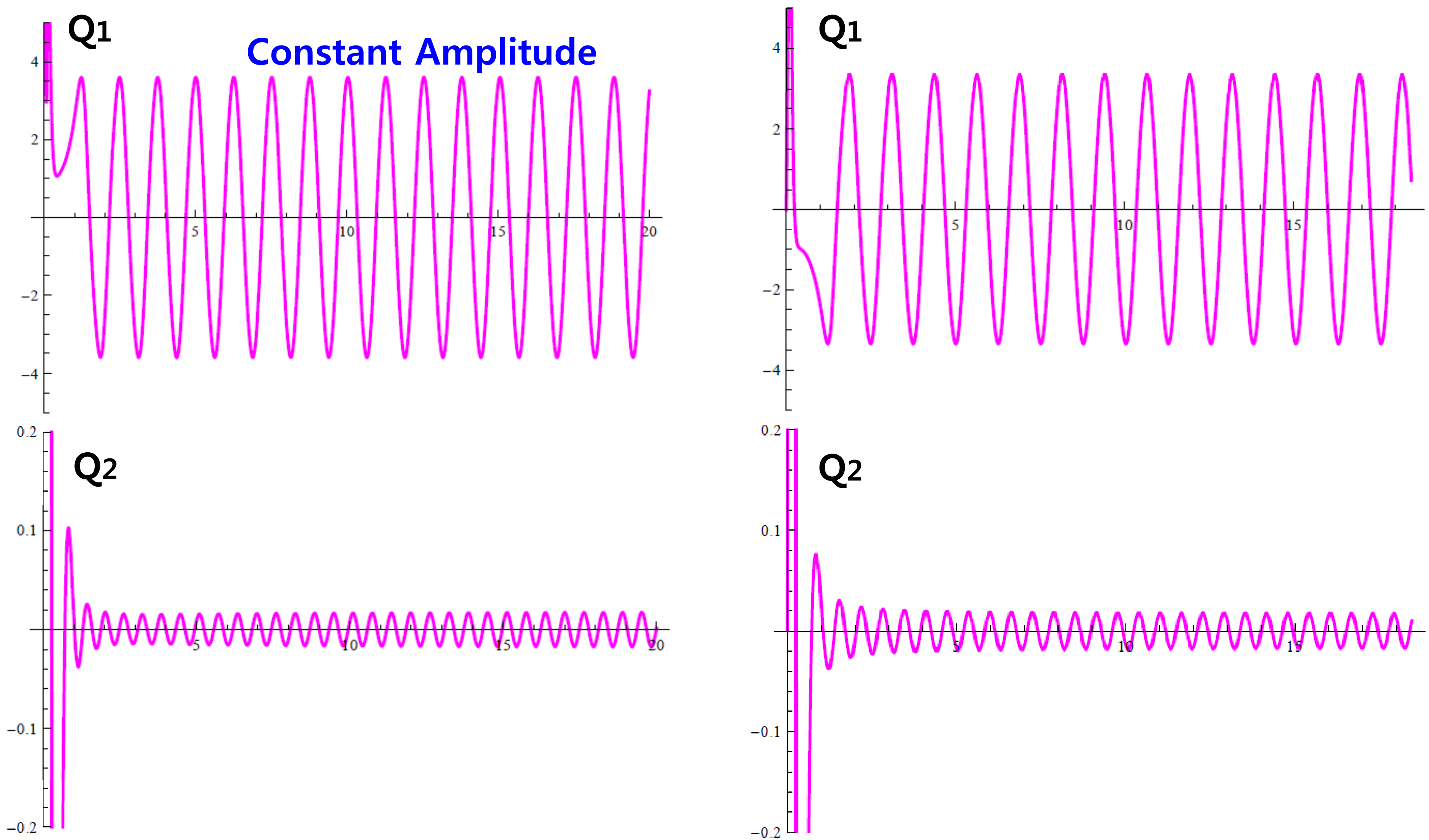
$$(2) \quad q_{23} = \frac{m_2}{m_1} q_{13} - \frac{(m_1^2 - m_2^2)k^2}{6 m_1 m_2} q_{11}$$

$q_{10},\ q_{12}\ (q_{11},\ q_{13})$ **: free to fix → Shooting Parameter**

Numerical Solutions

(i) even case

(ii) odd case



t<0 region is evenly- or oddly-symmetric

Numerical Solutions

(1) Dominant : gives Q ~ Constant Oscillation

$$\ddot{Q}_1 + \frac{2}{t}\dot{Q}_1 + \left(\frac{k^2}{\alpha^2 t^{\frac{4}{3}}} + m_1^2\right) Q_1 - \frac{4m_1}{t}\sin(2m_1 t + 2\alpha_1) Q_1 \simeq 0.$$

(2) Sub-dominant : controls Q ~ 1/t Damed Oscillation

→ Decay/Growing-Mode → Initial Vacuum

(3) When k-term is comparable to (2) : gives $\boxed{Q_1 \sim \frac{1}{t^{1/3}}}$ Damped Oscillation

→ only appears during intermediate period for large k

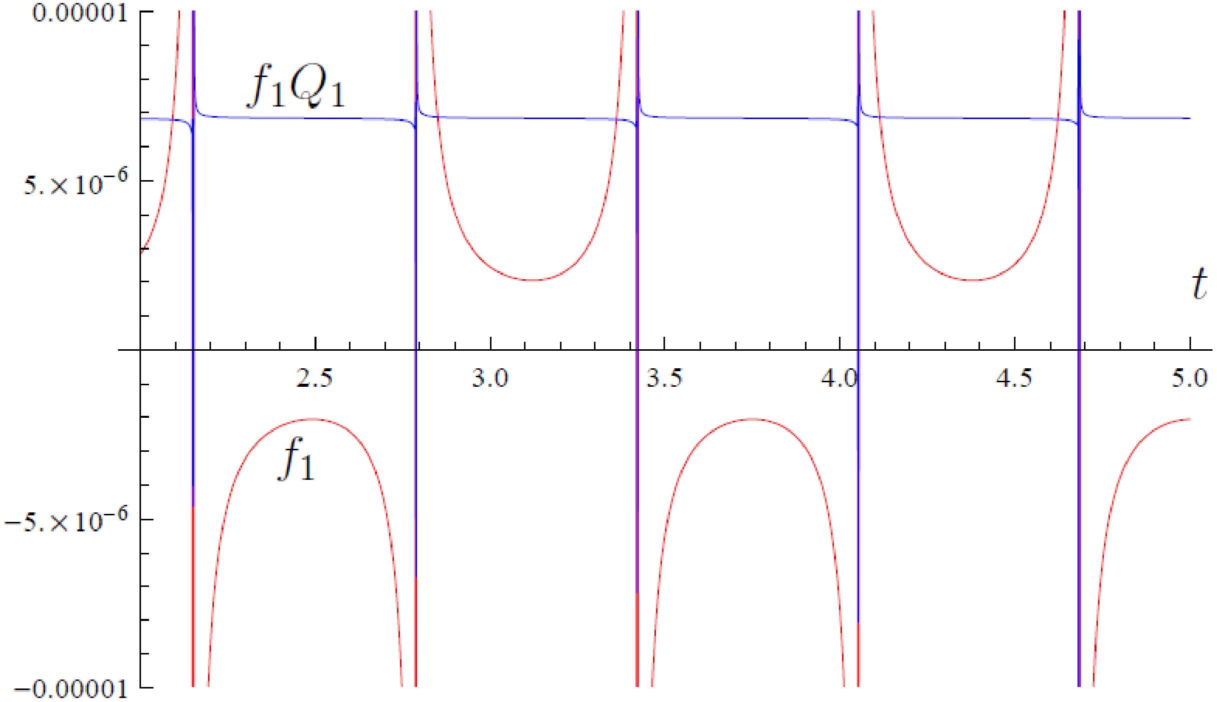
Comoving Curvature

$$\mathcal{R} = \frac{H}{\dot\varphi_1^2 - \dot\varphi_2^2}(\dot\varphi_1 Q_1 - \dot\varphi_2 Q_2) \equiv f_1 Q_1 - f_2 Q_2.$$

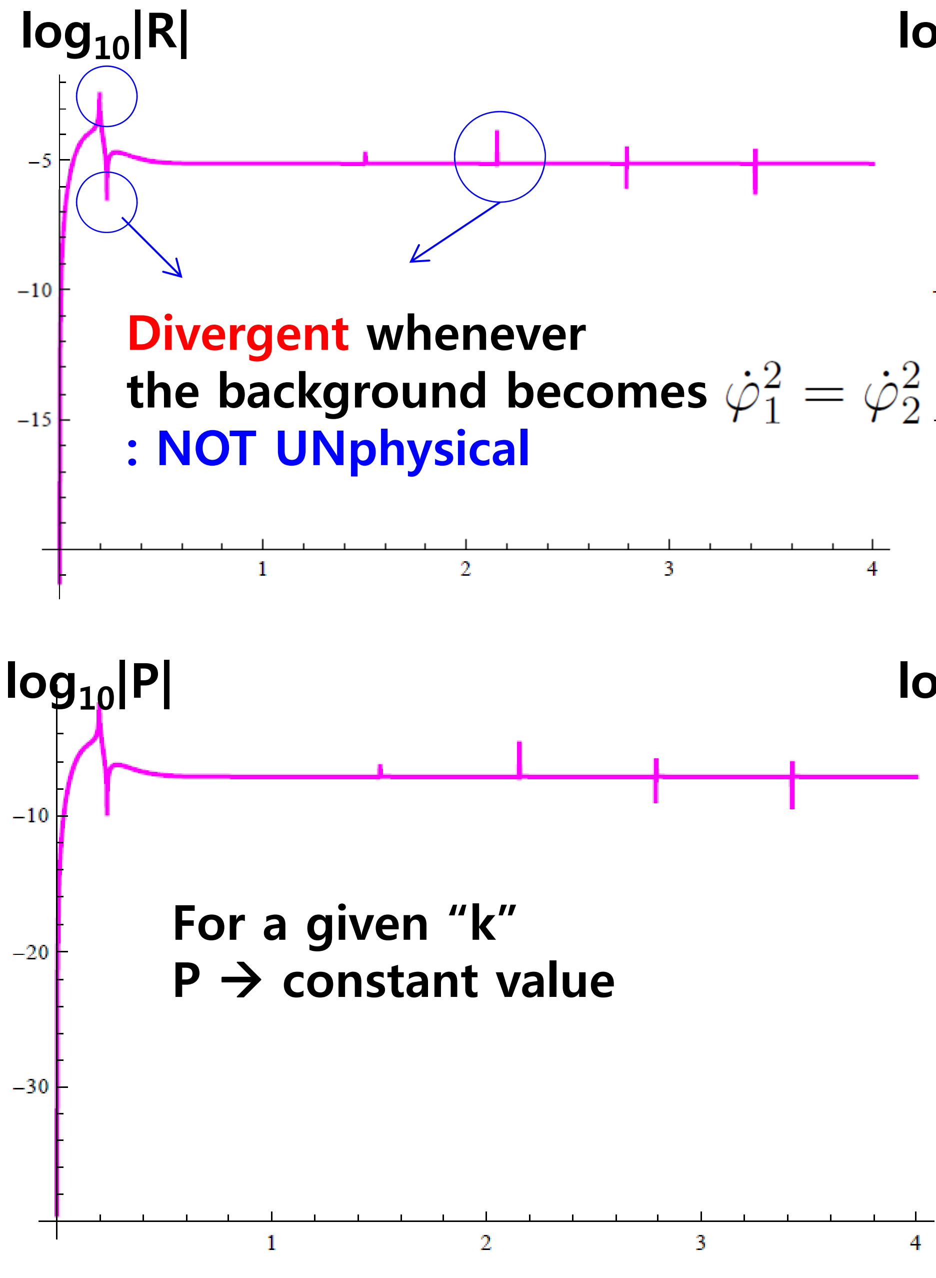
Q(even) → R(even)
Q(odd) → R(odd)

Even function

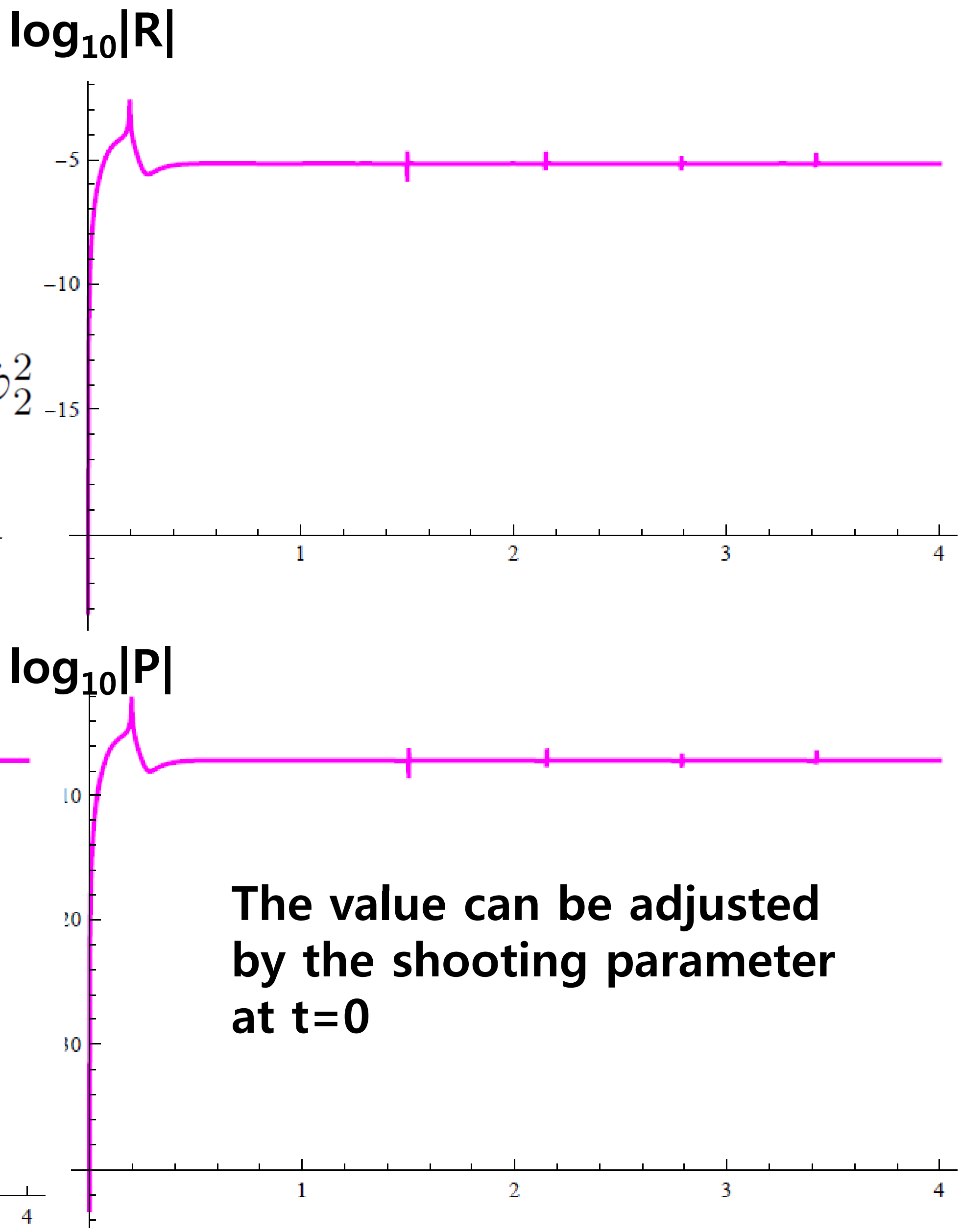
Periodic function at |t|>>
Does NOT change



(i) even case

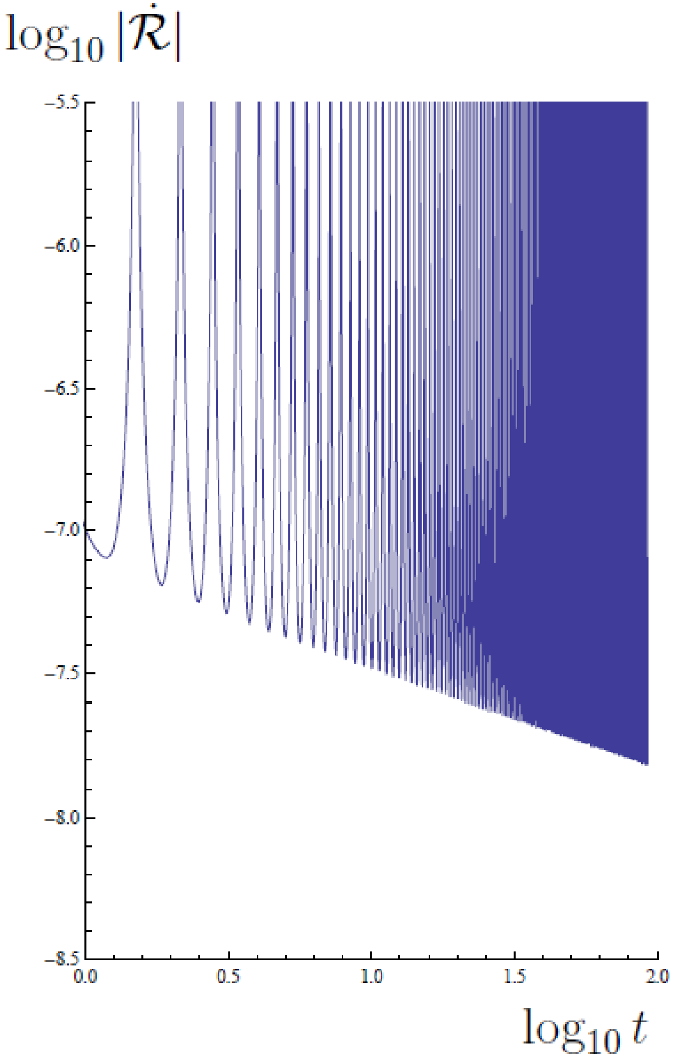


(ii) odd case



So, is R completely CONSTANT ???

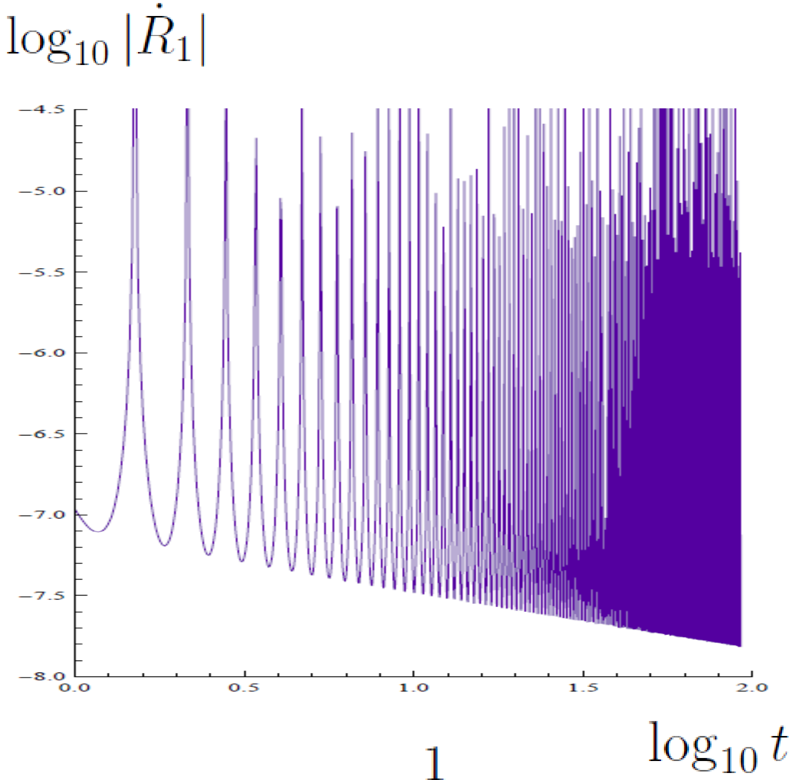
k=30



$\mathcal{R} \propto \dfrac{1}{t^{1/3}}$ **: still k-term is dominant**

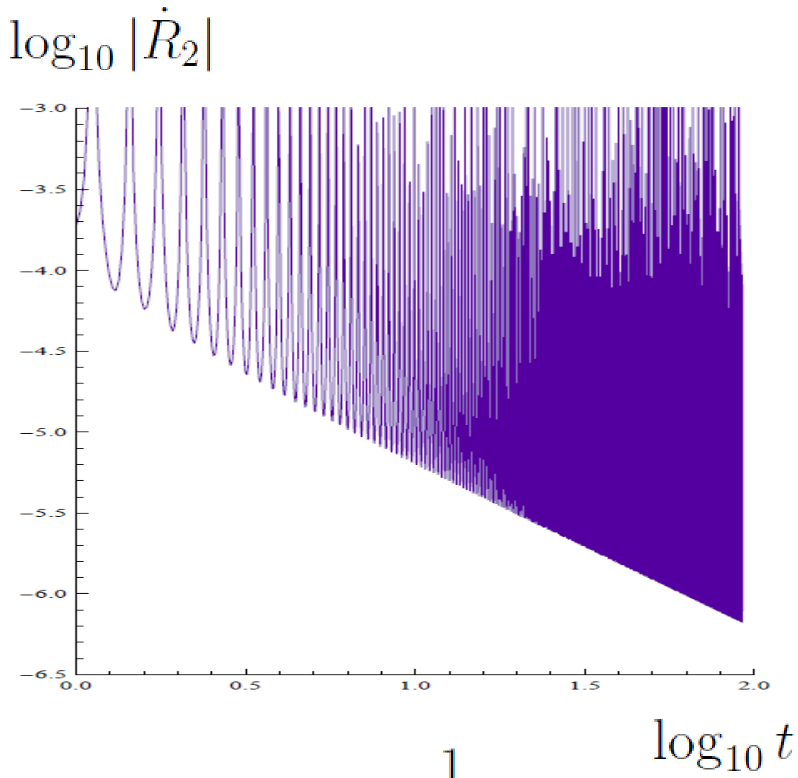
k=30

$$R_n = \frac{H}{\dot{\varphi}_n} Q_n$$



$$Q_1 \propto \frac{1}{t^{1/3}}$$

: k-term dominant period



$$Q_2 \propto \frac{1}{t}$$

: k-term is negligible

→ Expected also for Q_1 at t>>

In general, the scalar perturbation consists of linearly independent
Constant- and Decaying-mode

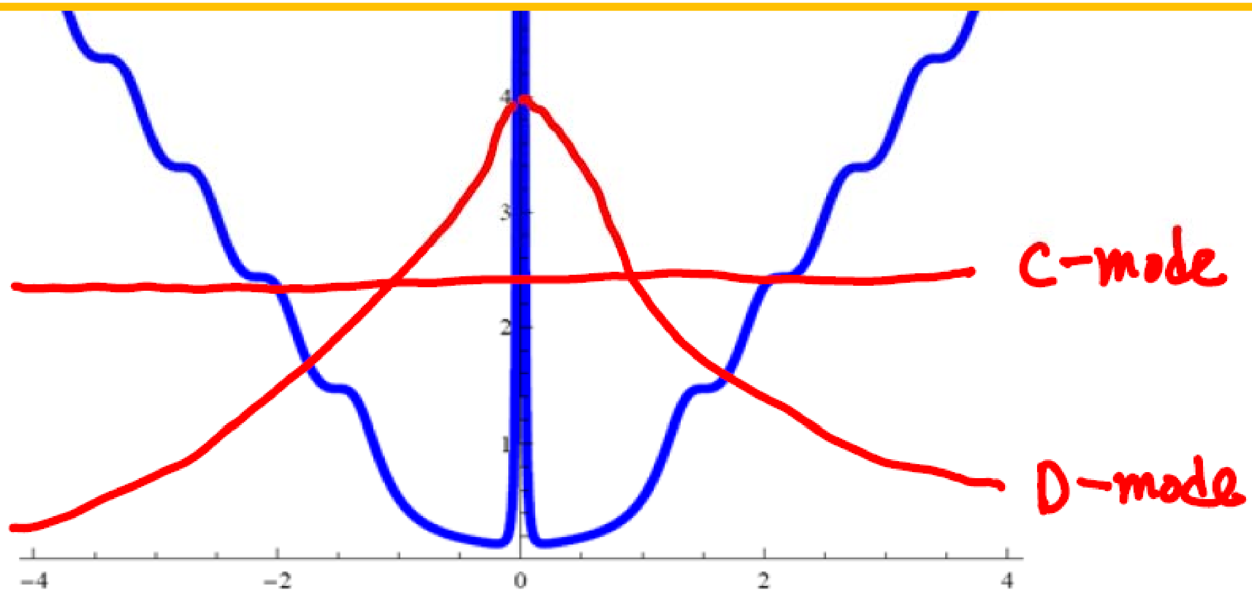
Even- and Odd-mode : also linearly independent

$$\begin{pmatrix} \text{Constant} \\ \text{Decaying} \end{pmatrix} = \begin{pmatrix} \odot & \otimes \\ \oplus & \amalg \end{pmatrix} \begin{pmatrix} \text{Even} \\ \text{Odd} \end{pmatrix}$$: related by a linear combination

Need to extract and study C- & D-mode from E- & O-mode

D-mode: "Growing-mode" during contracting phase (t<0)

(For massless ghost, C- and D-mode were studied by '04 Wands, '09 Hwang)



Normalization and Vacuum Solution

Conformal Transformation: $\boxed{dt = ad\eta.}$

Introduce New Variables: $\boxed{v = aQ_1,}$ $\quad z = \dfrac{a\varphi'}{\mathcal{H}}$

Action:

$$S = \int d\eta dx^3 \left[\frac{1}{2}(\partial_\eta \tilde{v})^2 - \frac{1}{2}(\partial_i \tilde{v})^2 + \frac{1}{2}\frac{z''}{z}\tilde{v}^2\right]$$

$$\tilde{v}(\eta, \vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[v(\eta; k)a_{\vec{k}} + v^*(\eta; k)a^\dagger_{-\vec{k}}\right] e^{i\vec{k}\cdot\vec{x}}$$

Field Equation:

$$\boxed{v'' + \left(k^2 - \frac{z''}{z}\right) v = 0}$$

Normalization from Canonical Quantization: $\boxed{vv^{*\prime} - v^* v' = i.}$

At $|\eta| \gg$, the field equation is approximated by

$$\boxed{v''(\eta) + \left[k^2 \boxed{- \frac{2}{\eta^2}} + \boxed{\frac{m_1^2\alpha^6}{81}\eta^4 - \frac{4m_1\alpha^3}{3}\eta \sin\left(\frac{2m_1\alpha^3}{27}\eta^3 + 2\alpha_1\right)}\right] v(\eta) \approx 0,}$$

In the subhorizon limit $|k\eta| \gg$, the dominant terms in Eq. are

$$\boxed{\frac{d^2v(\eta)}{d(k\eta)^2} + \frac{m_1^2\alpha^6}{81k^6}(k\eta)^4 v(\eta) \approx 0.}$$

The solution is given by

$$\boxed{v(\eta) = \sqrt{\frac{\pi\eta}{12}}\left[A_1 H^{(1)}_{\frac{1}{6}}\left(\frac{m_1\alpha^3}{27}\eta^3\right) + A_2 H^{(2)}_{\frac{1}{6}}\left(\frac{m_1\alpha^3}{27}\eta^3\right)\right]}$$

where $|A_2|^2 - |A_1|^2 = 1$

Take positive-energy mode in the beginning of perturbation ($\eta \ll 0$)

$$\Leftrightarrow \quad A_1 = 0,$$

$$\Rightarrow \quad \boxed{v(\eta) \approx \sqrt{\frac{9}{2m_1\alpha^3}}\frac{1}{\eta}\exp\left[-i\left(\frac{m_1\alpha^3}{27}\eta^3 - \frac{\pi}{3}\right)\right]}$$

$$\Leftrightarrow \quad Q_1(t) = \frac{v}{a} \propto \frac{1}{t}e^{-im_1 t}$$

New Type of
Vacuum Solution:
INITIAL Perturbation

$$\boxed{\mathcal{R}^{\text{growing}} \approx f_1 Q_1^{\text{growing}} \propto \frac{1}{t}.}$$

Schematic Picture

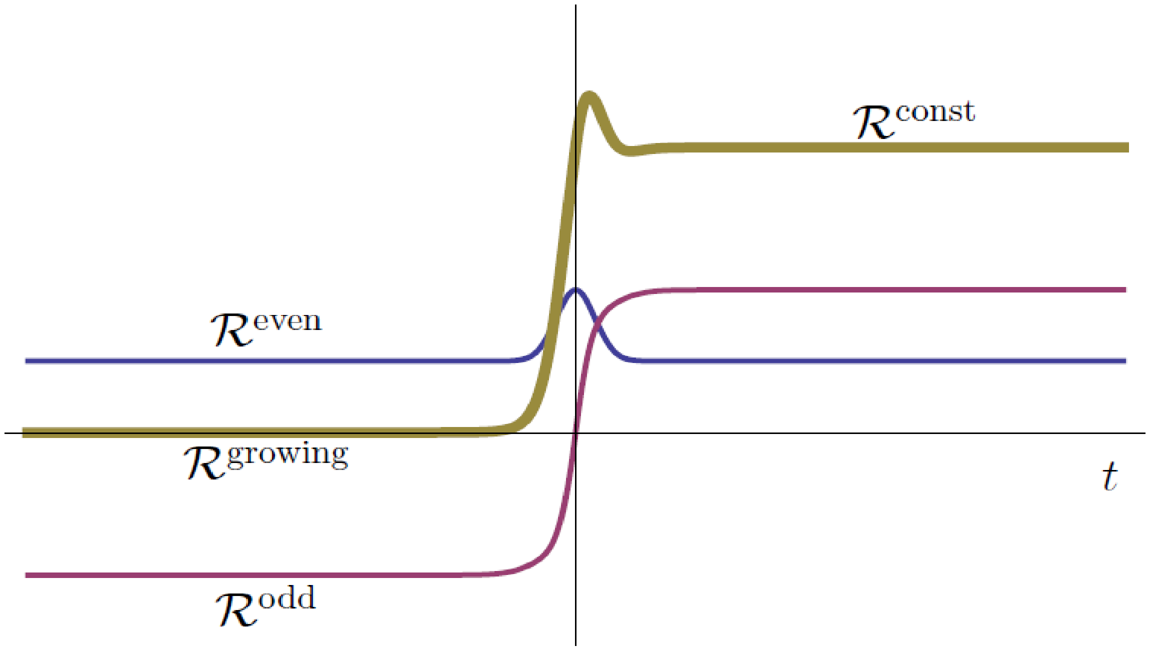
Comoving Curvature

t<0

To have "Normalized Growing Mode" at initial moment (t<<),
→Linearly combine "even" and "odd" mode of R
→ Remove "constant" mode in R

$$\begin{aligned}
\mathcal{R}(t \ll 0) &= c_1 \mathcal{R}^{\text{even}} + c_2 \mathcal{R}^{\text{odd}} \\
&= c_1 \left[\mathcal{R}^{\text{even-growing}} + \cancel{\mathcal{R}^{\text{even-const}}}\right] + c_2 \left[\mathcal{R}^{\text{odd-growing}} + \cancel{\mathcal{R}^{\text{odd-const}}}\right] \\
&= c_1 \mathcal{R}^{\text{even-growing}} + c_2 \mathcal{R}^{\text{odd-growing}} \\
&\equiv \boxed{\mathcal{R}^{\text{growing}},}
\end{aligned}$$

:→ This should meet the "Normalization Value"



t>0

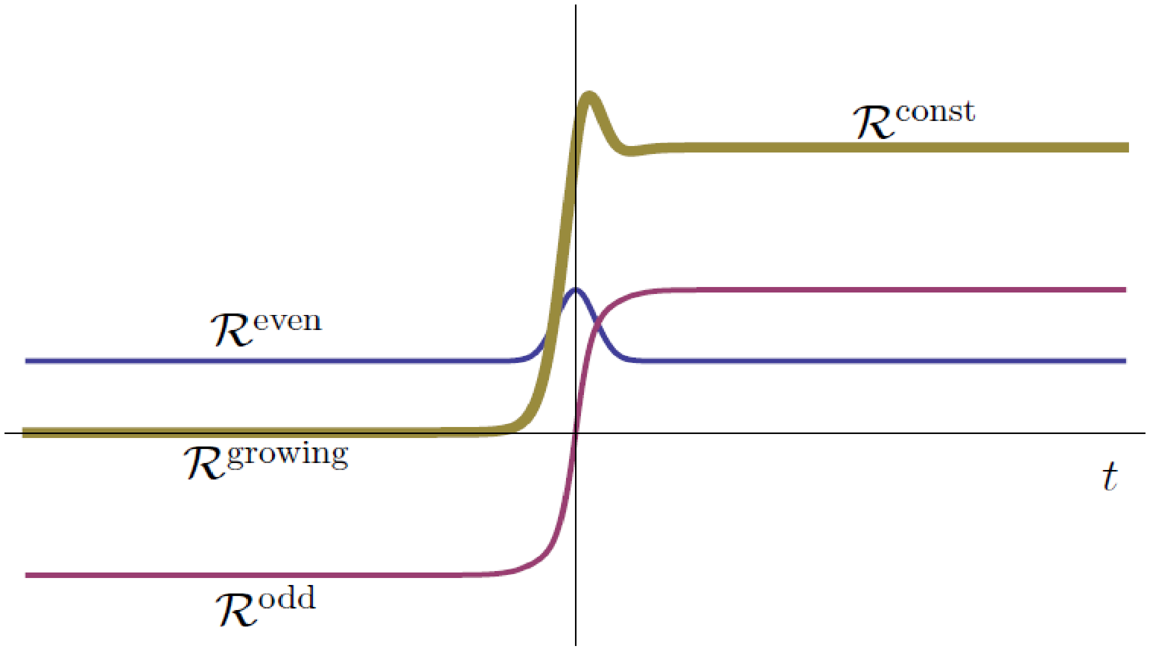
At $t > 0$, since $\mathcal{R}^{\mathrm{odd-const}}(t > 0) = -\mathcal{R}^{\mathrm{odd-const}}(t < 0)$,

the "constant mode" does NOT disappear!!

$$\begin{aligned}\mathcal{R}(t \gg 0) &= c_1\mathcal{R}^{\mathrm{even}} + c_2\mathcal{R}^{\mathrm{odd}} \\ &= c_1\left[\mathcal{R}^{\mathrm{even-decaying}} + \mathcal{R}^{\mathrm{even-const}}\right] + c_2\left[\mathcal{R}^{\mathrm{odd-decaying}} + \mathcal{R}^{\mathrm{odd-const}}\right] \\ &= \left[c_1\mathcal{R}^{\mathrm{even-decaying}} + c_2\mathcal{R}^{\mathrm{odd-decaying}}\right] + \left[c_1\mathcal{R}^{\mathrm{even-const}} + c_2\mathcal{R}^{\mathrm{odd-const}}\right] \\ &\equiv \mathcal{R}^{\mathrm{decaying}} + \mathcal{R}^{\mathrm{const}} \approx \boxed{\mathcal{R}^{\mathrm{const}}.}\end{aligned}$$

Survive !!

:→ This should provide 10^{-9} Power-Spectrum



Tensor Perturbation

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (\delta_{ij} + \boxed{\bar{h}_{ij}})dx^i dx^j\right]$$

$$\bar{h}_{ij}(\eta, \vec{x}) = \sum_{\lambda=+,-} \int \frac{d^3k}{(2\pi)^{3/2}}\, h_\lambda(\eta, \vec{k})\, \epsilon_{ij}^{\lambda}\, e^{i\vec{k}\cdot\vec{x}}$$

Let $\mu_\lambda \equiv a h_\lambda$, the field equation becomes

$$\boxed{\mu_\lambda'' + \left(k^2 - \frac{a''}{a}\right)\mu_\lambda = 0}$$

When the background settles down to matter-dominated expansion at $|\eta| \gg$,

$$a \approx \frac{\alpha^3}{9}\eta^2$$

then

$$\boxed{\mu_\lambda'' + \left(k^2 - \frac{2}{\eta^2}\right)\mu_\lambda \approx 0}$$

This is exactly same with the cases of inflation !!!

The asymptotic solution becomes

$$\begin{aligned}\mu_\lambda(\eta) &\approx B_\lambda(k)e^{-ik\eta}\left(1+\frac{1}{ik\eta}\right)+A_\lambda(k)e^{ik\eta}\left(1-\frac{1}{ik\eta}\right)\\ &= (A+B)\left[\cos(k\eta)-\frac{\sin(k\eta)}{k\eta}\right]+i(A-B)\left[\sin(k\eta)-\frac{\cos(k\eta)}{k\eta}\right]\end{aligned}$$

"Power Spectrum"

$$\mathcal{P}_\mathrm{T}=\frac{64\pi G k^3}{2\pi^2}|h_\lambda|^2 \propto \left|\frac{\mu_\lambda}{a}\right| \propto \left|\frac{\mu_\lambda}{\eta^2}\right| \quad : \text{ Damps as } |\eta| \text{ increases}$$

Killing $A=0$ ⇔ Picking **"positive-energy mode"** :

$$\mu_\lambda(\eta)=B\left\{\left[\cos(k\eta)-\frac{\sin(k\eta)}{k\eta}\right]-i\left[\sin(k\eta)-\frac{\cos(k\eta)}{k\eta}\right]\right\}$$

Taking the deep sub-horizon limit, $|k\eta| \gg 2 \Rrightarrow$ **"Initial Perturbation"**

$$\mu_\lambda(\eta)\approx B\left[\cos(k\eta)-i\sin(k\eta)\right]=Be^{-ik\eta}$$

So, the tensor perturbation initially starts as this at η << 0

$$\boxed{\mu_\lambda(\eta) \approx B\left[\cos(k\eta) - i\,\sin(k\eta)\right] = Be^{-ik\eta}}$$

Then, what about at η >> 0 ???

Odd mode amplitude is reversed → |amplitude|^2 will be different ???

: No.....

Since the perturbation is "oscillatory",
the reversed amplitude gives the same magnitude...

Conclusions

1. Obtained Transformations among HD, AF, and LW
2. Investigated N=2 Lee-Wick Bouncing Universe Model
   for strictly Symmetric Case
3. Scalar Perturbation was studied in a different scope
   : Even and Odd Modes → analyzed Constant and Decay Modes
4. Found New Type of Initial Vacuum Solution for scalar perturbation
5. Tensor Perturbation Damps