Poincaré Gauge Theory with Coupled Even and Odd Parity Dynamic Spin-0 Modes: Dynamical Isotropic Bianchi Cosmologies

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Abstract and Outline

- We are investigating the dynamics of **a new Poincaré gauge theory of gravity model**, **the BHN PG model** which has **cross coupling** between the **spin-0^+ and spin-0^-** modes, in a situation which is simple, non-trivial, and yet may give physically interesting results that might be observable.
- To this end we here consider a very appropriate situation—**homogeneous-isotropic cosmologies**—which is relatively simple, and yet all the modes have non-trivial dynamics which reveals physically interesting and possibly observable results.
- More specifically we consider manifestly **isotropic Bianchi class A** cosmologies; for this case we find an **effective Lagrangian and Hamiltonian** for the dynamical system. The Lagrange equations for these models lead to a set of first order equations that are compatible with those found for the FLRW models and provide a foundation for further investigations.
- The first order equations are **linearized**. **Numerical evolution** confirms the late time asymptotic approximation and shows the expected effects of the cross parity pseudoscalar coupling. We can fine tune our model by these coupling parameters to fit our accelerating universe.

Background and Motivation

- All the known physical interactions (strong, weak, electromagnetic and not excepting gravity) can be formulated in a common framework as *local gauge theories*:

  In Electrodynamics: field strength $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ can be specified as

  $$\vec{E} = -\nabla\Phi - \frac{\partial \vec{A}}{\partial t} \quad \text{and} \quad \vec{B} = \nabla \times \vec{A},$$

  where Φ and $\vec{A}$ are potentials. $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are invariant under transformation of the Φ and the $\vec{A}$ (*gauge* freedom),

  $$\Phi' = \Phi + \frac{\partial \Lambda}{\partial t} \quad \text{and} \quad \vec{A}' = \vec{A} - \nabla\Lambda,$$

  i.e. a *gauge* transformation, where Λ is an arbitrary scalar function.

- However the standard theory of gravity, Einstein's GR, based on the spacetime metric, is a rather unnatural gauge theory

- Physically (and geometrically) it is reasonable to consider gravity as a gauge theory of the local Poincaré symmetry of Minkowski spacetime

- There is no fundamental reason to expect gravity to be parity invariant so no fundamental reason to exclude odd parity coupling terms

- Accelerating universe

The Poincaré gauge theory

In the Poincaré gauge theory of gravity (PG Theory)
[Hehl '80, Hayashi & Shirafuji '80],
the local gauge potentials are, for translations, the orthonormal co-frame,
(which determines the metric):

$$\vartheta^\alpha = e^\alpha{}_i \mathrm{d}x^i \to g_{ij} = e^\alpha{}_i e^\beta{}_j \eta_{\alpha\beta}, \quad \eta_{\alpha\beta} = \text{diag}(-1, +1, +1, +1),$$

and, for Lorentz/rotations, the metric-compatible (Lorentz) connection

$$\Gamma^{\alpha\beta}{}_i \mathrm{d}x^i = \Gamma^{[\alpha\beta]}{}_i \mathrm{d}x^i.$$

The associated field strengths are the *torsion* and *curvature*:

$$T^\alpha \quad := \quad \mathrm{d}\vartheta^\alpha + \Gamma^\alpha{}_\beta \wedge \vartheta^\beta = \frac{1}{2} T^\alpha{}_{\mu\nu} \vartheta^\mu \wedge \vartheta^\nu,$$

$$R^{\alpha\beta} \quad := \quad \mathrm{d}\Gamma^{\alpha\beta} + \Gamma^\alpha{}_\gamma \wedge \Gamma^{\gamma\beta} = \frac{1}{2} R^{\alpha\beta}{}_{\mu\nu} \vartheta^\mu \wedge \vartheta^\nu,$$

which satisfy the respective Bianchi identities:

$$DT^\alpha \equiv R^\alpha{}_\beta \wedge \vartheta^\beta, \qquad DR^\alpha{}_\beta \equiv 0.$$

General PG Lagrangian

- The general quadratic PG Lagrangian density has the form (see [Baekler, Hehl and Nester PRD 2011])

$$\mathscr{L}[\vartheta, \Gamma] \quad \sim \quad \kappa^{-1}[\Lambda + \text{curvature} + \text{torsion}^2] + \varrho^{-1}\text{curvature}^2,$$

  where Λ is the cosmological constant, $\kappa = 8\pi G/c^4$, ϱ^{-1} has the dimensions of action.

- Gravitational field eqns are 2nd order eqns for the gauge potentials:

$$\begin{aligned} \delta\vartheta^{\alpha}{}_{i} : &\qquad \Lambda + R + DT + T^2 + R^2 \sim \text{energy-momentum density} \\ \delta\Gamma^{\alpha\beta}{}_{k} : &\qquad T + DR \sim \text{source spin density}, \end{aligned}$$

  where R and T represent curvature and torsion.
  Bianchi identities $\Longrightarrow$ conservation of source energy-momentum & angular momentum.

good dynamic modes

- Investigations of the linearized theory identified six possible dynamic connection modes carrying spin-$2^{\pm}$, $1^{\pm}$, $0^{\pm}$.
  [Hayashi & Shirafuji '80, Sezgin & van Nuivenhuizen '80]
- A good dynamic mode transports positive energy at speed $\leq c$.
  At most three modes can be simultaneously dynamic;
  all the cases were tabulated;
  many combinations are satisfactory to linear order.
  The Hamiltonian analysis revealed the related constraints
  [Blagojević & Nicolić, 1983].
- Then detailed investigations
  [Hecht, Nester & Zhytnikov '96, Chen, Nester & Yo '98, Yo & Nester '99, '02]
  concluded that effects due to nonlinearities could be expected to render all of these cases physically unacceptable—
  except for the two "scalar modes": spin-0^{+} and spin-0^{-}.

BHN Lagrangian

- Generalizing [Shie, Nester & Yo PRD '08], we considered two dynamic spin-0^+ and spin-0^- modes [Chen et al JCAP '09].
- Now, the model has been extended to include parity violating terms by [BHN PRD '11].
- The Lagrangian of the BHN model is

$$\mathcal{L}[\vartheta,\Gamma] = \frac{1}{2\kappa}\left[-2\Lambda + a_0 R - \frac{1}{2}\sum_{\mathsf{n}=1}^{3} a_{\mathsf{n}} \overset{(\mathsf{n})}{T}{}^2 + b_0 X + 3\sigma_2 V_\mu A^\mu\right] + \frac{1}{2\varrho}\left[\frac{w_6}{12}R^2 + \frac{w_3}{12}X^2 + \frac{\mu_3}{12}RX\right],$$

where R & $X = 6R_{[0123]}$ are the scalar & pseudoscalar curvatures, $V_\mu \equiv T^\alpha{}_{\alpha\mu}$, $A_\mu \equiv \frac{1}{2}\epsilon_{\mu\nu}{}^{\alpha\beta}T^\nu{}_{\alpha\beta}$ are the torsion trace & axial vectors and b_0 & σ_2 & μ_3 are the odd parity coupling constants.

Cosmological model

- Earlier PGT cosmology: Minkevich [e.g., '80, '83, '95, '07] and Goenner & Müller-Hoissen ['84];
  recent: Shie, Nester & Yo ['08], Wang & Wu ['09], Chen et al ['09], Li, Sun & Xi ['09ab], Ao, Li & Xi ['10, '11], Baekler, Hehl & Nester ['11].

- Homogeneous isotropic cosmology is the ideal place to study the dynamics of the spin-$0^{\pm}$ modes of the BHN model.
- Here, we consider the homogeneous, isotropic Bianchi I & IX cosmological model. The isotropic orthonormal coframe:

$$\vartheta^0 := dt, \qquad \vartheta^a := a\sigma^a,$$

  where $a = a(t)$ is the scale factor and σ^j depends on the (never needed) spatial coordinates in such a way that

$$d\sigma^i = \zeta\epsilon^i{}_{jk}\sigma^j \wedge \sigma^k,$$

  where $\zeta = 0$ for Bianchi I (equivalent to the FLRW $k = 0$ case, which appears to describe our physical universe) and $\zeta = 1$ for Bianchi IX, thus $\zeta^2 = k$.

- isotropy $\Longrightarrow$ non-vanishing connection one-form coefficients

$$\Gamma^a{}_0 = \psi(t)\,\sigma^a, \quad \Gamma^a{}_b = \chi(t)\epsilon^a{}_{bc}\,\sigma^c,$$

$\Longrightarrow$ nonvanishing curvature components:

$$R^{a0}{}_{b0} = \frac{\dot\psi\delta^a_b}{a}, \quad R^{ab}{}_{0c} = \frac{\dot\chi\epsilon^{ab}{}_c}{a},$$

$$R^{a0}{}_{bc} = \frac{2\psi(\chi-\zeta)\epsilon^a{}_{bc}}{a^2}, \quad R^{ab}{}_{cd} = \frac{(\psi^2-\chi^2+2\chi\zeta)\delta^{ab}_{cd}}{a^2}.$$

$\Longrightarrow$ scalar and pseudoscalar curvatures:

$$\begin{aligned} R &= 6[a^{-1}\dot\psi + a^{-2}(\psi^2 - [\chi-\zeta]^2) + \zeta^2], \\ X &= 6[a^{-1}\dot\chi + 2a^{-2}\psi(\chi-\zeta)]. \end{aligned}$$

- isotropy $\Longrightarrow$ nonvanishing torsion tensor components

$$T^{a}{}_{b0} = u(t)\delta^{a}_{b}, \qquad T^{a}{}_{bc} = -2x(t)\epsilon^{a}{}_{bc}.$$

they depend on the gauge variables:

$$u = a^{-1}(\dot{a} - \psi), \qquad x = a^{-1}(\chi - \zeta).$$

- isotropy $\Longrightarrow$ energy-momentum tensor has the perfect fluid form with an energy density and pressure: ρ, p.
  - We assume that the source spin density vanishes.
  - When $p = 0$, the gravitating material behaves like dust with

$$\rho a^{3} = \text{constant}.$$

effective Lagrangian, eqns

- The dynamical equations for the homogeneous cosmology can be obtained by imposing the Bianchi symmetry on the field equations found by BHN from the BHN *Lagrangian density*$\Longrightarrow$
- *These same dynamical equations* can be obtained directly (and independently) from a classical mechanics type effective *Lagrangian* (a variational principle), which in this case can be simply obtained by restricting the BHN Lagrangian density to the Bianchi symmetry.
- This procedure is known to be successful for all Bianchi class A models (which includes our cases) in GR, and it is conjectured to also be true for the PG theory. [Our calculations will explicity verify this for isotropic Bianchi I and IX models.]

- The effective Lagrangian $L_{\text{eff}} = L_{\text{G}} + L_{\text{int}}$ includes the *interaction* Lagrangian:

$$L_{\text{int}} \;=\; pa^3, \qquad p = p(t) \qquad \text{pressure,}$$

  and the *gravitational* Lagrangian:

$$\begin{aligned} L_{\text{G}} \;=\;& \frac{a^3}{\kappa}\left[-\Lambda + \frac{a_0}{2}R + \frac{b_0}{2}X - \frac{3}{2}a_2u^2 + 6a_3x^2 + 6\sigma_2 ux\right] \\ &+ \frac{a^3}{\varrho}\left[-\frac{w_6}{24}R^2 + \frac{w_3}{24}X^2 - \frac{\mu_3}{24}RX\right] \end{aligned}$$

  with $a_2 < 0$, $w_6 < 0$, $w_3 > 0$, $-4w_3w_6 - \mu^2 > 0$, these signs are physically necessary for least action.

- In the following we often take for simplicity units such that $\kappa = 1 = \varrho$.

- For convenience we introduce the *modified* parameters $\tilde{a}_2, \tilde{a}_3, \tilde{\sigma}_2$ with the definitions

$$\tilde{a}_2 := a_2 - 2a_0, \qquad \tilde{a}_3 := a_3 - \frac{1}{2}a_0, \qquad \tilde{\sigma}_2 := \sigma_2 + b_0.$$

- The energy function obtained from $L_{\rm G}$ is an effective energy, it is just the "00 constraint", Hamiltonian with magnitude $-\rho a^3$,

$$
\begin{aligned}
\mathcal{E} \quad = \quad & a^3 \Bigg\{ \frac{3}{2}\tilde{a}_2 u^2 - 3a_0 H^2 - 6\tilde{a}_3 x^2 - 3\tilde{a}_2 u H + \Lambda \\
& + 6\tilde{\sigma}_2 x (H-u) - 3a_0 \frac{\zeta^2}{a^2} \\
& - \frac{w_6}{24}\left[R^2 - 12R\left\{(H-u)^2 - x^2 + \frac{\zeta^2}{a^2}\right\}\right] \\
& + \frac{w_3}{24}\left[X^2 + 24Xx(H-u)\right] \\
& - \frac{\mu_3}{24}\left[RX - 6X\left\{(H-u)^2 - x^2 + \frac{\zeta^2}{a^2}\right\} + 12Rx(H-u)\right]\Bigg\},
\end{aligned}
$$

- it satisfies

$$\frac{d(\rho a^3)}{dt} = -p\frac{da^3}{dt},$$

so ρa^3 is a constant when $p = 0$.

The Dynamical Equations

- the Lagrange eqns, ψ, χ and a:

$$
\begin{aligned}
\frac{d}{dt}\frac{\partial L_G}{\partial \dot{\psi}} &= \frac{d}{dt}\left(a^2\left[3a_0 - \frac{w_6}{2}R - \frac{\mu_3}{4}X\right]\right) = \frac{\partial L_G}{\partial \psi} \\
&= 3(a_2 u - 2\sigma_2 x)a^2 + \left[6a_0 - w_6 R - \frac{\mu_3}{2}X\right] a\psi \\
&\quad + \left[6b_0 - \frac{\mu_3}{2}R + w_3 X\right] a(\chi - \zeta), \qquad \Longrightarrow \dot{R}, \dot{X}.
\end{aligned}
$$

$$
\begin{aligned}
\frac{d}{dt}\frac{\partial L_G}{\partial \dot{\chi}} &= \frac{d}{dt}\left(a^2\left[3b_0 - \frac{\mu_3}{4}R + \frac{w_3}{2}X\right]\right) = \frac{\partial L_G}{\partial \chi} \\
&= -6(2a_3 x + \sigma_2 u)a^2 - \left[6a_0 - w_6 R - \frac{\mu_3}{2}X\right] a(\chi - \zeta) \\
&\quad + \left[6b_0 - \frac{\mu_3}{2}R + w_3 X\right] a\psi, \qquad \Longrightarrow \dot{R}, \dot{X}.
\end{aligned}
$$

$$
\begin{aligned}
\frac{d}{dt}\frac{\partial L_G}{\partial \dot{a}} &= \frac{d}{dt}\left(-a^2 3[a_2 u - 2\sigma_2 x]\right) = \frac{\partial L_G}{\partial a} + \frac{\partial L_{int}}{\partial a} \\
&= 3a^{-1}L - \left(\frac{a_0}{2} - \frac{w_6}{12}R - \frac{\mu_3}{24}X\right)[a^2 R + 6(\psi^2 - [\chi - \zeta]^2 + \zeta^2)] \\
&\quad - \left(\frac{b_0}{2} + \frac{w_3}{12}X - \frac{\mu_3}{24}R\right)[a^2 X + 12\psi(\chi - \zeta)] \\
&\quad + 3a^2(a_2 u - 2\sigma_2 x)u - 6a^2[2a_3 x + \sigma_2 u]x + 3pa^2, \implies \dot{u}, \dot{x}.
\end{aligned}
$$

- First order eqns from:

$$
\begin{aligned}
\dot{a} &= aH \\
\dot{x} &= -Hx - \frac{X}{6} - 2x(H - u), \\
\dot{H} - \dot{u} &= \frac{R}{6} - H(H - u) - (H - u)^2 + x^2 - \frac{\zeta^2}{a^2}.
\end{aligned}
$$

First order equations with parity coupling

$$
\begin{aligned}
\dot{a} &= aH, \\
\dot{H} &= \frac{1}{6a_2}(\tilde{a}_2 R - 2\tilde{\sigma}_2 X) - 2H^2 + \frac{\tilde{a}_2 - 4\tilde{a}_3}{a_2}x^2 - \frac{\zeta^2}{a^2} \\
&\quad + \frac{(\rho - 3p)}{3a_2} + \frac{4\Lambda}{3a_2}, \\
\dot{u} &= -\frac{1}{3a_2}(a_0 R + \tilde{\sigma}_2 X) - 3Hu + u^2 - \frac{4a_3}{a_2}x^2 \\
&\quad + \frac{(\rho - 3p)}{3a_2} + \frac{4\Lambda}{3a_2}, \\
\dot{x} &= -\frac{X}{6} - (3H - 2u)x, \\
-\frac{w_6}{2}\dot{R} - \frac{\mu_3}{4}\dot{X} &= \left[3\tilde{a}_2 + w_6 R + \frac{\mu_3}{2}X\right]u - \left[6\tilde{\sigma}_2 - \frac{\mu_3}{2}R + w_3 X\right]x \\
\frac{w_3}{2}\dot{X} - \frac{\mu_3}{4}\dot{R} &= \left[-6\tilde{\sigma}_2 + \frac{\mu_3}{2}R - w_3 X\right]u - \left[12\tilde{a}_3 + w_6 R + \frac{\mu_3}{2}X\right]x
\end{aligned}
$$

For our numerical evolution we consider only the case of dust $p = 0$, (a good approximation except at early times).

Hamiltonian formulation

- canonical conjugate momentum

$$
\begin{aligned}
P_a &\equiv \frac{\partial L}{\partial \dot{a}} = -3a^2 \left[a_2 u - 2\sigma_2 x\right], \\
P_\psi &\equiv \frac{\partial L}{\partial \dot{\psi}} = a^2 \left[3a_0 - \frac{w_6}{2} R - \frac{\mu_3}{4} X\right], \\
P_\chi &\equiv \frac{\partial L}{\partial \dot{\chi}} = a^2 \left[3b_0 + \frac{w_3}{2} X - \frac{\mu_3}{4} R\right].
\end{aligned}
$$

- the effective Hamiltonian

$$
\begin{aligned}
H_{\rm eff} &= P_a\dot{a} + P_\psi\dot{\psi} + P_\chi\dot{\chi} - L_{\rm eff} \\
&= a^3(\Lambda - p) - 6aa_3(\chi-\zeta)^2 + \frac{3\sigma_2^2 a^2(\chi-\zeta)}{a_2} \\
&\quad + \frac{3a^3}{2\alpha}(w_3a_0^2 - w_6b_0^2 + \mu_3b_0a_0) \\
&\quad + P_a\left[\frac{\sigma_2}{a_2}\left(\frac{a}{2} - \chi + \zeta\right) + \psi\right] \\
&\quad + P_\psi\left[-\psi^2 + (\chi-\zeta)^2 - \zeta^2 - \frac{(b_0\mu_3 - 2a_0w_3)a^2}{2\alpha}\right]\frac{1}{a} \\
&\quad + P_\chi\left[-2\psi(\chi-\zeta) - \frac{(a_0\mu_3 + 2b_0w_6)a^2}{2\alpha}\right]\frac{1}{a} \\
&\quad + P_\psi P_\chi\left[\frac{\mu_3}{6\alpha}\right]\frac{1}{a} + P_\psi^2\left[\frac{w_3}{6\alpha}\right]\frac{1}{a} + P_\chi^2\left[-\frac{w_6}{6\alpha}\right]\frac{1}{a} + P_a^2\left[-\frac{1}{6a_2}\right]\frac{1}{a},
\end{aligned}
$$

where $\alpha := -w_3w_6 - \frac{\mu_3^2}{4}$.

- the six Hamilton equations are

$$\dot{a} = \frac{\partial \mathcal{H}}{\partial P_a} = \left[\frac{\sigma_2}{a_2}\left(\frac{a}{2} - \chi + \zeta\right) + \psi\right] - \frac{P_a}{3a_2 a}$$

$$\dot{\psi} = \frac{\partial \mathcal{H}}{\partial P_\psi} = \frac{1}{a}\left[-\psi^2 + (\chi - \zeta)^2 - \zeta^2 - \frac{\mu_3(3a^2 b_0 - P_\chi) - 2w_3(3a^2 a_0 + P_\psi)}{6\alpha}\right]$$

$$\dot{\chi} = \frac{\partial \mathcal{H}}{\partial P_\chi} = \frac{1}{a}\left[-2\psi(\chi - \zeta) - \frac{\mu_3(3a^2 a_0 - P_\psi) + 2w_6(3a^2 b_0 + P_\chi)}{6\alpha}\right]$$

$$\dot{P}_a = -\frac{\partial \mathcal{H}}{\partial a} = \frac{\mathcal{H} - P_a\left[\frac{\sigma_2}{a_2}(a - \chi + \zeta) + \psi\right]}{a} - 4a^2\left[\frac{3(w_3 a_0^2 - w_6 b_0^2 + \mu_3 b_0 a_0)}{2\alpha} + (\Lambda - p)\right]$$

$$+2\left[6a_3(\chi - \zeta)^2 + P_\psi \frac{(b_0\mu_3 - 2a_0 w_3)}{4\alpha} + P_\chi \frac{(a_0\mu_3 + 2b_0 w_6)}{4\alpha}\right] - \frac{9\sigma_2^2 a(\chi - \zeta)}{a_2}$$

$$\dot{P}_\psi = -\frac{\partial \mathcal{H}}{\partial \psi} = -P_a + \frac{2}{a}\left[P_\psi \psi + P_\chi(\chi - \zeta)\right]$$

$$\dot{P}_\chi = -\frac{\partial \mathcal{H}}{\partial \chi} = 12 a a_3 \chi - \frac{3\sigma_2^2 a^2}{a_2} + P_a \frac{\sigma_2}{a_2} + \frac{2}{a}[P_\chi \psi - P_\psi(\chi - \zeta)].$$

Linearize and Normal Modes

- By dropping higher than linear order terms in $\{H, u, x, R, X\}$, we can lead our model to the first order *linearized* versions of equations

$$\dot{a} = aH, \tag{1}$$

$$3a_2\dot{H} = \frac{1}{2}\tilde{a}_2 R - \tilde{\sigma}_2 X, \tag{2}$$

$$3a_2\dot{u} = -a_0 R - \tilde{\sigma}_2 X, \tag{3}$$

$$\dot{x} = -\frac{X}{6}, \tag{4}$$

$$-\frac{w_6}{2}\dot{R} - \frac{\mu_3}{4}\dot{X} = 3\tilde{a}_2 u - 6\tilde{\sigma}_2 x, \tag{5}$$

$$-\frac{\mu_3}{4}\dot{R} + \frac{w_3}{2}\dot{X} = -6\tilde{\sigma}_2 u - 12\tilde{a}_3 x, \tag{6}$$

with the associated (to lowest, i.e., quadratic, order) “energy”:

$$\mathcal{E} = a^3\Bigg\{-\frac{3}{2}\tilde{a}_2 u^2 - 3a_0 H^2 - 6\tilde{a}_3 x^2 - 3uH\tilde{a}_2 + 6\tilde{\sigma}_2 x(H-u) - \frac{w_6}{24}R^2 + \frac{w_3}{24}X^2 - \frac{\mu_3}{24}RX\Bigg\}.$$

- The odd parity coupling terms lead to mixing of the even (R, u) and odd (X, x) dynamical variables; this is especially apparent in (5), (6). We can see the acceleration is now driven by the odd pseudoscalar curvature.
- To analyze this system we first introduce a new variable combination:

$$z := a_0 H + \frac{\tilde{a}_2}{2} u - \tilde{\sigma}_2 x, \tag{7}$$

which to linear order from (2)–(4) is *constant*:

$$\dot{z} = a_0 \dot{H} + \frac{\tilde{a}_2}{2} \dot{u} - \tilde{\sigma}_2 \dot{x} = 0. \tag{8}$$

This is, to linear order, a *zero frequency normal mode*.

Late time asymptotical expansion

- At late times the scale factor a is large. For $\Lambda = 0$ the quadratic terms will dominate, then H, u, x, R, and X should have a $a^{-3/2}$ fall off. Let

$$H = \underline{H}a^{-3/2},\ u = \underline{u}a^{-3/2},\ x = \underline{x}a^{-3/2},\ R = \underline{R}a^{-3/2},\ X = \underline{X}a^{-3/2},$$

dropping higher order terms, gives the 6 linear equations with odd parity coupling:

$$\dot{a} = a^{-1/2}\underline{H}, \qquad \dot{\underline{H}} = \frac{1}{6a_2}[\tilde{a}_2\underline{R} - 2\tilde{\sigma}_2\underline{X}],$$

$$\dot{\underline{x}} = -\frac{\underline{X}}{6}, \qquad \dot{\underline{u}} = -\frac{1}{3a_2}[a_0\underline{R} + \tilde{\sigma}_2\underline{X}],$$

$$\dot{\underline{R}} = \frac{6}{\alpha}\left[(w_3\tilde{a}_2 - \mu_3\tilde{\sigma}_2)\underline{u} - 2(w_3\tilde{\sigma}_2 + \mu_3\tilde{a}_3)\underline{x}\right],$$

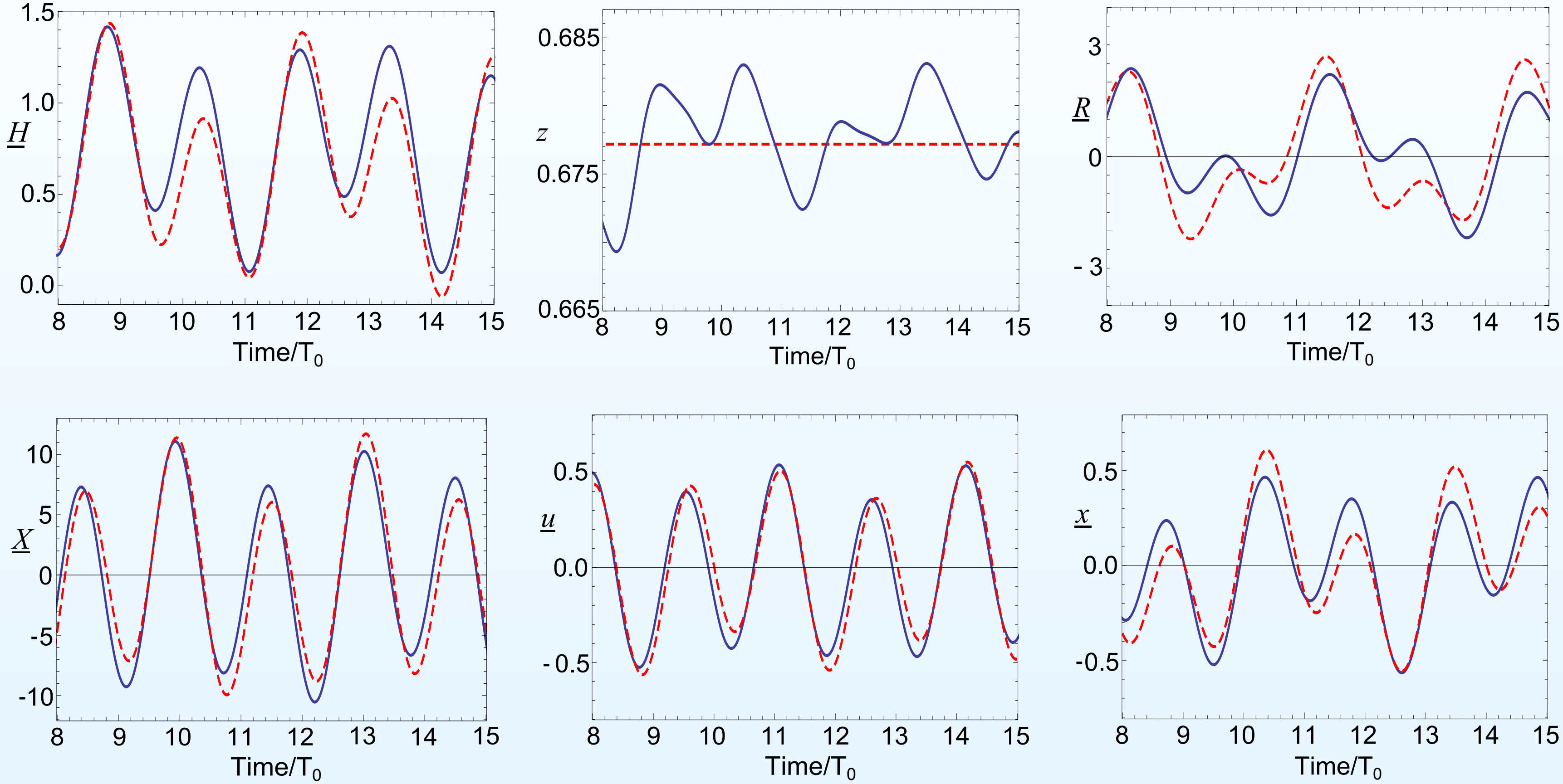
$$\dot{\underline{X}} = \frac{6}{\alpha}[(2w_6\tilde{\sigma}_2 + \frac{1}{2}\mu_3\tilde{a}_2)\underline{u} + (4w_6\tilde{a}_3 - \mu_3\tilde{\sigma}_2)\underline{x}],$$

plus the energy constraint

$$-a^3\kappa\rho = \frac{3\tilde{a}_2}{2}(\underline{H} - \underline{u})^2 - \frac{3}{2}a_2\underline{H}^2 + 6\tilde{\sigma}_2\underline{x}(\underline{H} - \underline{u}) - 6\tilde{a}_3\underline{x}^2 + \frac{w_3}{24}\underline{X}^2 - \frac{w_6}{24}\underline{R}^2.$$

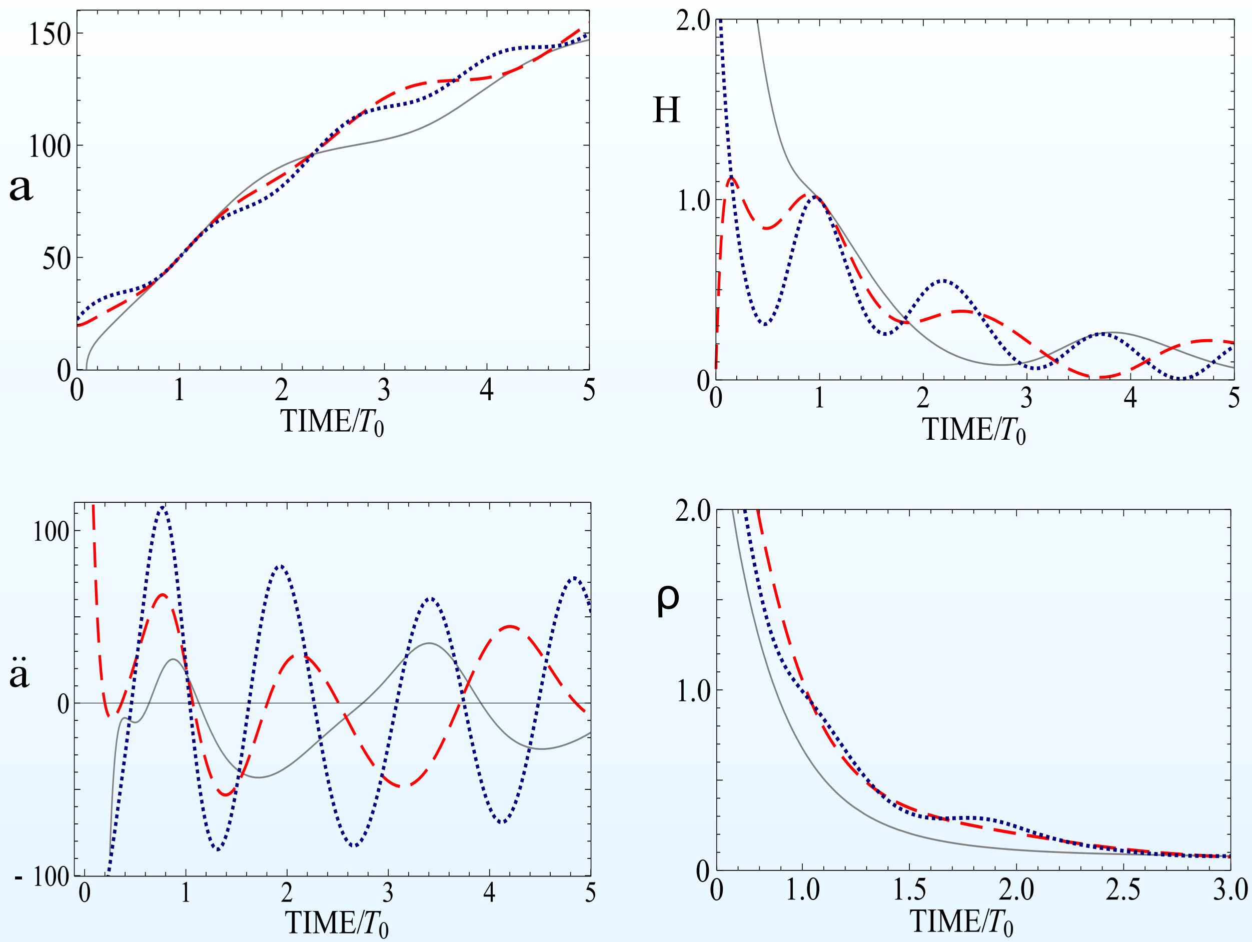
Linearized vs. late time evolution

a_2	a_3	w_6	w_3	σ_2	μ_3
-0.83	-0.35	-1.1	0.091	0.4	-0.07



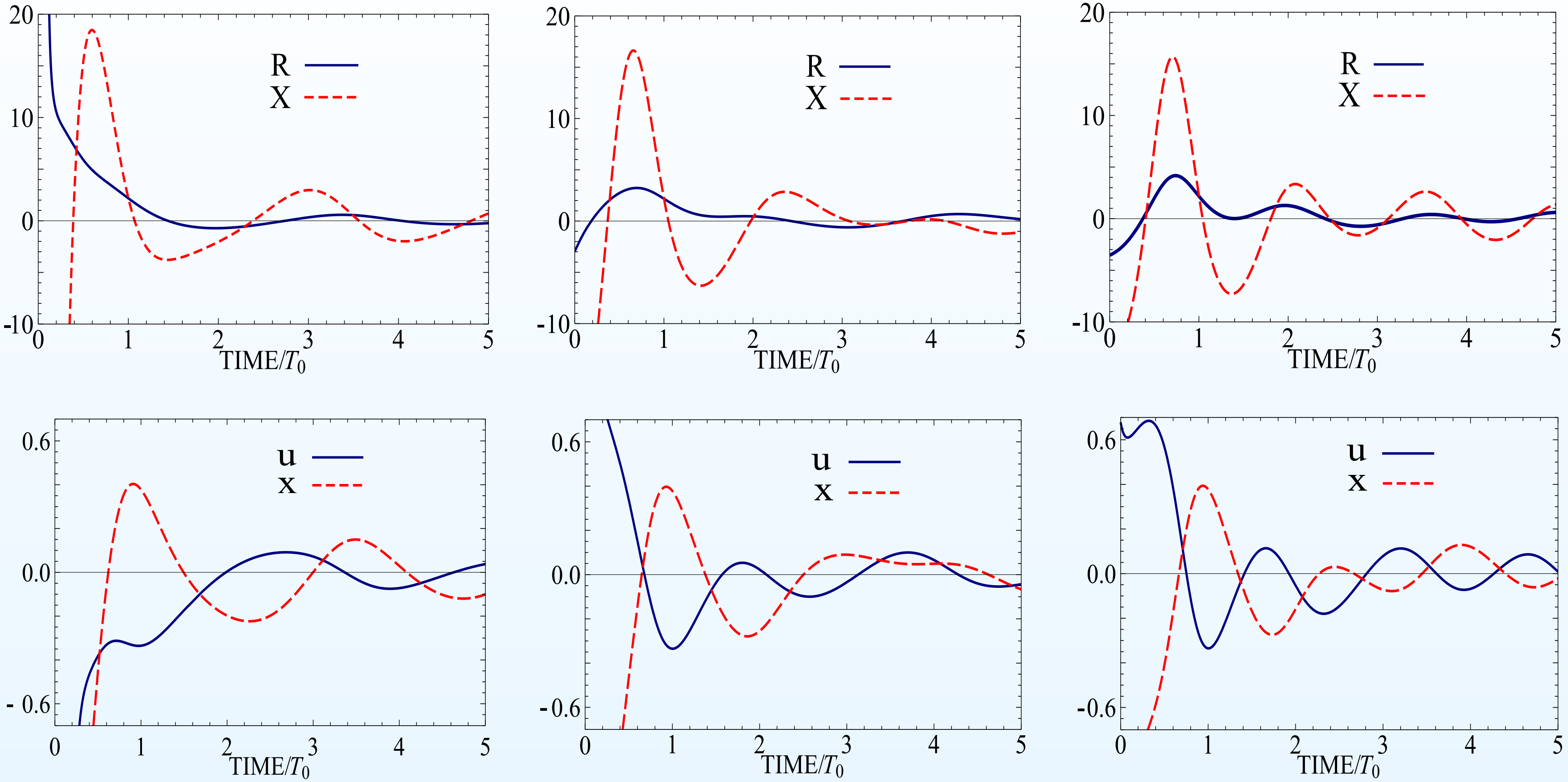
Hubble function $\underline{H}$, "constant mode" z, scalar curvature $\underline{R}$, pseudoscalar curvature $\underline{X}$, scalar torsion $\underline{u}$ and pseudoscalar torsion, $\underline{x}$. The blue (solid) lines represent the rescaled late time evolution and the red (dashed) lines represent the linear approximation evolution.

The effect of odd coupling parameters (I):



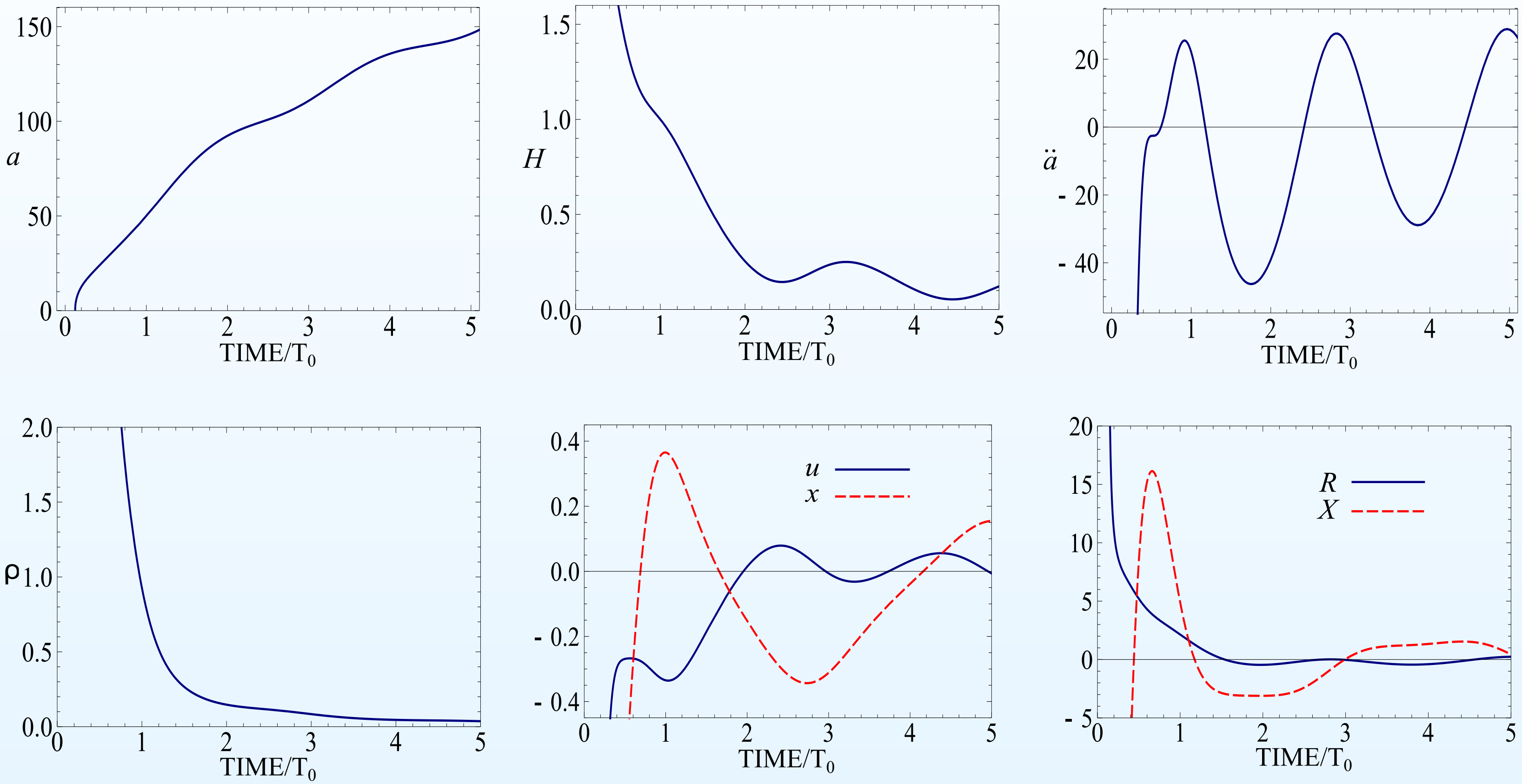
(I)The effect of the cross coupling odd parity parameters σ_2 and μ_3. The red (dashed) line represents the evolution with the parameter σ_2 activated. The blue (doted) line represents the evolution including both pseudoscalar parameters σ_2 and μ_3.

The effect of odd coupling parameters (II):



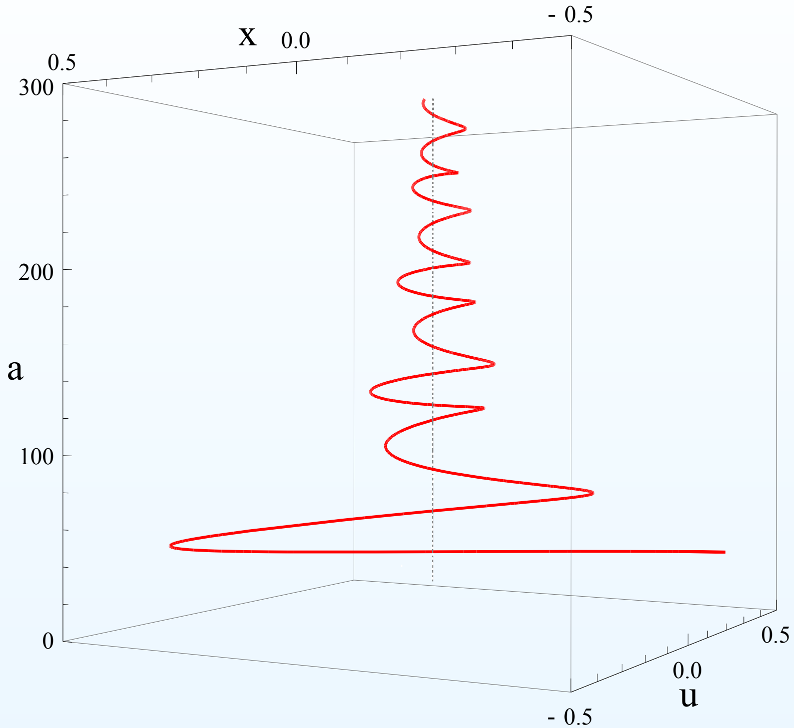
(II)The effect of the cross odd parity parameters σ_2 and μ_3. In the first line we compare the scalar curvature, R and the pseudoscalar curvature, X in different situations. In the second line we compare the torsion, u and the axial torsion, x. The first column is the evolution with vanishing pseudoscalar parameters, σ_2 and μ_3, the second column, with parameter σ_2, the third column, with both pseudoscalar parameters, σ_2 and μ_3.

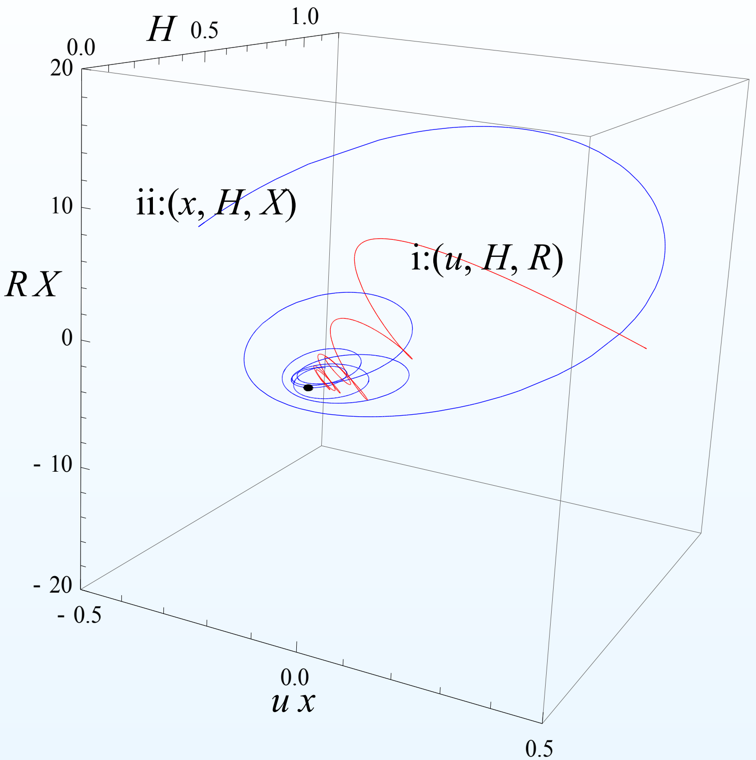
case	a_2	a_3	w_6	w_3	σ_2	μ_3	$u(1)$	$x(1)$	$R(1)$	$X(1)$
I	-0.845	-0.45	-1.2	0.081	0.097	-0.43	-0.3349	0.365	2.144	4.9
II	-0.905	-0.35	-1.1	0.091	0.097	-0.068	-0.3349	0.378	2.164	2.21



The full evolution. Shown are the expansion factor a, the Hubble function, H, the 2nd time derivative of the expansion factor, $\ddot{a}$, the energy densities, ρ, the scalar and the pseudoscalar torsion components, u and x, the affine scalar curvature and the pseudoscalar curvature, R, X with the parameter choice and the initial data for Case I.

3D Phase Diagram for case I





The two figures are for the phase diagrams for Case I. The left 3D diagram of (x, u, a) is shown in this panel. The (red) solid line is the trajectory of the (x, u, a) evolution starting from the initial value (0.365, -0.3349, 50). The (gray) doted line is the convergence line (0, 0, a) for this diagram. The right 3D diagram of (u, H, R) and of (x, H, X) are shown in this panel. The i (red) line is the trajectory of the (u, H, R) evolution starting from the initial value (-0.3349, 1, 2.144), the ii (blue) line is the trajectory of the (x, H, X) evolution starting from the initial value (0.365, 1, 4.9) and the (filled) black point marks the asymptotic focus point (0, 0, 0).

Summary

- Here we have considered the dynamics of the **BHN model** in the context of manifestly homogeneous and isotropic Bianchi I and IX cosmological models.
- The BHN cosmological model system of ODEs resemble those of a particle with 3 degrees of freedom. Imposing the **homogeneous-isotropic Bianchi I and IX symmetry** into the BHN PG theory Lagrangian density, the evolution equations can be obtained directly from a variational principle. The Hamilton equations can be obtained also.
- Imposing symmetries and variations do not commute in general. However, for GR they are known to commute for all **Bianchi class A cosmologies**. We verify this for our models for isotropic Bianchi I and IX. Our isotopic Bianchi I and IX models are both class A. They correspond to the FLRW $k = 0$ and $k = +1$ models. The FLRW $k = -1$ model can be represented by Bianchi V or VII models, however the representation cannot be *manifestly* isotropic. One can of course get the FLRW $k = -1$ dynamical *equations* from our dynamical equations just by simply replacing ζ^2 with -1.

- The system of first order equations obtained from an effective Lagrangian was **linearized**, **the normal modes** were identified, and it was shown analytically how they control the **late time asymptotics**.
- The analysis of the equations confirms certain expected effects of the pseudoscalar coupling constants—which provide a direct interaction between the even and odd parity modes. In these models, at late times the acceleration oscillates. It can be positive at the present time.
- As far as we know the scalar torsion mode does not directly couple to any known form of matter, but we noted that it does **couple directly to the Hubble expansion**, and thus it can directly influence the acceleration of the universe. On the other hand, the pseudoscalar torsion couples directly to fundamental fermions; with the newly introduced pseudoscalar coupling constants it too can directly influence the cosmic acceleration.