Quantum Stress Tensor Fluctuations and Cosmology - Part 1

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Roles of the stress tensor $T_{\mu\nu}(x)$

1) Describes energy densities, fluxes, and forces on material objects

2) Acts as the source of gravity through Einstein's equation

3) In quantum field theory, it becomes an operator

Physical states are not eigenstates of $T_{\mu\nu}(x)$

$\Longrightarrow$ stress tensor fluctuations

Effects of stress tensor fluctuations:

1) Force fluctuations on material bodies

2) Passive quantum fluctuations of spacetime geometry

Distinct from the active fluctuations from the dynamical degrees of freedom of gravity itself, but still a quantum gravity effect

Stress tensor correlation function

(noise kernal)

$$K_{\mu\nu\rho\sigma} = \langle T_{\mu\nu}(x) T_{\rho\sigma}(x')\rangle - \langle T_{\mu\nu}(x)\rangle\langle T_{\rho\sigma}(x')\rangle$$

Independent of renormalization

Singular in the coincidence limit: $K_{\mu\nu\rho\sigma} \sim \dfrac{1}{(x-x')^8}$

Observables are integrals of $K_{\mu\nu\rho\sigma}$

Well-defined as a distribution

E.g., define integrals by integration by parts.

Some features of stress tensor fluctuations:

Subtle correlations and anticorrelations

Negative energy fluctuations

Negative power spectra

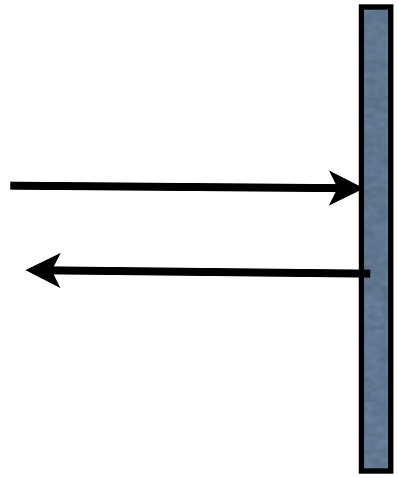
Radiation Pressure Fluctuations

Can be viewed in two equivalent ways:

1) Uncertainty in the number of photons hitting a mirror (Caves)

2) The effect of quantum stress tensor fluctuations (C-H Wu & LF)

$$v = \frac{1}{m} \int_0^{\tau} dt \int_A da \; T_{xx}$$



Stress tensor fluctuations lead to velocity fluctuations:

$$\langle \triangle v^2 \rangle = \frac{1}{m^2} \int_0^\tau dt \int_0^\tau dt' \int_A da \int_A da' \, [\langle : T_{xx}(x) :: T_{xx}(x') : \rangle - \langle : T_{xx}(x) : \rangle \langle : T_{xx}(x') : \rangle]$$

Result (coherent state): $$\langle \triangle v^2 \rangle = 4 \, \frac{A\omega\rho}{m^2} \, \tau$$

$\omega = \text{frequency of light}$

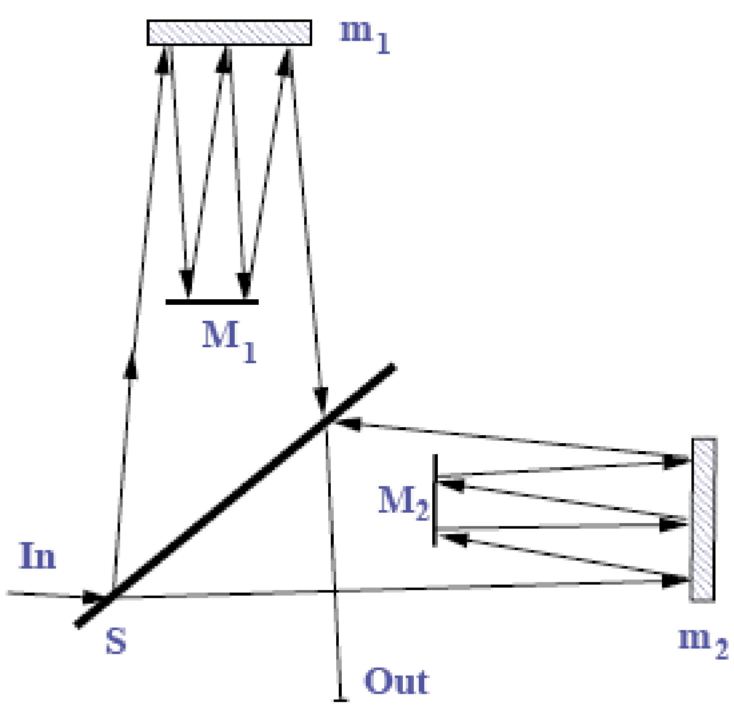
$\rho = \text{energy density of light}$

$A = \text{area illuminated}$

Effect comes from a "cross term":

$$\langle T_{\mu\nu}(x)T_{\rho\sigma}(x')\rangle_{cross} = \Sigma[\langle \text{state dependent}\rangle \; \langle \text{vacuum}\rangle]$$

The vacuum part enforces correlations between different bounces of a beam in an interferometer:



Fluctuations grow linearly (not quadratically) with the number of illuminated spots.

Vacuum fluctuations: $\langle 0| : T_{\mu\nu} : |0\rangle = 0$

Mean value of zero means both positive and negative fluctuations

Probability distribution for quantum stress tensor fluctuations

Need to look at an operator averaged over a sampling function.

Must be a skewed, non-Gaussian distribution

Odd moments are nonzero

The probability distribution will have a lower cutoff at the quantum inequality bound, the lowest eigenvalue of the averaged operator.

Quantum inequalities - bounds on expectation values in any quantum states

$$\int \langle T_{\mu\nu} \rangle \, u^{\mu} \, u^{\nu} \, g(\tau, \tau_0) \, d\tau \geq -\frac{C}{\tau_0^d}$$

$g(\tau, \tau_0)$ = sampling function

τ_0 = sampling time

C = positive constant

d = spacetime dimension

Quantum inequalities place strong limits on negative energy density and its physical effects:

Prevent violations of the 2nd law of thermodynamics

Strongly constrain traversable wormholes, warp drive spacetimes, and time machines

Let $u = \int T_{tt}\, g(t,\tau)\, dt$ averaged energy density

A result for vacuum fluctuations in conformal field theory (2 spacetime dimensions)

$$u = \frac{1}{\sqrt{\pi}\tau} \int_{-\infty}^{\infty} T_{tt}(x,t)\, e^{-t^2/\tau^2}\, dt$$

C. Fewster, T. Roman & LF

Probability distribution:

$$P(x) = \frac{\pi^{c/24}}{\Gamma(c/24)}\, (x + x0)^{\frac{c}{24} - 1}\, e^{-\pi(x+x_0)}$$

$$x = u\,\tau^2$$

$P(x) = 0 \quad x < x_0$

x_0 = quantum inequality bound

c = central charge

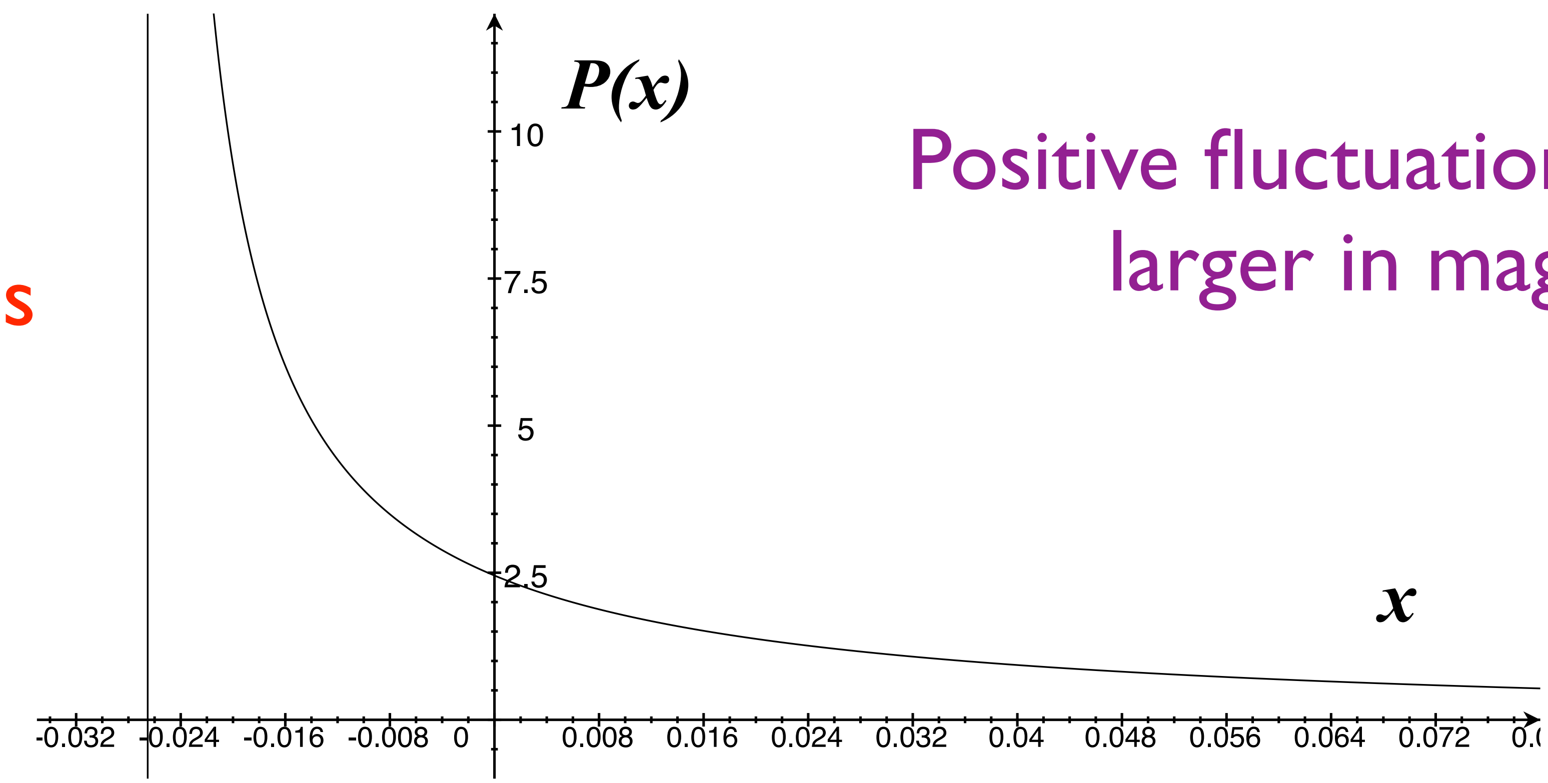
A massless scalar field in two dimensions (c = 1):

$$P(x) = \frac{\pi^{1/24}}{\Gamma(1/24)} \left(x + \frac{1}{24\pi} \right)^{-23/24} e^{-\pi(x+1/24\pi)}$$

$$P(x) = 0 \quad x < -\frac{1}{24\pi}$$

84% chance of finding u < 0

Negative energy is more likely than positive energy.



Positive fluctuations tend to be larger in magnitude.

Some results for 4D theories:

Qualitatively similar results - a lower bound and a long positive tail

As before, the lower bound of P(x) is at the quantum inequality bound for expectation values

This seems to prevent quantum nucleation of large wormholes ,ect.

However, the tail falls more slowly than in 2D

Hamburger moment condition is not fulfilled, so P(x) is not strictly determined by its moments

Lorentzian average of the EM energy density:

$$x = 16\pi\tau^5 \int_{-\infty}^{\infty} dt\, \frac{T_{tt}(\mathbf{x}, t)}{t^2 + \tau^2}$$

Inferred asymptotic form for the tail:

$$P(x) \sim x^{-2}\, \mathrm{e}^{-a x^{1/3}} \qquad x \gg 1$$

$$a \approx 0.76$$

Probability falls more slowly than exponentially

Vacuum effects eventually dominate thermal fluctuations

Rare positive fluctuations are enhanced

Estimated black hole nucleation rate:

one 400 Planck mass BH per cubic cm per second

Nucleation of observers (“Boltzmann brains)?

May complicate attempts at anthropic explanation

Page’s estimate for the nucleation rate:

$$R \approx \mathrm{e}^{-I} \approx \mathrm{e}^{-10^{50}}$$

$I = Mt = \text{action} = (1\,\mathrm{kg})(0.3\,\mathrm{s})$ (Units irrelevant!)

Our revised estimate: $R \approx \mathrm{e}^{-10^{26}}$

Much larger!

“Boltzmann Brains”

Stress tensor and expansion fluctuations

u^α = 4-velocity of a congruence of timelike geodesics

$\theta = u^\alpha_{;\alpha}$ = expansion of the congruence

Raychaudhuri equation

$$\frac{d\theta}{d\lambda} = -R_{\mu\nu}u^\mu u^\nu - \frac{1}{3}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu}$$

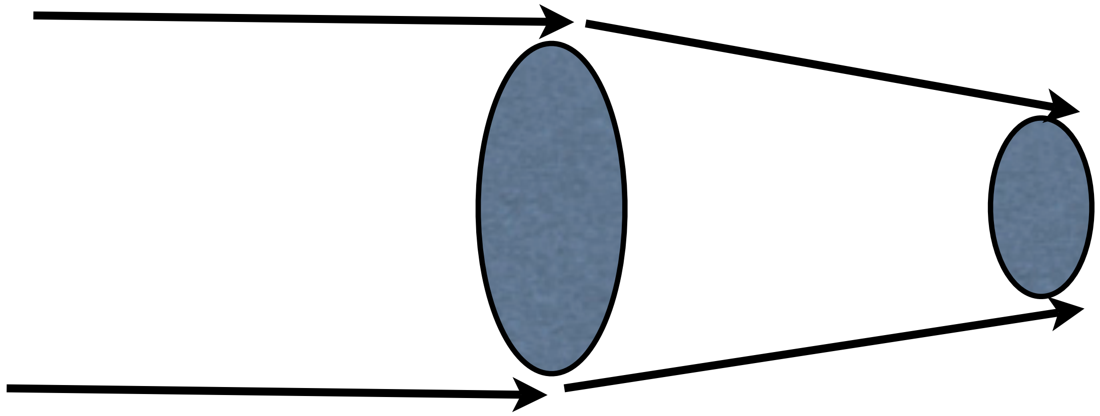
coefficient of 1/2 for null rays

$$R_{\mu\nu} = 8\pi(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T)$$

Ordinary matter: focussing

Expansion as the logarithmic derivative of the cross sectional area of bundle of rays:

$$\theta = \frac{d \log A}{d\lambda}$$



$T_{\mu\nu}$ Fluctuations $\longrightarrow$ Ricci tensor Fluctuations

$\longrightarrow$ θ Fluctuations $\longrightarrow$ Luminosity Fluctuations

luminosity variance as an integral of the θ correlation function:

$$(\Delta L)^2 = \int d\lambda \int d\lambda' \left[\langle \theta(\lambda)\, \theta(\lambda') \rangle - \langle \theta(\lambda) \rangle \langle \theta(\lambda') \rangle\right] \; L^2$$

Ignore the quadratic terms in the Raychaudhuri equation, so

$$\frac{d\theta}{d\lambda} = -R_{\mu\nu} u^\mu u^\nu$$

Write the θ correlation function as integrals of the Ricci tensor correlation function.

Normally a small effect in the present universe; an estimate for the effects of a thermal bath on light rays:

$$\left(\frac{\Delta L}{L}\right)_{rms} = 0.02\,\left(\frac{s}{10^{28}\mathrm{cm}}\right)^{\frac{3}{2}}\,\left(\frac{T}{10^6 K}\right)^{\frac{7}{2}} = 10^{-3}\,\left(\frac{s}{10^6\mathrm{km}}\right)^{\frac{3}{2}}\,\left(\frac{T}{1\mathrm{GeV}}\right)^{\frac{7}{2}}$$

J. Borgman & LF

s = flight distance

Where the effects of θ fluctuations might be large:

1) Small scale structure of spacetime

Carlip, Mosna, Pitelli arXiv:1103.5993

Effects of the rare, positive stress tensor fluctuations seems to cause lightcones to close on scales of about 10 Planck lengths - “asymptotic silence”.

2) Early universe

3) Cases where cancellation of anticorrelated fluctuations does not occur

Role of anticorrelations in quantum field fluctuations:

Limit the growth of fluctuations and enforce energy conservation

Example: a charged particle coupled to the quantized electromagnetic field in flat spacetime

Time dependence can upset the anticorrelations and provide an external energy source

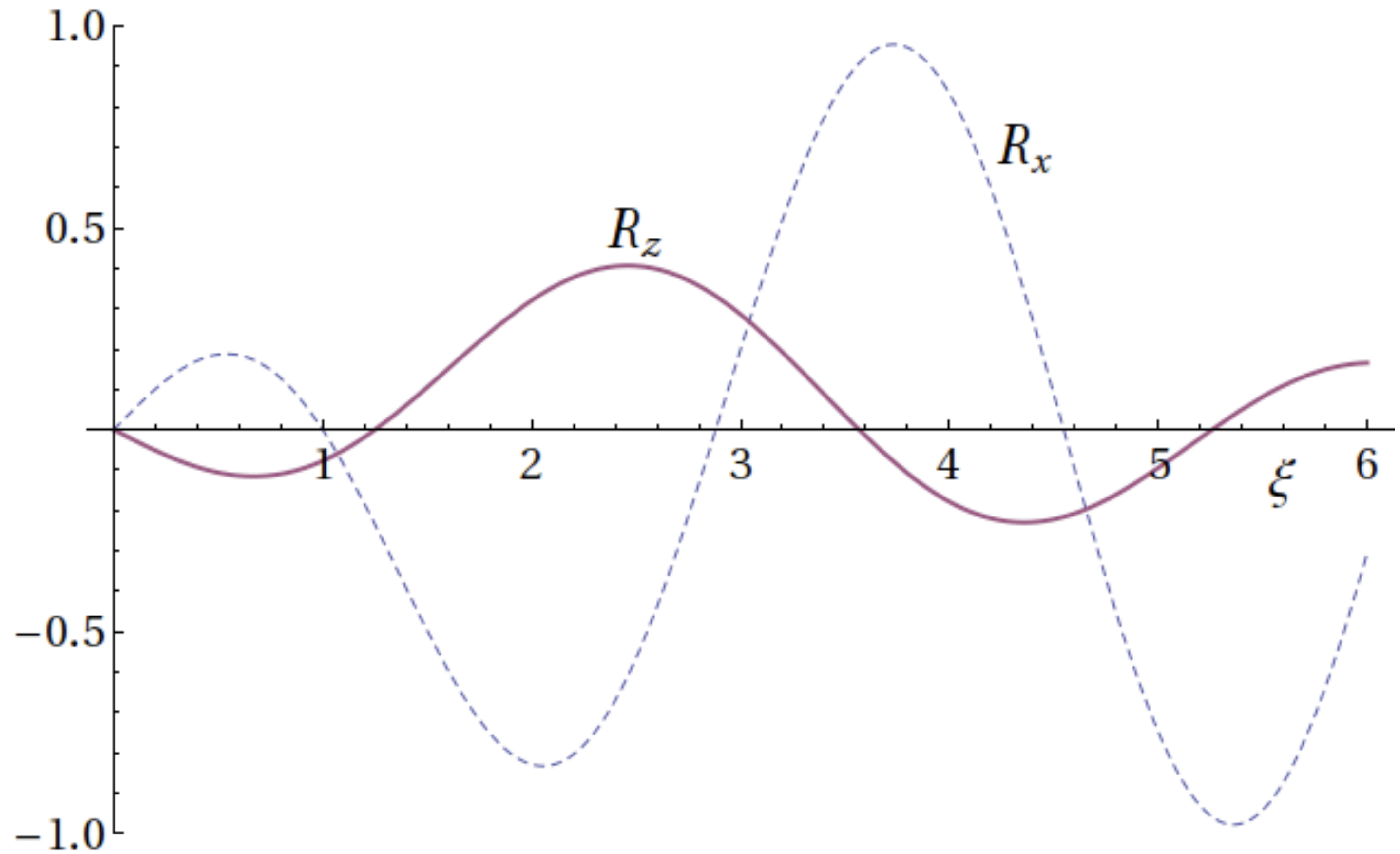
Example: an oscillating charge near a mirror

V. Parkinson & LF

Rate of change of velocity variance:

$$R_i(\xi) = \frac{16\pi m^4 d}{q^4 E_0^2} \frac{d\langle \Delta v_i^2 \rangle}{dt}$$

$$\xi = \omega\, d$$



A model with rapid scale factor oscillations:

$$ds^2 = a^2(\eta)(-d\eta^2 + dx^2 + dy^2 + dz^2)$$

$$a(\eta) = 1 + A\cos(\omega\,\eta) \qquad A \ll 1$$

Luminosity fluctuations of a distant source grow with distance, s:

$$\left(\frac{\Delta L}{L}\right)^2 \propto A^2\,\ell_p^4\,s^3\,\omega^5\,\tau_0^{-2}$$

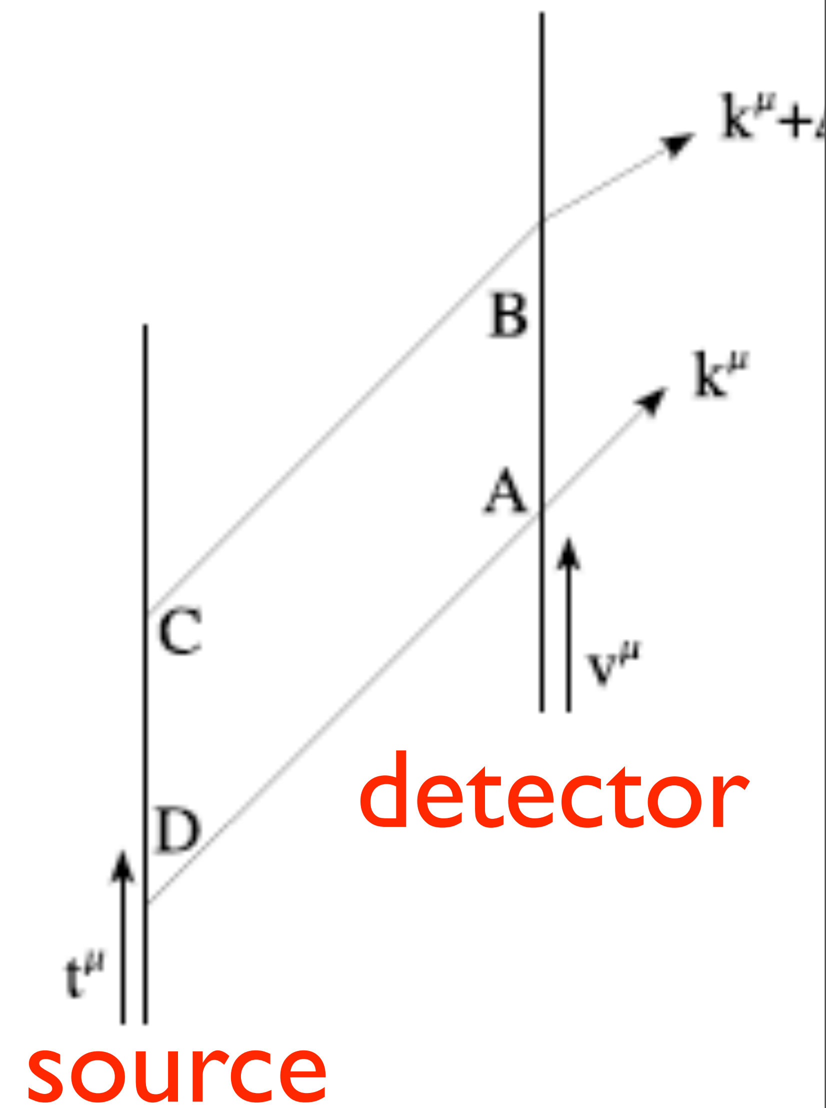
ℓ_p = Planck length

τ_0 = sampling time

Other physical effects of curvature fluctuations:

Spectral line broadening

Angular blurring of images



k^μ = 4-momentum of a photon exchanged by two inertial observers

$t^\mu,\ v^\mu$ = observers' 4-velocity

fractional redshift due to curvature:

$$\frac{\Delta\omega(\lambda_0)}{\omega_0} = \frac{\omega(\tau_2,\lambda_0) - \omega(\tau_1,\lambda_0)}{\omega_0} = -v_\mu \Delta k^\mu = v_\mu \int_{\tau_1}^{\tau_2} d\tau \int_0^{\lambda_0} d\lambda\, R^\mu{}_{\alpha\nu\beta} k^\alpha t^\nu k^\beta$$

angular shift in the direction of s^μ

$$\Delta\Theta = s_\mu \Delta k^\mu = \int da\, R_{\alpha\beta\mu\nu} s^\alpha k^\beta t^\mu k^\nu .$$

Both frequency and angle fluctuations can be written as integrals of the Riemann tensor correlation function.

These effects can arise from either active or passive quantum gravity fluctuations.

As there situations where these effects can accumulate over cosmological distances?

Role of stress tensor fluctuations in inflation

Density perturbations

K.W. Ng, C.H. Wu &LF, PRD 75 103502 (2007)

S.P. Miao, K.W. Ng, R.P. Woodard, C.H. Wu &LF, PRD 82 043501 (2010)

Talk by C-H Wu

Gravity Waves

C-H Wu, J-T Hsiang, K-W Ng, &LF, PRD 84 103515 (2011)

Talk by J-T Hsiang

Summary

Stress tensor fluctuations produce force fluctuations and passive quantum gravity effects

Skewed, highly non-Gaussian probability distributions

Possible applications to anthropic reasoning and small scale structure of spacetime

Stress tensor fluctuations produce fluctuations of luminosity, line broadening, and angular blurring

In some cases, it is possible to enhance these effects by preventing cancellation of anticorrelated fluctuations.