Cosmological perturbations in nonlinear massive gravity

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Why massive gravity?

1. Is there a massive gravity theory which reduces smoothly to GR in the massless limit? Are the predictions of GR stable against small graviton mass?
2. Galactic curves, supernovae $\Rightarrow$ new types of (dark) matter and energy. Alternative approach: can these components be associated with the gravity sector, by large distance modifications of GR?

Massive extension of GR?

- Linear mass terms (Fierz, Pauli '39)
  $\Rightarrow$ Discontinuity with GR in the limit $m_g \to 0$ (van Dam, Veltman '70; Zakharov '70)
- Nonlinear effects can recover continuity (Vainshtein '72)
- Nonlinear extensions have generically an additional ghost degree. (Boulware, Deser '72)

(*See G. Gabadadze's lectures on Sat and Sun*)

Nonlinear massive gravity

de Rham, Gabadadze, Tolley '10

- Gauge invariant, nonlinear mass term:

$$S_m[g_{\mu\nu}, f_{\mu\nu}] = M_p^2 m_g^2 \int d^4x \sqrt{-g}\,(\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4)$$

$$\mathcal{L}_2 = \frac{1}{2}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right), \qquad \mathcal{L}_3 = \frac{1}{6}\left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right),$$

$$\mathcal{L}_4 = \frac{1}{24}\left([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]\right),$$

$$\mathcal{K}^\mu_\nu \equiv \delta^\mu_\nu - \left(\sqrt{g^{-1}f}\right)^\mu_\nu, \qquad [\cdots] \equiv \mathrm{Tr}(\cdots), \qquad f_{\mu\nu} \equiv \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$$

- $f_{\mu\nu}$: fiducial metric; ϕ^a : Stückelberg fields.
  $< \phi^a >$ breaks the general coordinate invariance.
  Unitary gauge: $\phi^a = \delta^a_\mu x^\mu$, $f_{\mu\nu} = \eta_{\mu\nu}$.
- By construction, free of BD ghost in the decoupling limit.
- For generic $f_{\mu\nu}$, free of BD ghost away from the decoupling limit.

Hassan, Rosen '11

Cosmological backgrounds

- $f_{\mu\nu}$ with FRW symmetry $\Rightarrow$ cosmological solutions

$$f_{\mu\nu} = -n^2(\varphi^0)\partial_\mu\varphi^0\partial_\nu\varphi^0 + \alpha^2(\varphi^0)\Omega_{ij}(\varphi^k)\partial_\mu\varphi^i\partial_\nu\varphi^j$$

$$\Omega_{ij}(\{\varphi^k\}) = \delta_{ij} + \frac{K\,\delta_{il}\delta_{jm}\varphi^l\,\varphi^m}{1 - K\,\delta_{lm}\varphi^l\varphi^m}$$

- $\varphi^a = \delta^a_\mu x^\mu$ in the unitary gauge.
- Metric ansatz: $g_{\mu\nu}dx^\mu dx^\nu = -N(t)^2dt^2 + a(t)^2\Omega_{ij}dx^idx^j$

Equations of motion for $\phi^a \Rightarrow$ 3 branches of solutions

- Branch I : $\dot{a}/N = \dot{\alpha}/n \Longrightarrow$ Trivial, evolution determined by $f_{\mu\nu}$.
- Branches $\mathrm{II}_\pm$: Two cosmological branches

$$\alpha(t) = X_\pm\, a(t)\,,$$

$$\text{with } X_\pm \equiv \frac{1 + 2\alpha_3 + \alpha_4 \pm \sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}}{\alpha_3 + \alpha_4} = \text{constant}$$

Background equations of motion

- Dynamics of Branch $\mathrm{II}_\pm$, with generic (conserved) matter source:

$H \equiv \frac{\dot{a}}{a\,N}$

$$3\,H^2 + \frac{3\,K}{a^2} = \Lambda_\pm + \frac{1}{M_{Pl}^2}\rho\,, \qquad -\frac{2\dot{H}}{N} + \frac{2\,K}{a^2} = \frac{1}{M_{Pl}^2}(\rho + P),$$

$$\Lambda_\pm \equiv -\frac{m_g^2}{(\alpha_3+\alpha_4)^2}\left[(1+\alpha_3)\left(2+\alpha_3+2\,\alpha_3^2-3\,\alpha_4\right) \pm 2\left(1+\alpha_3+\alpha_3^2-\alpha_4\right)^{3/2}\right]$$

- For Minkowski $f_{\mu\nu}$, only $K < 0$ solutions exists. $\longleftarrow$ *Chunshan Lin's talk*
- For dS fiducial, flat/open/closed FRW are allowed.

Perturbing the solution

- Lack of BD ghost does not guarantee stability.
  e.g. Higuchi's ghost (Higuchi '87)
- Scalar sector may include additional couplings, giving rise to potential conflict with observations.
- Can we distinguish massive gravity from other models of dark energy/modified gravity?

Introducing perturbations

- $f_{\mu\nu}$ does not depend on physical metric. FRW symmetry is preserved even when ϕ^a are perturbed.
- Perturbations in the metric, Stückelberg fields and matter fields:

$$g_{00} = -N^2(t)\,[1+2\phi]\,, \quad g_{0i} = N(t)a(t)\beta_i\,, \quad g_{ij} = a^2(t)\left[\Omega_{ij}(x^k) + h_{ij}\right]$$

$$\varphi^a = x^a + \pi^a + \frac{1}{2}\pi^b\partial_b\pi^a + O(\epsilon^3)\,, \qquad \sigma_I = \sigma_I^{(0)} + \delta\sigma_I$$

- Matter sector: a set of independent degrees of freedom $\{\sigma_I\}$.

Gauge invariant variables

- Scalar-vector-tensor decomposition:

$$\left.\begin{aligned} \beta_i = D_i\beta + S_i\,, \qquad \pi_i = D_i\pi + \pi_i^T\,, \\ h_{ij} = 2\psi\Omega_{ij} + \left(D_iD_j - \tfrac{1}{3}\Omega_{ij}\triangle\right)E + \tfrac{1}{2}(D_iF_j + D_jF_i) + \gamma_{ij} \end{aligned}\right\} \begin{matrix} D_i \leftarrow \Omega_{ij}\,,\ \triangle \equiv \Omega^{ij}D_iD_j \\ D^iS_i = D^i\pi_i^T = D^iF_i = 0 \\ D^i\gamma_{ij} = \gamma^i_i = 0 \end{matrix}$$

- Gauge invariant variables without Stückelberg fields:

Originate from $g_{\mu\nu}$ and matter fields $\delta\sigma_I$

$$\begin{aligned} Q_I &\equiv \delta\sigma_I - \mathcal{L}_Z\sigma_I^{(0)}\,, \\ \Phi &\equiv \phi - \tfrac{1}{N}\partial_t(NZ^0)\,, \\ \Psi &\equiv \psi - \tfrac{\dot a}{a}Z^0 - \tfrac{1}{6}\triangle E\,, \\ B_i &\equiv S_i - \tfrac{a}{2N}\dot F_i\,, \end{aligned} \qquad \left(\begin{matrix} Z^0 \equiv -\frac{a}{N}\beta + \frac{a^2}{2N^2}\dot E \\ Z^i \equiv \frac{1}{2}\Omega^{ij}(D_jE + F_j) \\ \text{Under } x^\mu \to x^\mu + \xi^\mu : \\ Z^\mu \to Z^\mu + \xi^\mu \end{matrix}\right)$$

- However, we have 4 more degrees of freedom:

Associated with Stückelberg fields

$$\psi^\pi \equiv \psi - \frac{1}{3}\triangle\pi - \frac{\dot a}{a}\pi^0\,, \qquad E^\pi \equiv E - 2\,\pi\,, \qquad F_i^\pi \equiv F_i - 2\,\pi_i^T$$

Quadratic action

- After using background constraint for Stückelberg fields:

$$S^{(2)} = \underbrace{S^{(2)}_{\rm EH} + S^{(2)}_{\rm matter} + S^{(2)}_{\Lambda_\pm}}_{\text{depend only on } Q_I,\, \Phi,\, \Psi,\, B_i,\, \gamma_{ij}} + \underbrace{\tilde{S}^{(2)}_{\rm mass}}_{\tilde{S}^{(2)}_{\rm mass} = S^{(2)}_{\rm mass} - S^{(2)}_{\Lambda_\pm}}$$

- The first part is equivalent to GR + $\Lambda_\pm$+ Matter fields σ_I.
- The additional term:

$$\tilde{S}^{(2)}_{\rm mass} = M_p^2 \int d^4x\, N\, a^3 \sqrt{\Omega}\, M_{GW}^2 \left[3(\psi^\pi)^2 - \frac{1}{12} E^\pi \triangle(\triangle + 3K) E^\pi + \frac{1}{16} F^i_\pi (\triangle + 2K) F^\pi_i - \frac{1}{8} \gamma^{ij}\gamma_{ij} \right]$$

- The only common variable is γ_{ij}.
- E^π, ψ^π, F_i^π have no kinetic term! We treat them as nondynamical.
- Scalar and vector sector $\Rightarrow$ same dynamics as GR, with additional cosmological constant $\Lambda_\pm$ and same matter content.
- The only modification at linear order is in the tensor sector:

$$\tilde{S}^{(2)}_{\rm mass} = -\frac{M_p^2}{8} \int d^4x\, N\, a^3 \sqrt{\Omega}\, M_{GW}^2 \gamma^{ij}\gamma_{ij}$$

Tensor modes

- Assuming no tensor contribution from matter sector,

$$S^{(2)}_{tensor} = \frac{M_{Pl}^2}{8}\int d^4x\, N\, a^3\sqrt{\Omega}\left[\frac{1}{N^2}\dot{\gamma}^{ij}\dot{\gamma}_{ij} + \frac{1}{a^2}\gamma^{ij}(\triangle - 2K)\gamma_{ij} - M_{GW}^2\gamma^{ij}\gamma_{ij}\right],$$

- The mass function M_{GW}^2 is time dependent:

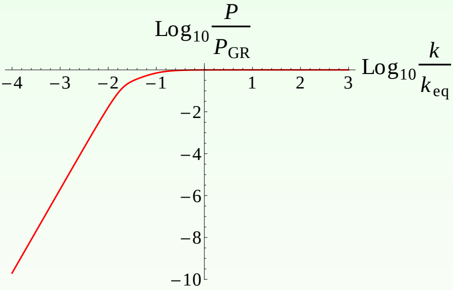
$$M_{GW}^2 \equiv \pm(r-1)m_g^2 X_{\pm}^2\sqrt{1+\alpha_3+\alpha_3^2-\alpha_4}$$

Time dependence provided by $r \equiv \frac{na}{N\alpha} = \frac{1}{X_{\pm}}\frac{H}{H_f}$, $\left(H \equiv \frac{\dot{a}}{Na}, \quad H_f \equiv \frac{\dot{\alpha}}{n\alpha}\right)$

- Stability is determined by the sign of $(r-1)m_g^2$.
- Fiducial metric $f_{\mu\nu} \to$ Evolution of r.
  eg.1: Minkowski fiducial $\Rightarrow r \propto \dot{a}$
  eg.2: dS fiducial $\Rightarrow r \propto \dot{a}/a$

Possible signals?

- For $M_{GW}^2 > 0$, the spectrum of stochastic GW will undergo a suppression (w.r.t GR) when $(k/a)^2 \lesssim M_{GW}^2$.
- For $M_{GW} \sim \mathcal{O}(H_0)$, the suppression may be observed.



- Example for $M_{GW}^2 = \text{constant}$.
- Assumed initial scale invariance.
- Small scales: Same as GR signal.
- Larges scales: Suppression.
- Frequency dominated by M_{GW}^2 at large scales.
- Cutoff: $k/k_{\mathrm{eq}} = (M_{GW}/2H_{\mathrm{eq}})^{1/3}$ (Here: $\sim .02$)

Work in progress, `with S. Kuroyanagi, C. Lin, S. Mukohyama, N. Tanahashi`

Summary/Discussion

- Gauge invariant study of perturbations of self-accelerating cosmological solutions in potentially ghost-free nonlinear massive gravity.
- Dynamics of scalar and vector modes are same as in GR, at the level of quadratic action.
  $\Rightarrow$ No stability issues in scalar/vector sectors.
- Tensor sector acquires a time dependent mass.
  $\Rightarrow$ Modification of stochastic GW spectrum, CMB B–mode polarization at large scales.

- Expected 5 degrees for massive spin 2
  $\Rightarrow$ Only 2 degrees (2 GW polarizations). Cancellation of kinetic terms at quadratic level. Possible connection with the cosmological branch of solutions?
- Strong coupling vs Nondynamical?
  $\Rightarrow$ Need to go beyond perturbation theory.
- Radiative stability?
  $\Rightarrow$First step: strong coupling scale in the cosmological branch?