Trispectrum estimation in various models of equilateral type non-Gaussianity

Phys.Rev. D85 (2012) 023521

Keisuke Izumi

(Leung center for Cosmology and Particle Astrophysics, National Taiwan Uni.)

Collaborator: Shuntaro Mizuno (Orsay LPT & APC Paris)
Kazuya Koyama (Portsmouth)

Introduction

Initial condition of Big Bang Theory

Flatness problem, Horizon Problem, Monopole problem

Inflation can solve these problems
and predict perturbation of CMB is almost Gaussian and
almost scale invariant

observed

Which inflation??

More detailed information of CMB perturbation

Deviation from Gaussianity
Deviation from scale invariance
Different scale from CMB

Gravitational wave

Topic of my talk: Non-Gaussianity

Bispectrum $\sim \quad < \phi\phi\phi >$

Momentum dependence

$\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3}$ — 9 variables

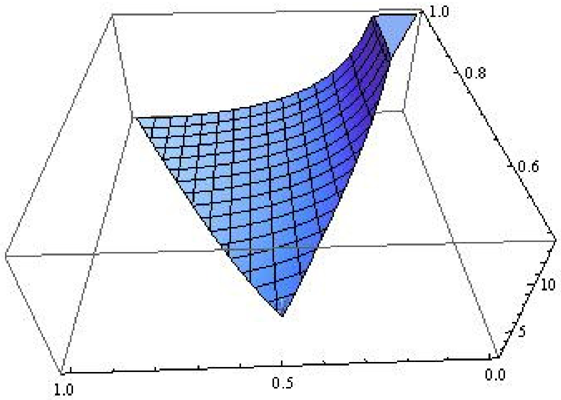
Constraint from symmetry on background

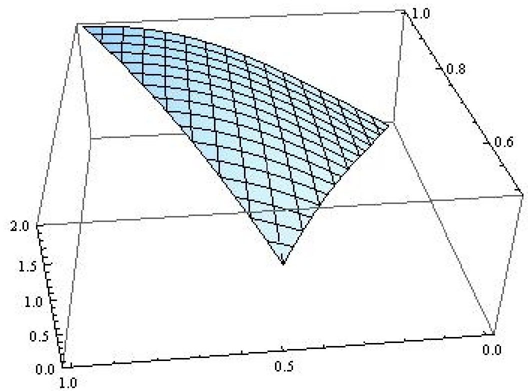
Homogeneity — 3 constraints

Isotropy — 3 constraints

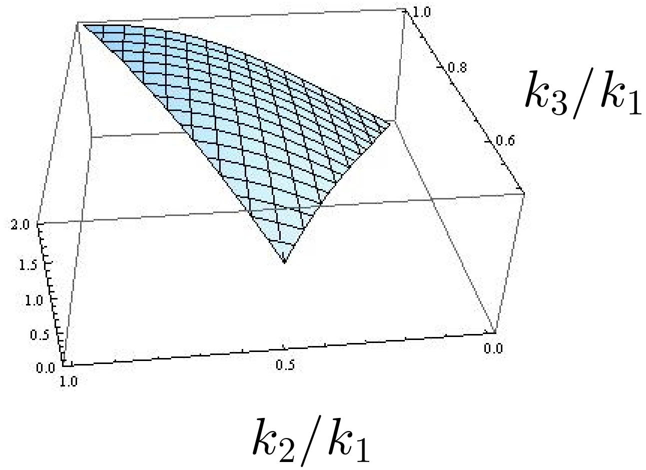
Bispectrum depends on 3 variables

Additionally, assume scale independence → 2 variables $k_2/k_1, k_3/k_1$





Equilateral shape



Maximum at equilateral point

$$k_2/k_1 = k_3/k_1$$

Derivative coupling gives equilateral shape

DBI inflation, ghost inflation, Lifshitz-scalling scalar

In these model, the shapes of bispectrum are similar.

Hard to discriminate by observation

Higher order correlation function

The next order is Trispectrum $\sim \quad < \phi\phi\phi\phi >$

Trispectrum $\sim \quad < \phi\phi\phi\phi >$

Momentum dependence

$\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3}, \mathbf{k_4}$ 12 variables

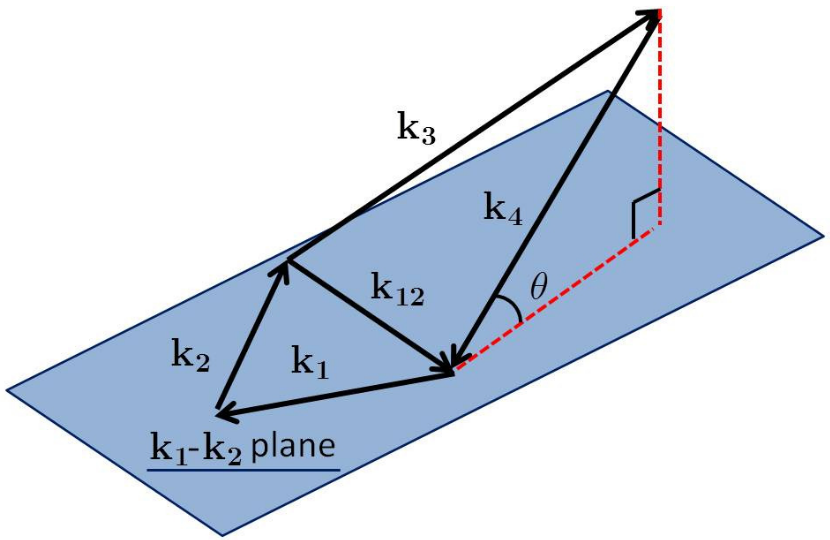
Constraint from symmetry on background

Homogeneity 3 constraints

Isotropy 3 constraints

➡ Trispectrum depends on 6 variables

Additionally, assume scale independence ➡ 5 variables



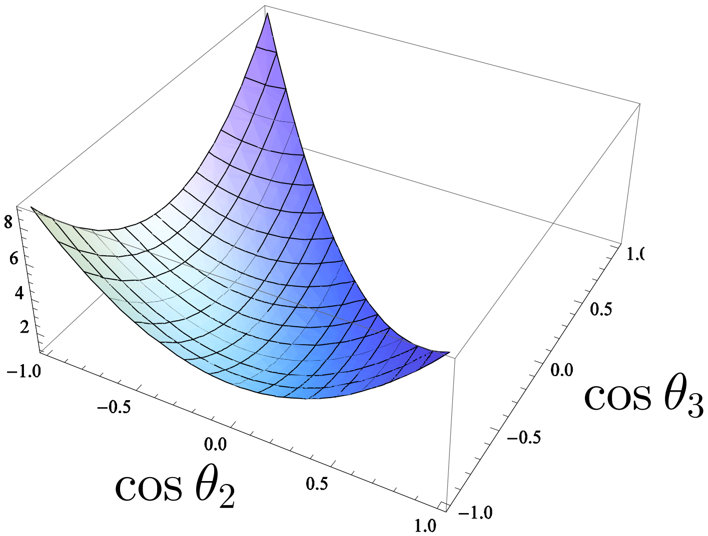
$$k_2/k_1, k_3/k_1, k_4/k_1,$$

$$k_{12}/k_1, \cos\theta_4$$

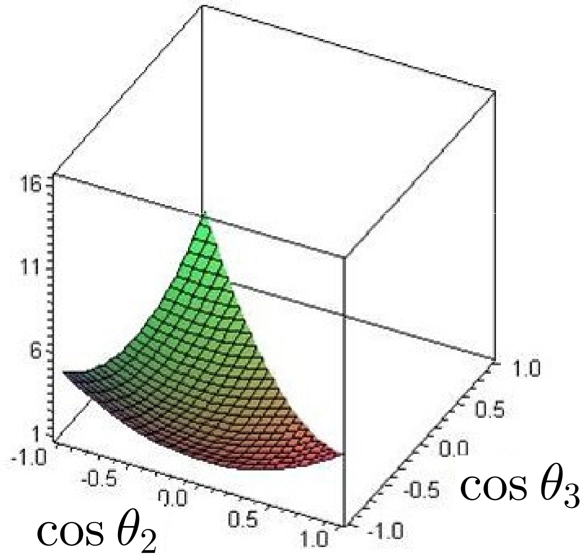
Shape of equilateral Trispectrum

One way to see difference among models visually is
fixing 3 of 5 variables

Example: equilateral case $k_1 = k_2 = k_3 = k_4 = k$



Ghost inflation
(K. I., S. Mukohyama 2010)



Multi DBI inflation
(S. Mizuno, F. Arroja, K. Koyama 2009)

$$\cos\theta_i = \mathbf{k_1}\cdot\mathbf{k_i}/k^2$$

$$\cos\theta_2 + \cos\theta_3 + \cos\theta_4 = 1$$

Correlator of Trispectrum shape

- For exact science, numerical comparison is needed.
- using all information is better.

⇓

Introduce inner product (D. M. Regan, E. P. S. Shellard and J. R. Fergusson 2010)

In Regan's paper,
Trispectrum in some of model can be decompose into sum of functions which depends on 5 variables

$$S_T(k_1, k_2, k_3, k_4, k_{12}) + S_T(k_1, k_2, k_3, k_4, k_{13}) + S_T(k_1, k_2, k_3, k_4, k_{14})$$

Reduced Trispectrum

Definition of inner product of $S_T(k_1, k_2, k_3, k_4, k_{12})$ and $S_T'(k_1, k_2, k_3, k_4, k_{12})$

$$\int dk_1 dk_2 dk_3 dk_4 dk_{12} W S_T S_T'$$

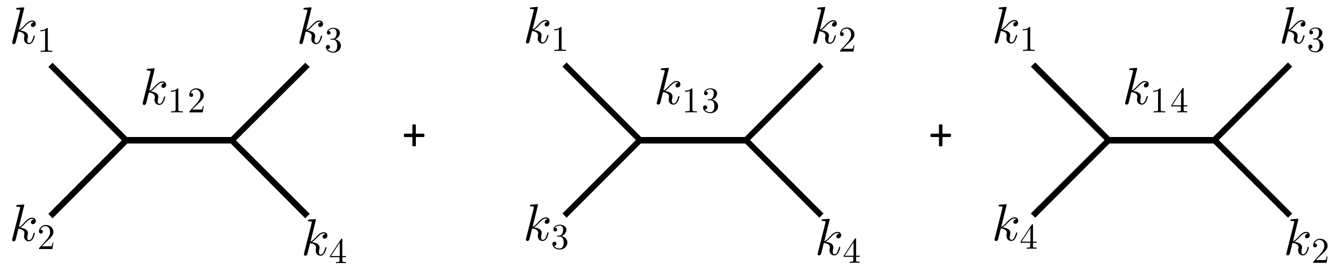
$W = W(k_1, k_2, k_3, k_4, k_{12})$: Window function

Decomposition of Trispectrum

$$S_T(k_1, k_2, k_3, k_4, k_{12}) + S_T(k_1, k_2, k_3, k_4, k_{13}) + S_T(k_1, k_2, k_3, k_4, k_{14})$$

Possible case

Trispectrum from scalar exchange



Impossible case

Trispectrum from higher derivative term

$$k_1 k_2 k_3 k_4 k_{12} k_{13}$$

We must use full Trispectrum

$$\int dk_1 dk_2 dk_3 dk_4 dk_{12} d\cos\theta_4 W F_T F_T'$$

$F_T = F_T(k_1, k_2, k_3, k_4, k_{12}, \cos\theta_4)$: Full Trispectrum

Difference between two definitions

Correlation by reduced Trispectrum

	$S_{\mathcal{T}}^{DBI(\sigma)}$	$S_{\mathcal{T}}^{DBI(s)}$	$S_{\mathcal{T}}^{ghost}$	$S_{\mathcal{T}}^{h(se,11)}$	$S_{\mathcal{T}}^{h(se,12)}$	$S_{\mathcal{T}}^{h(se,22)}$	$S_{\mathcal{T}}^{h(ci,1)}$	$S_{\mathcal{T}}^{h(ci,2)}$	$S_{\mathcal{T}}^{h(ci,3)}$
$S_{\mathcal{T}}^{c1}$	0.87	0.33	0.24	0.24	-0.62	0.95	-0.35	0.01	-0.53

Correlation by full Trispectrum

	$F_{T}^{DBI(\sigma)}$	$F_{T}^{DBI(s)}$	F_{T}^{ghost}	$F_{T}^{h(se,11)}$	$F_{T}^{h(se,12)}$	$F_{T}^{h(se,22)}$	$F_{T}^{h(ci,1)}$	$F_{T}^{h(ci,2)}$	$F_{T}^{h(ci,3)}$
F_{T}^{c1}	0.96	0.42	0.41	0.23	-0.70	0.99	-0.54	-0.01	-0.38

These two results are roughly equal.

If correlation by reduced Trispectrum is almost one,
correlation by full Trispectrum must be almost one.
The opposite is not always true because it depends on decomposition.

$$F_T = S_T(k_{12}) + S_T(k_{13}) + S_T(k_{14})$$

In rough estimation, using reduced Trispectrum might be better because of easiness of calculation.
For precise result, full Trispectrum is needed

Summary

High order correlation function of primordial perturbation gives the information of inflation epoch.

Trispectrum could give additional information of inflation.

In precise science, quantifying correlation must be needed.

⇓

By inner product, correlation can be quantified.

Reduced Trispectrum

In some model, Trispectrum can not be decomposed.

↕ Roughly equal

Full Trispectrum

Correlation can be defined in all models