Gregory Gabadadze (NYU) "Massive General Relativity"

Motivation
- Dark Energy
- Cosmological constant problem
- Field th. for IR modification of GR $\sim 10^{26}\,\mathrm{cm}$

5 d.o.f on nearly Minkowski background

massive spin $J=2$

Poincare inv $m \neq 0$. $2J+1=5$

GR: helicity ± 2

mGR: helicity $\pm 2, \pm 1, 0$

$= \mathrm{GR} + \text{vector} + \text{scalar}$

Linear th. Fierz-Pauli '39

· non-linear ext.

No-Go, Boulware-Deser '72

$5+1 \to$ BD ghost

· Particular ths evade the BD ghost

de Rham, GG

de Rham, GG, Tolley

$$g_{\mu\nu}(x) = \underbrace{\partial_\mu\varphi^a\partial_\nu\varphi^b\eta_{ab}}_{\text{ref metric}} + H_{\mu\nu}$$

$$K^\mu_\nu \equiv \delta^\mu_\nu - \sqrt{\delta^\mu_\nu - H^\mu_\nu} = \delta^\mu_\nu - \sqrt{\partial^\mu\varphi^a\partial_\nu\varphi^b\eta_{ab}}, \quad \partial^\mu = g^{\mu\alpha}\partial_\alpha$$

$$\mathcal{L} = \frac{M_P^2}{2}\sqrt{g}\left(R - m^2\left(U_2(K) + \alpha_3 U_3(K) + \alpha_4 U_4(K)\right)\right)$$

$$U_2(K) = K^\mu_\nu K^\nu_\mu = \tfrac{1}{2}\varepsilon_{\mu\alpha..}\varepsilon^{\nu\beta..}K^\mu_\nu K^\alpha_\beta$$

$$U_3(K) = 2(K^\mu_\nu)^3 - 3(K^\alpha_\beta)^2 K - K^3 = \varepsilon_{\mu\alpha\rho.}\varepsilon^{\nu\beta\sigma.}K^\mu_\nu K^\alpha_\beta K^\rho_\sigma$$

$$U_4(K) = 6(K^\mu_\nu)^4 \cdots = \varepsilon_{\mu\alpha\rho\lambda}\varepsilon^{\nu\beta\sigma\kappa}KKKK$$

$(m^2, \alpha_3, \alpha_4)$: three parameter family

U_1 is just c.c.

1. decoupling limit

Massive photon

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}^2 - \tfrac{1}{2}m^2 A_\mu^2$$

3 d.o.f.

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}^2 - \tfrac{1}{2}m^2\left(A_\mu - \partial_\mu\varphi/m\right)^2$$

$\delta A_\mu = \partial_\mu\alpha$. $\delta\varphi = m\alpha$: gauge sym.

$m \gg 0$. decoupling limit

$$\mathcal{L}_{dec} = -\tfrac{1}{4}F_{\mu\nu}^2 - \tfrac{1}{2}(\partial_\mu\varphi)^2$$

2 d.o.f. 1 d.o.f.

FP: $$\mathcal{L} = \frac{M_{pl}^2}{2}\left[h_{\mu\nu}(\hat{E}h)^{\mu\nu} - \frac{m^2}{4}(h_{\mu\nu}^2 - h^2)\right]$$

$$h_{\mu\nu} = \frac{\tilde h_{\mu\nu}}{M_{pl}} - \frac{\partial_\mu A_\nu + \partial_\nu A_\mu}{m M_{pl}} - \frac{\partial_\mu\partial_\nu\pi}{m^2 M_{pl}}$$

$\delta\tilde h_{\mu\nu} = \partial_{(\mu}J_{\nu)}$ $\quad \delta A_\nu = m J_\nu$

$\delta_g A_\nu = \partial_\nu\alpha$ $\quad \delta_g\pi = -m\alpha$

$m \to 0$ $M_{pl} \to \infty$

$$\mathcal{L} = \tilde h\hat{E}\tilde h - \tfrac{1}{4}F_{\mu\nu}^2(A) - \tilde h_{\mu\nu}(\partial_\mu\partial_\nu\pi - \eta_{\mu\nu}\partial^2\pi)$$

$2+2+1$ d.o.f.

$$= \tilde h\hat{E}\tilde h - \tfrac{1}{4}F_{\mu\nu}^2(A) - (\partial_\mu\pi)^2 + \tilde h_{\mu\nu}T^{\mu\nu} + \pi T$$

$$h_{\mu\nu} = \tilde h_{\mu\nu} + \eta_{\mu\nu}\pi$$

vDVZ $\to$ Vainshtein

$$\mathcal{L} = \frac{M_{pl}^2}{2}\sqrt{g}R - \frac{M_{pl}^2 m^2}{8}\sqrt{g}\left(H_{\mu\nu}^2 + H^2 + c_1 H_{\mu\nu}^3 + c_2 H_{\mu\nu}^2 H + c_3 H^3 + d_1 H_{\mu\nu}^4 + \cdots + d_5 H^4 + \cdots\right) \in U$$

$$\mathcal{L} = (\partial\pi)^2 + \frac{(\partial^2\pi)^3}{\Lambda^5} \to -(\partial\delta\pi)^2 + \frac{(\partial^2\pi_0)(\partial^2\delta\pi)^2}{\Lambda^5}$$

Bad because of ghost

necessary condition

$U|_{h=0}(\pi)$ = total derivative composed of $\Pi_{\mu\nu} = \partial_\mu\partial_\nu\pi$

$$U_2(\Pi) = (\partial_\mu\partial_\nu\pi)^2 - (\Box\pi)^2 = \varepsilon_{\mu\alpha..}\varepsilon^{\nu\beta..}\Pi^\mu_\nu\Pi^\alpha_\beta$$

$$U_3(\Pi) = \cdots = \varepsilon\varepsilon\Pi\Pi\Pi$$

$$U_4(\Pi) = \cdots = \varepsilon\varepsilon\Pi\Pi\Pi\Pi$$

$$\mathcal{L}_{dec} = h\hat{E}h + h_{\mu\nu}\overbrace{\left(X^{(1)}_{\mu\nu}(\pi) + \alpha X^{(2)}_{\mu\nu}(\pi) + \beta X^{(3)}_{\mu\nu}(\pi)\right)}^{X_{\mu\nu}(\pi)} + h_{\mu\nu}T^{\mu\nu}$$

$$X^{(1)}_{\mu\nu} = \partial_\mu\partial_\nu\pi - \eta_{\mu\nu}\partial^2\pi = \varepsilon_{\mu\alpha..}\varepsilon_{\nu\beta..}\Pi^{\alpha\beta}$$

$$X^{(2)}_{\mu\nu} = \cdots = \varepsilon\varepsilon\Pi\Pi$$

$$h_{\mu\nu} \to h_{\mu\nu} + \eta_{\mu\nu}\pi + \alpha\frac{\partial_\mu\pi\partial_\nu\pi}{\Lambda^3}$$

$$\mathcal{L} = hEh - (\partial\pi)^2 + \frac{\alpha(\partial\pi)^2\Box\pi}{\Lambda^3} + \frac{\alpha^2(\partial\pi)^2\left((\partial\partial\pi)^2 - (\Box\pi)^2\right)}{\Lambda^6} + \beta h X^{(3)}_{\mu\nu}(\pi) + \frac{(\alpha\beta\,\beta^2)}{\Lambda^6}(\partial\pi)^2(\ldots)$$

Galileon action appears

β term doesn't decouple

$\partial^\mu X^{(i)}_{\mu\nu} = 0$

important to maintain diffeo

Koyama, Niz, Tasinato

Mirbabayi

$$G^{Lin}_{\mu\nu} + X_{\mu\nu}(\pi) = 8\pi T_{\mu\nu}$$

$$\partial\partial h\left(1 + \alpha\partial^2\pi + \beta(\partial^2\pi)^2\right) = 0$$

SA: $\pi = q x_\mu^2$

$$\Rightarrow (1 + 8\alpha q + 64\beta q^2) = 0$$

$X_{\mu\nu}(\pi_a) \neq \Lambda^3\eta_{\mu\nu}$ background $\quad G_{\mu\nu} = \eta_{\mu\nu}\Lambda^3$ For $h=0$ bg.. decoupling doesn't happen

$$ds^2 = (\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu$$

$h_{\mu\nu} \sim \eta_{\mu\nu}x_\alpha^2 \quad R = m^2(\alpha, \beta)$

$h_{\mu\nu} = h^{cl}_{\mu\nu} + \chi_{\mu\nu} \quad \pi = \pi_{cl} + \varphi$

$$\mathcal{L} = \chi\hat{E}\chi + 0\,\cancel{\chi_{\mu\nu}X^{(1)}_{\mu\nu}(\varphi)} - (\alpha,\beta)(\partial\varphi)^2 + \chi_{\mu\nu}X^{(2)}_{\mu\nu}(\varphi) + \chi_{\mu\nu}T^{\mu\nu}$$

β cannot be 0

vector kinetic term $\neq 0$

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necessary condition

for $g_{\mu\nu} \to \eta_{\mu\nu}$, $K^\mu_\nu \to \partial^\mu\partial_\nu\pi$.

$\mathcal{L}$ should be total derivative

M. Mirbabayi (proven to be a sufficient condi.)

Self Acc. Sol. $\frac{1}{m}$ $\quad h_{\mu\nu} = m^2\eta_{\mu\nu}X_\alpha^2$, $\quad \pi/M_{pl} = m^2 X_\alpha^2$

$X^\mu \to X^\mu + C^\mu$: translation inv.

$$S_\pi = M_{pl}m^2(2C_\alpha X^\alpha + C^2)$$

Both are necessary

$\pi \to \pi + b_\mu X^\mu + C$ "Galilean"

How about in full theory?

Koyama, Niz, Tasinato

Th. Nieuwenhuizen

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + C(r)d\Omega^2$$

$$\varphi^a = (ct + T_0(r), X^i)$$

$$A = 1 - m^2r^2, \quad B = 1/A \quad C = r^2$$

$$V_0' = \frac{mr\sqrt{c - m^2r^2}}{1 - m^2r^2}$$

Under the transformation

$t \to F_t(\tau, \rho)$, $r \to F_r(\tau, \rho)$

$$ds^2 = -d\tau^2 + e^{m\tau}(d\rho^2 + \rho^2 d\Omega)$$

$\Rightarrow V_0(\tau, \rho)$, $V_r(\tau, \rho)$

this sector is not homogeneous

flat FRW? under complete homoge.

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2 \quad \varphi^0 = f(t) \quad \varphi^i = x^i$$

$$K^0_0 = 1 - \sqrt{g^{00}\partial_0\varphi^a\partial_0\varphi^b\eta_{ab}} = 1 - f'$$

$$K^i_j = \delta^i_j - \sqrt{\frac{\delta^i_j}{a^2}} = \delta^i_j\left(1 - \frac{1}{a}\right)$$

$$m^2\sqrt{g}\left((K_\mu^\nu)^2 - K^2\right) = m^2\dot{f}\left(a^3 - a^2\right) + m^2 g(a)$$

$$L = 3M_{pl}^2\left(-a\dot{a}^2 - m^2\dot{f}\left(a^3 - a^2\right) - m^2 g(a)\right)$$

$$\delta/\delta f \rightarrow \partial_0 m^2\left(a^3 - a^2\right) = 0$$

$$G_{\mu\nu} + m^2 X_{\mu\nu}(K) = 8\pi G_N T_{\mu\nu}$$

$$X_{\mu\nu} = \varepsilon_\mu \ldots \varepsilon_\nu^{\cdots} K^{\cdot}_{\cdot} + \alpha\,\varepsilon_\mu \ldots \varepsilon_\nu^{\cdots} K^{\cdot}_{\cdot} K^{\cdot}_{\cdot}$$
$$+ \beta\varepsilon\varepsilon KKK$$

$$m^2\nabla^\mu X_{\mu\nu}(K(\varphi, g)) = 0$$

generic ansatz

$$ds^2 = -dt^2 + C(t,r)\,dt\,dr + A^2(t,r)\,d\vec{x}^2$$

$$\varphi^a = u(t,r)$$
$$y^i = v(t,r)\frac{x^i}{r}$$

If $\rho \gg m^2 M_{pl}^2$, in small region

$$G_{\mu\nu} \simeq 8\pi G_N$$
$$A \simeq a(t) + a_1$$
$$C \simeq 0 + C_1(t,r)$$

Compting $X_{\mu\nu}$, corrections are calculable.

Gümrükçüoğlu, Lin, Mukohyama

open FRW can be constructed

$$H^2 = \frac{8\pi G_N \rho_m}{3} + \frac{1}{a^2} + m^2$$

strongcoupling issue

vector kinetic term

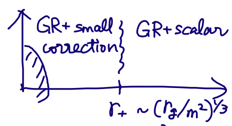
with Λ, $g_{\mu\nu} = \eta_{\mu\nu}$ is a sol.

but. Vainshtein mech. doesn't work.

Star ~ BH

asymptotic flat sol.

Gruzinov, Mirbabayi



$$\Delta\Phi_{DGP} \sim m^2 r^2 \left(\frac{r_*}{r}\right)^{3/2}$$

$$\Delta\Phi_m \sim m^2 r^2 \left(r_*/r\right)$$

power is different

what about hairs?

$$I^{ab} \equiv g^{\mu\nu}\partial_\mu\varphi^a\partial_\nu\varphi^b$$

, which should be non-singular

U.g. $g^{\mu\nu}\delta_\mu^a\delta_\nu^b = g^{ab}$

on horizon. singular

all solutions so far singular

BH sol. on S.A branch.

One should consider

GP coordinate

$$ds^2 = -dt^2 + \left(dr \pm \sqrt{f}dt\right)^2 + r^2 d\Omega^2$$

for $\beta = -\alpha^2/6$, we obtain Sch-dS.

$f = m^2 r^2 + r_g/r$ $\qquad \varphi^a \neq x^a$

Stückelberg sector is not unique

$$H_{GR} = N\left({}^{(3)}R + \pi^{i2}_{\ j} - \pi^2\right) + N_j D^i \pi_i^j$$

$$N = \frac{1}{\sqrt{-g^{00}}} \quad N_j = g_{0j} \quad \gamma_{ij} = g_{ij}$$

BD: Hamiltonian constraint cannot be kept.

Toy model: $H = N\left(\bar{R} + m^2 f(\gamma)\right) + N_j\left(R^j + m^2 Q^j(\gamma)\right)$
$$+ m^2\frac{N_j^2}{N}B(\gamma)$$

$$N_j = n_j N$$

$$H = N\left(\bar{R} + m^2 f(\gamma)\right) + n_j N\left(R^j + \cdots\right) + N[\qquad]$$

Similar way. (Hassan & Rosen)