Inflationary non-Gaussianities in the most general second-order scalar-tensor theories

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with Shinji Tsujikawa

Introduction

- Accelerated expansion at early times

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- Epoch: after QG and GUT (scale ?)
- New physics: usually scalar field (?)
- Or new gravity: 1st model Starobinsky model

General scalar-tensor theory

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_P^2 R + P(\phi, X) - G_3(\phi, X) \Box\phi + \mathcal{L}_4 + \mathcal{L}_5 \right]$$

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$$\mathcal{L}_4 \ = \ G_4(\phi, X) R + G_{4,X} \left[(\Box\phi)^2 - (\nabla_\mu \nabla_\nu \phi) (\nabla^\mu \nabla^\nu \phi) \right],$$

General scalar-tensor theory

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$$\begin{aligned}
\mathcal{L}_4 &= G_4(\phi,X)\,R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi)\right], \\
\mathcal{L}_5 &= G_5(\phi,X)\,G_{\mu\nu}\,(\nabla^\mu\nabla^\nu\phi) - \frac{1}{6}G_{5,X}[(\Box\phi)^3 \\
&\quad -3(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi) \\
&\quad +2(\nabla^\mu\nabla_\alpha\phi)\,(\nabla^\alpha\nabla_\beta\phi)\,(\nabla^\beta\nabla_\mu\phi)]. \quad X \equiv -\frac{1}{2}(\partial\phi)^2
\end{aligned}$$

Vacuum EOMs on flat FLRW

- $ds^2 = -dt^2 + a(t)^2 d\boldsymbol{x}^2$, $\phi = \phi(t)$

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\begin{aligned}
0 &= 3M_P^2 H^2 F + P + 6\,H\,G_{4,\phi}\dot{\phi} + (G_{3,\phi} - 12\,H^2 G_{4,X} + 9\,H^2 G_{5,\phi} - P_{,X})\dot{\phi}^2 \\
&\quad +(6\,G_{4,\phi X} - 3\,G_{3,X} - 5\,G_{5,X}H^2)H\dot{\phi}^3 + 3(G_{5,\phi X} - 2\,G_{4,XX})H^2\dot{\phi}^4 - H^3 G_{5,XX}\dot{\phi}^5\,, \\
0 &= 2[(G_{5,\phi} - 2\,G_{4,X})\dot{\phi}^2 - HG_{5,X}\dot{\phi}^3 + FM_P^2]\dot{H} + 3M_P^2 H^2 F + P + 4HG_{4,\phi}\dot{\phi} \\
&\quad +[2\,G_{4,\phi} + 4H(G_{5,\phi} - G_{4,X})\dot{\phi} + (2\,G_{4,\phi X} - G_{3,X} - 3\,H^2 G_{5,X})\dot{\phi}^2 + 2H(G_{5,\phi X} - 2\,G_{4,XX})\dot{\phi}^3 \\
&\quad + \left(2\,G_{4,\phi\phi} + 3\,H^2 G_{5,\phi} - G_{3,\phi} - 6\,H^2 G_{4,X}\right)\dot{\phi}^2 + 2H\left(G_{5,\phi\phi} - G_{5,X}H^2 - 2\,G_{4,\phi X}\right)\dot{\phi}^3 \\
&\quad - H^2 G_{5,\phi X}\dot{\phi}^4\,.
\end{aligned}
$$

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&\quad +(6\,G_{4,\phi X} - 3\,G_{3,X} - 5\,G_{5,X} H^2) H\dot\phi^3 + 3(G_{5,\phi X} - 2\,G_{4,XX}) H^2\dot\phi^4 - H^3 G_{5,XX}\dot\phi^5\,, \\
0 &= 2[(G_{5,\phi} - 2\,G_{4,X})\dot\phi^2 - HG_{5,X}\dot\phi^3 + FM_P^2]\dot H + 3M_P^2 H^2 F + P + 4HG_{4,\phi}\dot\phi \\
&\quad +[2\,G_{4,\phi} + 4H(G_{5,\phi} - G_{4,X})\dot\phi + (2\,G_{4,\phi X} - G_{3,X} - 3\,H^2 G_{5,X})\dot\phi^2 + 2H(G_{5,\phi X} - 2\,G_{4,XX})\dot\phi^3 \\
&\quad + \left(2\,G_{4,\phi\phi} + 3\,H^2 G_{5,\phi} - G_{3,\phi} - 6\,H^2 G_{4,X}\right)\dot\phi^2 + 2H\left(G_{5,\phi\phi} - G_{5,X} H^2 - 2\,G_{4,\phi X}\right)\dot\phi^3 \\
&\quad - H^2 G_{5,\phi X}\dot\phi^4\,.
\end{aligned}
$$

- $F = 1 + 2G_4/M_P^2$, and $\dot\phi E_3 = \dot E_1 + 3H(E_1 + E_2) = 0$.

- Removing P from E_2 by E_1

$$
\begin{aligned}
(1-4\delta_{G4X}-2\delta_{G5X}+2\delta_{G5\phi})\epsilon \quad = \quad & \delta_{PX}+3\delta_{G3X}-2\delta_{G3\phi}+6\,\delta_{G4X}-\delta_{G4\phi}-6\,\delta_{G5\phi}+3\,\delta_{G5X} \\
& +12\,\delta_{G4XX}+2\,\delta_{G5XX}-10\,\delta_{G4\phi X}+2\,\delta_{G4\phi\phi} \\
& -8\,\delta_{G5\phi X}+2\,\delta_{G5\phi\phi}-\delta_{\phi}(\delta_{G3X}+4\,\delta_{G4X}-\delta_{G4\phi}+8\,\delta_{G4XX} \\
& +3\,\delta_{G5X}-4\,\delta_{G5\phi}+2\,\delta_{G5XX}-2\delta_{G4\phi X}-4\,\delta_{G5\phi X})\,,
\end{aligned}
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\end{aligned}
$$

- Slow-roll parameters

$$
\epsilon=-\frac{\dot{H}}{H^2}\,,\quad \delta_{\phi}=\frac{\ddot{\phi}}{H\dot{\phi}}\,,\quad \delta_{PX}=\frac{P_{,X}X}{M_P^2H^2F}\,,\quad \delta_{G3X}=\frac{G_{3,X}\dot{\phi}X}{M_P^2HF}\,,\quad \delta_{G3\phi}=\frac{G_{3,\phi}X}{M_P^2H^2F}\,,\quad \delta_{G4X}=\frac{G_{4,X}X}{M_P^2F}\,,
$$

$$
\delta_{G4\phi}=\frac{G_{4,\phi}\dot{\phi}}{M_P^2HF}\,,\quad \delta_{G4\phi X}=\frac{G_{4,\phi X}\dot{\phi}X}{M_P^2HF}\,,\quad \delta_{G4\phi\phi}=\frac{G_{4,\phi\phi}X}{M_P^2H^2F}\,,\quad \delta_{G4XX}=\frac{G_{4,XX}X^2}{M_P^2F}\,,\quad \delta_{G5\phi}=\frac{G_{5,\phi}X}{M_P^2F}\,,
$$

$$
\delta_{G5X}=\frac{G_{5,X}H\dot{\phi}X}{M_P^2F}\,,\quad \delta_{G5XX}=\frac{G_{5,XX}H\dot{\phi}X^2}{M_P^2F}\quad \delta_{G5\phi X}=\frac{G_{5,\phi X}X^2}{M_P^2F}\,,\quad \delta_{G5\phi\phi}=\frac{G_{5,\phi\phi}\dot{\phi}X}{M_P^2HF}\,.
$$

- Perturbation theory / Uniform field gauge

$$ds^2 = -[(1+\alpha)^2 - a(t)^{-2}\, e^{-2\mathcal{R}}\, (\partial\psi)^2]\, dt^2 + 2\partial_i\psi\, dt\, dx^i + a(t)^2 e^{2\mathcal{R}} d\boldsymbol{x}^2\,,$$

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- Action at second order

$$S_2 = \int dt d^3x\, a^3 \left[-3w_1\dot{\mathcal{R}}^2 + \frac{1}{a^2}\,(2w_1\dot{\mathcal{R}} - w_2\alpha)\partial^2\psi - \frac{2w_1}{a^2}\,\alpha\partial^2\mathcal{R} + 3w_2\alpha\dot{\mathcal{R}} + \frac{1}{3}\,w_3\alpha^2 + \frac{w_4}{a^2}\,(\partial\mathcal{R})^2 \right]\,,$$

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$$\begin{aligned}
w_1 &= M_P^2 F - 4XG_{4,X} - 2HX\dot{\phi}G_{5,X} + 2XG_{5,\phi}\,,\\
w_2 &= 2M_P^2 HF - 2X\dot{\phi}G_{3,X} - 16H(XG_{4,X} + X^2G_{4,XX}) + 2\dot{\phi}(G_{4,\phi} + 2XG_{4,\phi X})\\
&\quad - 2H^2\dot{\phi}(5XG_{5,X} + 2X^2G_{5,XX}) + 4HX(3G_{5,\phi} + 2XG_{5,\phi X})\,,\\
w_3 &= -9M_{\rm pl}^2 H^2 F + 3(XP_{,X} + 2X^2P_{,XX}) + 18H\dot{\phi}(2XG_{3,X} + X^2G_{3,XX}) - 6X(G_{3,\phi} + XG_{3,\phi X})\\
&\quad +18H^2(7XG_{4,X} + 16X^2G_{4,XX} + 4X^3G_{4,XXX}) - 18H\dot{\phi}(G_{4,\phi} + 5XG_{4,\phi X} + 2X^2G_{4,\phi XX})\\
&\quad + 6H^3\dot{\phi}(15XG_{5,X} + 13X^2G_{5,XX} + 2X^3G_{,5XXX}) - 18H^2X(6G_{5,\phi} + 9XG_{5,\phi X} + 2X^2G_{5,\phi XX})\,,\\
w_4 &= M_P^2 F - 2XG_{5,\phi} - 2XG_{5,X}\ddot{\phi}\,.
\end{aligned}$$

Second order Lagrangian

- $\alpha = 2w_1\dot{\mathcal{R}}/w_2$

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Second order Lagrangian

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$$\begin{aligned} Q &= \frac{w_1(4w_1w_3 + 9w_2^2)}{3w_2^2} \, , \\ c_s^2 &= \frac{3(2w_1^2w_2H - w_2^2w_4 + 4w_1\dot{w}_1w_2 - 2w_1^2\dot{w}_2)}{w_1(4w_1w_3 + 9w_2^2)} \, . \end{aligned}$$

Three-point function

- Look for

$$\langle \mathcal{R}(\boldsymbol{k}_1)\mathcal{R}(\boldsymbol{k}_2)\mathcal{R}(\boldsymbol{k}_3)\rangle = -i\int_{\tau_i}^{\tau_f} d\tau\, a\, \langle 0|\left[\mathcal{R}(\tau_f,\boldsymbol{k}_1)\mathcal{R}(\tau_f,\boldsymbol{k}_2)\mathcal{R}(\tau_f,\boldsymbol{k}_3), \mathcal{H}_{\rm int}(\tau)\right]|0\rangle\,,$$

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- 3rd order action

$$\begin{aligned} S_3 \quad = \quad & \int dt\, d^3x\, a^3 \,\{a_1\,\alpha^3 + \alpha^2\,(a_2\,\mathcal{R} + a_3\,\dot{\mathcal{R}} + a_4\,\partial^2\mathcal{R}/a^2 + a_5\partial^2\psi/a^2) \\ & + \alpha\,[a_6\,\partial_i\mathcal{R}\partial_i\psi/a^2 + a_7\,\dot{\mathcal{R}}\mathcal{R} + a_8\,\dot{\mathcal{R}}\partial^2\mathcal{R}/a^2 + a_9\,(\partial_i\partial_j\psi\partial_i\partial_j\psi - \partial^2\psi\partial^2\psi)/a^4 \\ & + a_{10}(\partial_i\partial_j\psi\partial_i\partial_j\mathcal{R} - \partial^2\psi\partial^2\mathcal{R})/a^4 + a_{11}\,\mathcal{R}\,\partial^2\psi/a^2 + a_{12}\,\dot{\mathcal{R}}\,\partial^2\psi/a^2 + a_{13}\,\mathcal{R}\,\partial^2\mathcal{R}/a^2 \\ & + a_{14}\,(\partial\mathcal{R})^2/a^2 + a_{15}\dot{\mathcal{R}}^2] \\ & + b_1\,\dot{\mathcal{R}}^3 + b_2\,\mathcal{R}\,(\partial\mathcal{R})^2/a^2 + b_3\dot{\mathcal{R}}^2\,\mathcal{R} + c_1\,\dot{\mathcal{R}}\partial_i\mathcal{R}\partial_i\psi/a^2 + c_2\dot{\mathcal{R}}^2\partial^2\psi/a^2 + c_3\dot{\mathcal{R}}\,\mathcal{R}\,\partial^2\psi/a^2 \\ & + (d_1\dot{\mathcal{R}} + d_2\mathcal{R})\,(\partial_i\partial_j\psi\partial_i\partial_j\psi - \partial^2\psi\partial^2\psi)/a^4 + d_3\partial_i\mathcal{R}\partial_i\psi\,\partial^2\psi/a^4\}\,, \end{aligned}$$

Reducing the action

- S_3:

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$$\begin{aligned}
f_1 &\equiv \left(A_1 + A_3\frac{Q}{w_1} - A_5\frac{Q^2}{w_1^2}\right)\dot{\mathcal{R}}^3 + \left(A_4 - A_6\frac{Q^2}{w_1^2}\right)\mathcal{R}\dot{\mathcal{R}}^2 + A_9\frac{Q}{w_1^2}\dot{\mathcal{R}}\partial_i\mathcal{R}\partial_i\mathcal{X} \\
&+ \frac{1}{w_1^2}\left(A_5\dot{\mathcal{R}} + A_6\mathcal{R}\right)(\partial_i\partial_j\mathcal{X})(\partial_i\partial_j\mathcal{X})\,, \\
f_2 &\equiv \left(A_2 - A_3L_1 + A_5\frac{2L_1Q}{w_1} - A_7\frac{Q}{w_1}\right)\dot{\mathcal{R}}^2\partial^2\mathcal{R} + A_6\frac{2L_1Q}{w_1}\mathcal{R}\dot{\mathcal{R}}\partial^2\mathcal{R} + A_8\mathcal{R}(\partial\mathcal{R})^2 - A_9\frac{L_1Q}{w_1}\dot{\mathcal{R}}(\partial\mathcal{R})^2 \\
&+ \frac{A_7 - 2A_5L_1}{w_1}\dot{\mathcal{R}}(\partial_i\partial_j\mathcal{R})(\partial_i\partial_j\mathcal{X}) - \frac{2A_6L_1}{w_1}\mathcal{R}(\partial_i\partial_j\mathcal{R})(\partial_i\partial_j\mathcal{X}) - \frac{A_9L_1}{w_1}\partial^2\mathcal{R}\partial_i\mathcal{R}\partial_i\mathcal{X}\,, \\
f_3 &\equiv \left(A_5L_1^2 - A_7L_1\right)\dot{\mathcal{R}}\left[(\partial_i\partial_j\mathcal{R})(\partial_i\partial_j\mathcal{R}) - (\partial^2\mathcal{R})^2\right] + A_6L_1^2\mathcal{R}\left[(\partial_i\partial_j\mathcal{R})(\partial_i\partial_j\mathcal{R}) - (\partial^2\mathcal{R})^2\right] \\
&+ A_9L_1^2(\partial\mathcal{R})^2\partial^2\mathcal{R}\,.
\end{aligned}$$

Reducing the action — 2

- Finally $S_3 = \int dt\, \mathcal{L}_3$

$$\begin{aligned}\mathcal{L}_3 \quad = \quad & \int d^3x \Big\{ a^3 \mathcal{C}_1 M_P^2 \mathcal{R}\dot{\mathcal{R}}^2 + a\, \mathcal{C}_2 M_P^2 \mathcal{R}(\partial\mathcal{R})^2 + a^3 \mathcal{C}_3 M_P \dot{\mathcal{R}}^3 + a^3 \mathcal{C}_4 \dot{\mathcal{R}}(\partial_i \mathcal{R})(\partial_i \mathcal{X}) \\ & + a^3 (\mathcal{C}_5/M_P^2) \partial^2\mathcal{R}(\partial\mathcal{X})^2 + a\mathcal{C}_6 \dot{\mathcal{R}}^2 \partial^2 \mathcal{R} + \mathcal{C}_7 \left[\partial^2\mathcal{R}(\partial\mathcal{R})^2 - \mathcal{R}\partial_i\partial_j(\partial_i\mathcal{R})(\partial_j\mathcal{R})\right]/a \\ & + a(\mathcal{C}_8/M_P)\left[\partial^2\mathcal{R}\partial_i\mathcal{R}\partial_i\mathcal{X} - \mathcal{R}\partial_i\partial_j(\partial_i\mathcal{R})(\partial_j\mathcal{X})\right] + \mathcal{F}_1 \frac{\delta\mathcal{L}_2}{\delta\mathcal{R}}\bigg|_1 \Big\},\end{aligned}$$

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- Looking for

$$\langle\mathcal{R}(\boldsymbol{k}_1)\mathcal{R}(\boldsymbol{k}_2)\mathcal{R}(\boldsymbol{k}_3)\rangle = -i\int_{\tau_i}^{\tau_f} d\tau\; a\,\langle 0|\left[\mathcal{R}(\tau_f,\boldsymbol{k}_1)\mathcal{R}(\tau_f,\boldsymbol{k}_2)\mathcal{R}(\tau_f,\boldsymbol{k}_3),\mathcal{H}_{\rm int}(\tau)\right]|0\rangle\,,$$

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$$\langle \mathcal{R}(\boldsymbol{k}_1)\mathcal{R}(\boldsymbol{k}_2)\mathcal{R}(\boldsymbol{k}_3)\rangle = (2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)(\mathcal{P}_{\mathcal{R}})^2 B_{\mathcal{R}}(k_1,k_2,k_3)\,,$$

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$$B_{\mathcal{R}}(k_1,k_2,k_3) = \frac{(2\pi)^4}{\prod_{i=1}^3 k_i^3}\mathcal{A}_{\mathcal{R}}\,.$$

$$
\begin{aligned}
\mathcal{A}_{\mathcal{R}} \quad = \quad & \frac{M_P^2}{Q}\Bigg\{\frac{1}{4}\left(\frac{2}{K}\sum_{i>j}k_i^2k_j^2-\frac{1}{K^2}\sum_{i\neq j}k_i^2k_j^3\right)\mathcal{C}_1+\frac{1}{4c_s^2}\left(\frac{1}{2}\sum_i k_i^3+\frac{2}{K}\sum_{i>j}k_i^2k_j^2-\frac{1}{K^2}\sum_{i\neq j}k_i^2k_j^3\right)\mathcal{C}_2 \\
& +\frac{3}{2}\frac{H}{M_P}\frac{(k_1k_2k_3)^2}{K^3}\mathcal{C}_3+\frac{1}{8}\frac{Q}{M_P^2}\left(\sum_i k_i^3-\frac{1}{2}\sum_{i\neq j}k_ik_j^2-\frac{2}{K^2}\sum_{i\neq j}k_i^2k_j^3\right)\mathcal{C}_4 \\
& +\frac{1}{4}\left(\frac{Q}{M_P^2}\right)^2\frac{1}{K^2}\left[\sum_i k_i^5+\frac{1}{2}\sum_{i\neq j}k_ik_j^4-\frac{3}{2}\sum_{i\neq j}k_i^2k_j^3-k_1k_2k_3\sum_{i>j}k_ik_j\right]\mathcal{C}_5 \\
& +\frac{3}{c_s^2}\left(\frac{H}{M_P}\right)^2\frac{(k_1k_2k_3)^2}{K^3}\mathcal{C}_6 \\
& +\frac{1}{2c_s^4}\left(\frac{H}{M_P}\right)^2\frac{1}{K}\left(1+\frac{1}{K^2}\sum_{i>j}k_ik_j+\frac{3k_1k_2k_3}{K^3}\right)\left[\frac{3}{4}\sum_i k_i^4-\frac{3}{2}\sum_{i>j}k_i^2k_j^2\right]\mathcal{C}_7 \\
& +\frac{1}{8c_s^2}\frac{H}{M_P}\frac{Q}{M_P^2}\frac{1}{K^2}\left[\frac{3}{2}k_1k_2k_3\sum_i k_i^2-\frac{5}{2}k_1k_2k_3K^2-6\sum_{i\neq j}k_i^2k_j^3-\sum_i k_i^5+\frac{7}{2}K\sum_i k_i^4\right]\mathcal{C}_8\Bigg\}\,.
\end{aligned}
$$

Observable

- Parameter $f_{\rm NL}$

$$f_{\rm NL} = \frac{10}{3} \frac{\mathcal{A}_{\mathcal{R}}}{\sum_{i=1}^{3} k_i^3} \, .$$

Observable

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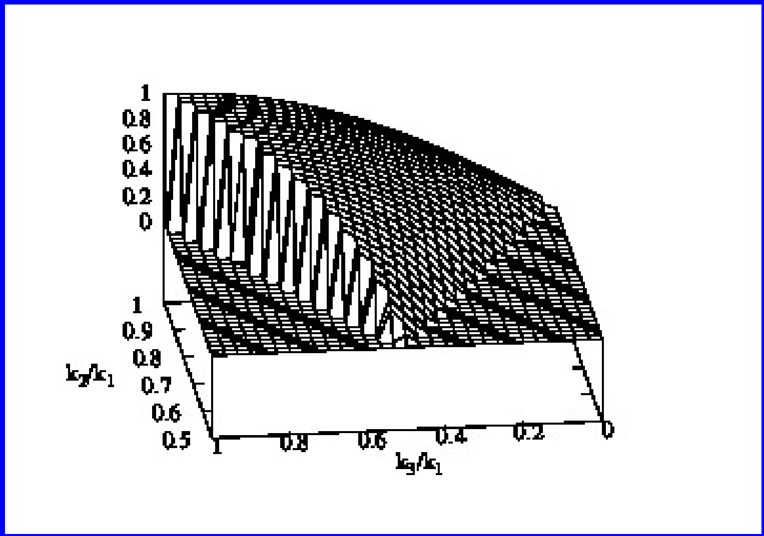
$$f_{\rm NL} = \frac{10}{3}\frac{\mathcal{A}_{\mathcal{R}}}{\sum_{i=1}^{3} k_i^3}\,.$$

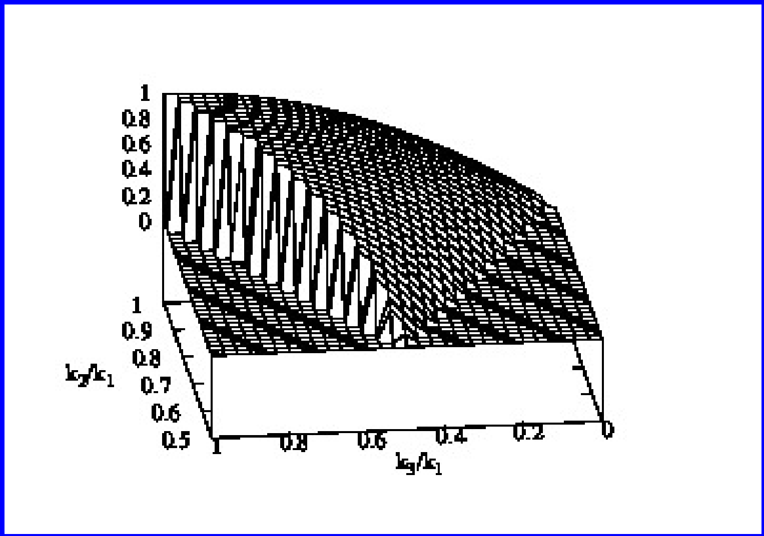
- Equilater configuration $k_1 = k_2 = k_3 = k$, $K = 3k$

$$\begin{aligned} f_{\rm NL} \quad = \quad & \frac{40}{9}\frac{M_P^2}{Q}\left[\frac{1}{12}\mathcal{C}_1 + \frac{17}{96c_s^2}\mathcal{C}_2 + \frac{1}{72}\frac{H}{M_P}\mathcal{C}_3 - \frac{1}{24}\frac{Q}{M_P^2}\mathcal{C}_4 - \frac{1}{24}\left(\frac{Q}{M_P^2}\right)^2\mathcal{C}_5 + \frac{1}{36c_s^2}\left(\frac{H}{M_P}\right)^2\mathcal{C}_6\right. \\ & \left. -\frac{13}{96c_s^4}\left(\frac{H}{M_P}\right)^2\mathcal{C}_7 - \frac{17}{192c_s^2}\frac{H}{M_P}\frac{Q}{M_P^2}\mathcal{C}_8\right]. \end{aligned}$$

Shape

- Maximum on the equilateral configuration





Slow-roll approximation

- Assuming slow roll

$$
\begin{aligned}
f_{\rm NL}^{\rm equil} \quad = \quad & \frac{85}{324}\left(1-\frac{1}{c_s^2}\right)-\frac{10}{81}\frac{\lambda}{\Sigma}+\frac{55}{36}\frac{\epsilon_s}{c_s^2}+\frac{5}{12}\frac{\eta_s}{c_s^2}-\frac{85}{54}\frac{s}{c_s^2}+\left(\frac{20}{81}\frac{1+\lambda_{3X}}{\epsilon_s}+\frac{65}{162c_s^2\epsilon_s}\right)\delta_{G3X} \\
& +\left(\frac{80}{81}\frac{3+2\lambda_{4X}}{\epsilon_s}+\frac{65}{27c_s^2\epsilon_s}\right)\delta_{G4XX}+\left(\frac{20}{81\epsilon_s}+\frac{65}{162c_s^2\epsilon_s}\right)\delta_{G5X} \\
& +\left(\frac{20}{81}\frac{5+2\lambda_{5X}}{\epsilon_s}+\frac{65}{162c_s^2\epsilon_s}\right)\delta_{G5XX}\,.
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\end{aligned}
$$

$$
\lambda_{3X}\equiv\frac{XG_{3,XX}}{G_{3,X}}\,,\qquad \lambda_{4X}\equiv\frac{XG_{4,XXX}}{G_{4,XX}}\,,\qquad \lambda_{5X}\equiv\frac{XG_{5,XXX}}{G_{5,XX}}\,.
$$

Examples

- k-inflation, $\epsilon = P_{,X}X/(3M_P^2H^2)$, $\lambda_{PX} = XP_{,XX}/P_{,X}$, $c_s^2 = 1/(1+2\lambda_{PX})$, $\lambda_{PXX} = X^2P_{,XXX}/P_{,X}$

$$f_{\rm NL}^{\rm equil} \simeq \frac{5}{324}\left(1-\frac{1}{c_s^2}\right)(17+4c_s^2) - \frac{20}{243}\frac{\lambda_{PXX}}{1+2\lambda_{PX}} + \frac{55}{36}\frac{\epsilon_s}{c_s^2} + \frac{5}{12}\frac{\eta_s}{c_s^2} - \frac{85}{54}\frac{s}{c_s^2}\,.$$

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- k-inflation $+$ G_i $(i = 3,4,5)$: $\big(P = -X + X^2/(2M^4)$, $G_3 = \mu X^2/M^4$, $G_4 = \mu X^2/M^7$, $G_5 = \mu X^2/M^{10}\big)$

$G_3:\ f_{\rm NL} \simeq 4.62 r^{-2/3}$, $G_4:\ f_{\rm NL} \simeq 1.28 r^{-2/3}$, $G_5:\ f_{\rm NL} \simeq 0.17 r^{-2/3}$.

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- Ph.D. scholarships for students: contact me!