Hawking Radiation around the Wormhole

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Motivation

- Recent active researches on dynamical horizon
- Morris-Thorne wormhole throat is a double trapping horizon
- Potential form around the throat is similar to that of event horizon
- Temperature of wormhole issue is revisited

Surface Gravity

- Several Definitions
  - Killing vector definition (Wald)

$$k^a \nabla_a k^b = \kappa k^b$$

  - Acceleration (Abreu & Visser)

$$\kappa = \sqrt{g_{ij}\kappa^i \kappa^j} = e^{-\Psi}\sqrt{g_{ij}A^i A^j}.$$

  - 2D Expansion (Jacobson & Parentani)

$$\kappa = (u \cdot \chi)\, \theta_{2D}^{\mathrm{hor}}$$

  - Minimality condition (Hayward)

$$\kappa = \tfrac{1}{2} * d * dr$$

- Recent review (Nielson & Yoon; Pielahn, Kunstatter, & Nielson)

Wormhole temperature

- By Killing vector definition (Hong & Kim, 2006)

$$k_{\mathrm{S}} = \Phi'(r)e^{\Phi(r)}\left(1 - \frac{b(r)}{r}\right)^{1/2}$$

- Negative temperature

$$T < T_{\mathrm{H}} = \frac{1}{4\pi r^2}\frac{b(r) - 8\pi G r^3 \tau(r)}{\left(1 - \frac{b(r)}{r}\right)^{1/2}} e^{\Phi(r)}$$

with exotic matter

Trapping Horizon

- A sphere of radius $r=(A/4\pi)^{1/2}$ is
  - Untrapped for spatial $g^{-1}(dr)$
  - Marginal for null $g^{-1}(dr)$
  - Trapped for temporal $g^{-1}(dr)$
- Trapping horizon: A hypersurface foliated by marginal spheres

Surface gravity at trapping horizon (Hayward, 1998)

- Kodama vector: preferred flow of time

$$k = g^{-1}(*dr)$$

- Normal to sphere of symmetry

$$k \cdot dr = 0, \qquad g(k,k) = -g^{-1}(dr,dr)$$

- Surface gravity

$$\kappa = \tfrac{1}{2} {*d{*dr}}$$

- Spherically symmetric metric

$$ds^2 = r^2 d\Omega^2 - 2e^{2\varphi} dx^+ dx^-$$

$$k = e^{-2\varphi}(\partial_+ r \partial_- - \partial_- r \partial_+) \qquad \kappa = -e^{-2\varphi}\partial_+ \partial_- r$$

Wormhole Case

- Morris-Thorne wormhole:

$$ds^2 = r^2 d\Omega^2 + (1 - 2m/r)^{-1} dr^2 - e^{2\varphi} dt^2$$

- Regge-Wheeler tortoise coordinate form

$$ds^2 = r^2 d\Omega^2 + e^{2\varphi} (dr_*{}^2 - dt^2)$$

- Surface gravity

$$2\kappa = \frac{m}{r^2} - \frac{\partial_r m}{r} + \left(1 - \frac{2m}{r}\right) \partial_r \varphi$$

- By Einstein's equation ($2m \neq 8\pi r^3 \tau$)

$$\kappa = 2\pi r(\tau - \rho)$$

- Flare-out condition

$$\kappa > 0$$

Hamilton-Jacobi equation (1/3)

$$ds^2 = r^2 d\Omega^2 + (1 - 2m/r)^{-1} dr^2 - e^{2\varphi} dt^2$$

- With redefinition of $v = t_* + r_*$ as

$$dt = C^{1/2} dt_*,\ dr = e^{\varphi} C dr_*,\ C = 1 - 2m/r$$

- Advanced Eddington-Finkelstein form ($\varphi = \Psi$)

$$ds^2 = r^2 d\Omega^2 + 2e^{\Psi} dv dr - e^{2\Psi} C dv^2$$

- WKB approximation of the tunneling probability Γ

$$\Gamma \propto \exp\left(-2\frac{\Im I}{\hbar}\right)$$

$\Im I$ is the imaginary part of action I

Hamilton-Jacobi equation (2/3)

- In thermal form

$$\Gamma \propto \exp\left(-\frac{\omega}{T}\right)$$

- Action

$$I = \int \omega e^{\Psi} dv - \int k dr$$

- Energy and momentum

$$\omega = K \cdot dI = e^{-\Psi} \partial_v I, \; k = -\partial_r I$$

- Hamilton-Jacobi equation

$$g^{-1}(\nabla I, \nabla I) = 0 \quad \text{or} \quad 2\omega k - C k^2 = 0$$

- Solution for outgoing mode

$$k = 2\omega / C$$

- I has a pole

$$C \approx (r - r_0)\partial_r C$$

Hamilton-Jacobi equation (3/3)

- For the case of $(2m=8\pi r^{3}\tau)$

$$k \approx \omega/\kappa(r - r_0)$$

- The imaginary part of the action

$$\Im I \cong \frac{\pi\omega}{\kappa}$$

- By comparing with the thermal form

$$T \cong \frac{\kappa}{2\pi}$$

Summary

- Various definitions of the surface gravity
- Wormhole’s Hawking temperature
- Checking by Hamilton-Jacobi tunneling method, we can consider the Hawking radiation