Effects of the quantum conformal matter on metric perturbations

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introduction

- inflation beautifully explains what the universe looks like today,
- but it is not clear what actually drives inflation,
- nor about how long it lasts, as long as inflation is over 60 e-foldings,
- it implies that the scales we are interested right now may derive from transplanckian scales in the beginning of inflation,
- this observation may offer to tool to probe transplackian physics.

introduction

- we will look into some of these issues by examining backreactions of the matter on the gravity waves.
- other than (active) quantum fluctuations of the metrics, there is an additional (passive) component induced by quantum fluctuations of matter,
- quantum fluctuations of the matter results in fluctuations of its energy stress tensor,
- in turn, by the Einstein equation, the stress tensor fluctuations drive (passive) metric fluctuations.

introduction

- it turns out that this induced component of the metric fluctuations may distort the tensor mode power spectrum of the CMB in the high frequency end,
- the extent of correction depends on duration of inflation,
- besides, it contains contributions from the (trans)plackain modes of the matter field,
- so it can used as a testground of ultra-high energy physics, when combined with the future observation data from the PLANCK or LISA-type experiments.

configuration

consider a spatially flat de Sitter universe

$$ds^2 = a^2(\eta)\left[-d\eta^2 + d\mathbf{x}^2\right].$$

with η the conformal time and $a(\eta) = -(H\eta)^{-1}$, $\eta < 0$.

- let $g_{\mu\nu} = \gamma_{\mu\nu} + h_{\mu\nu}$, where $\eta_{\mu\nu}$ is the background de Sitter metric, and $h_{\mu\nu}$ is the metric perturbation – tensor modes.
- choose the transverse-tracefree (TT) gauge:

$$h^{\mu\nu}{}_{;\nu} = 0\,, \qquad h = h^{\mu}{}_{\mu} = 0\,, \qquad h_{\mu\nu}u^{\nu} = 0\,,$$

  - ";" denotes covariant derivative wrpt the bkgd metric,
  - u^{ν} is some timelike vector.

(active) gravity waves

Lifshitz (1946) showed the tensor modes in a spatially flat universe behave as massless scalars,

$$\Box_s h^{\mu}{}_{\nu} = 0\,.$$

The $\Box_s$ is a scalar wave operator.

- gravitons are equivalent to a pair of minimally coupled massless scalar fields.
- the corresponding quantum fluctuations are so-called "active".

generation of gravity waves

on the other hand, gravity waves may be generated by a source,

$$\Box_s h^{\mu}{}_{\nu} = -16\pi G_N S^{\mu}{}_{\nu} \, .$$

here $S^{\mu}{}_{\nu}$ is the transverse-tracefree part of the source stress tensor.

in the semiclassical theory, they may be generated by

- the renormalized expectation value of its stress tensor (PRD **83**, 084027).
- quantum fluctuations of the matter (PRD **84**, 103515).

quantum fluctuations of the stress tensor

integrating linearized Einstein equation

$$\Box_s h^{\mu}{}_{\nu} = -16\pi G_N S^{\mu}{}_{\nu} \, ,$$

by the retarded Green's function

$$\Box_s G_R(x, x') = -\frac{\delta(x - x')}{\sqrt{-\gamma}} \, ,$$

gives the induced metric perturbation/fluctuation

$$h^{\mu}{}_{\nu}(x) = 16\pi G_N \int d^4x' \sqrt{-\gamma'} \; G_R(x, x') S^{\mu}{}_{\nu}(x') \, .$$

we may form the metric correlation function $K^{\mu}{}_{\nu}{}^{\rho}{}_{\sigma}$

$$K^{\mu}{}_{\nu}{}^{\rho}{}_{\sigma}(x, x') = (16\pi)^2 \int d^4y \sqrt{-\gamma} \int d^4y' \sqrt{-\gamma'}\; G_R(x, y) G_R(x', y') C^{\mu}{}_{\nu}{}^{\rho}{}_{\sigma}(y, y') \,,$$

in terms of a stress tensor correlation function $C^{\mu}{}_{\nu}{}^{\rho}{}_{\sigma}$, where

$$K^{\mu}{}_{\nu}{}^{\rho}{}_{\sigma}(x, x') = \langle h^{\mu}{}_{\nu}(x) h^{\rho}{}_{\sigma}(x')\rangle - \langle h^{\mu}{}_{\nu}(x)\rangle\langle h^{\rho}{}_{\sigma}(x')\rangle \,,$$
$$C^{\mu}{}_{\nu}{}^{\rho}{}_{\sigma}(x, x') = \langle S^{\mu}{}_{\nu}(x) S^{\rho}{}_{\sigma}(x')\rangle - \langle S^{\mu}{}_{\nu}(x)\rangle\langle S^{\rho}{}_{\sigma}(x')\rangle \,.$$

- the evolution of metric nonlocally depends on matter (history dependent).
- for a conformally invariant field

$$C^{FRW}_{\mu\nu\rho\sigma}(x,x') = a^{-4}(\eta)a^{-4}(\eta')\,C^{Mink}_{\mu\nu\rho\sigma}(x,x')\,.$$

- define the power spectrum by the Fourier transform of the equal-time correlation function,

$$P(k) = \int \frac{d^3\mathbf{R}}{(2\pi)^3}\, e^{i\mathbf{k}\cdot\mathbf{R}} K(\eta=\eta',\mathbf{R})$$

- it is related to the power spectrum in cosmology by $\mathcal{P}(k) = 4\pi k^3 P(k)$.

sudden switching

1st scenario: if quantum fluctuations of matter couple w/ gravity at the onset of inflation $\eta = \eta_0$, and then integrate forward in time to the end of inflation at $\eta = \eta_r$, then we have

$$\mathcal{P}_s(k) = -\frac{4H^2}{3\pi} S^2 k^2 \left(1 + k^2 H^{-2}\right) , \qquad\qquad k|\eta_0| \gg 1 ,$$

S is the expansion factor during inflation.

- negative power spectra,
- blue tilt $\mathcal{P}(k) \propto k^4$,
- grows as S^2.

exponential switching

2nd scenario: the coupling to the fluctuating stress tensor is switched on gradually with a switching function $e^{\lambda\eta}$.

note that λ^{-1} is the approximate conformal time at which the interaction begins.

$$\mathcal{P}_e(k) = -\frac{3H^3}{8\pi} S\, k\left(1 + k^2 H^{-2}\right)$$

S is the expansion factor during inflation.

- negative power spectra,
- blue tilt $\mathcal{P}(k) \propto k^3$,
- grows as S^1.

as for the negative power spectra: the Wiener-Khinchine theorem requires a non-negative spectrum for a regular correlation function.

however, for quadratic quantum operators, such as a stress tensor, the positive definite quantity in this theorem may not exist b/c the corresponding correlation function is highly singular.

this allows for negative power spectra. (Phys. Lett. A**375**, 2296.)

another example: for the flat-space EM energy density

$$P(k) = -\frac{k^5}{960\pi^5}.$$

numerical estimation

- consider perturbations of the order of the present horizon size, $\ell \approx 10^{61} \ell_p$.
- WMAP constrain these perturbations to satisfy $h \leq 10^{-5}$.
- $|K_{now}| \leq 10^{-10}$.
- this limits, for exponential switching,

$$S_e < 10^{40} \left(\frac{10^{16}\mathrm{GeV}}{E_R} \right)^7$$

- it is compatible with adequate inflation $S \geq 10^{23}$ for the flatness problem.
- $\because \mathcal{P} < 0$, quantum stress tensor fluctuations during inflation tends to produce ANTI-correlated gravity wave fluctuations.

summary

- the gravity waves are generated by quantum stress tensor fluctuations during inflation.
- this induced gravity waves tend to anti-correlated.
- its power spectra are negative, nonscale-invariant.
- this spectrum also depends on the duration of inflation.
- the effect is in principle observable in that gravity wave modes are no longer exactly solutions of the Lifshitz equation.
- this possibility does require the contribution of modes which were transplanckian at the beginning of inflation.
- if we apply similar considerations to different inflation models/alternative gravity theories, together with observation data from LISA or BBO, it may improve our understanding of inflation/transplankian physics.