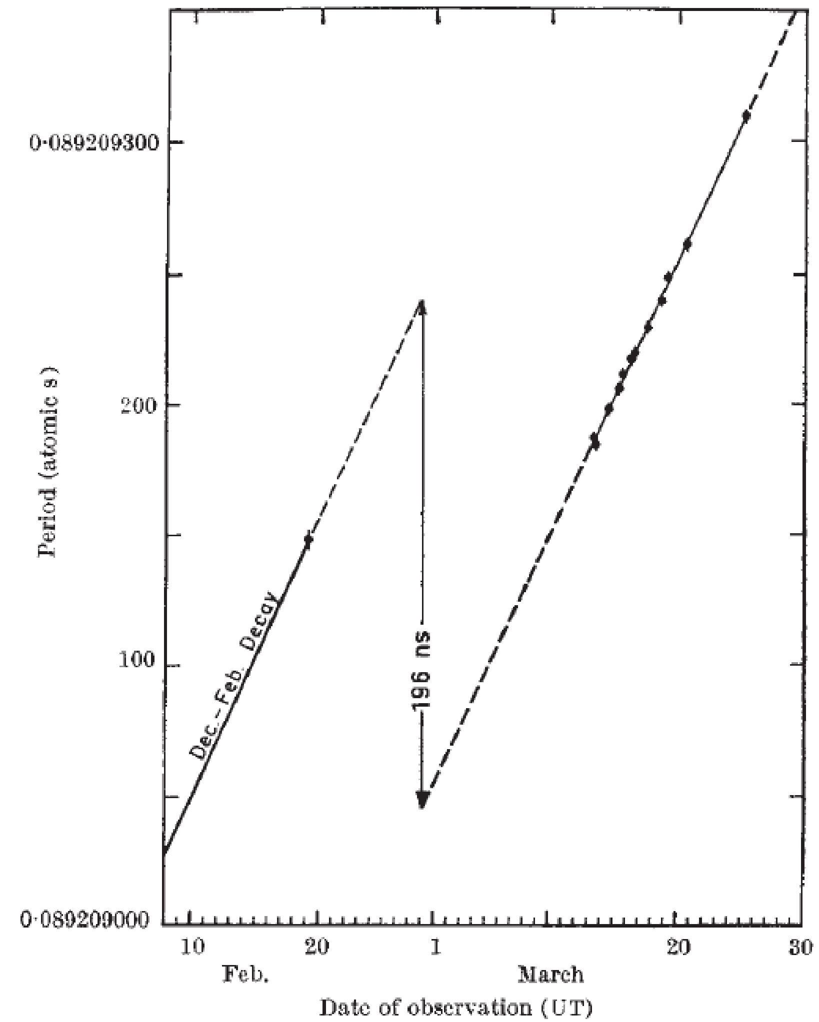
Gravitational Wave Emission from Pulsar Glitches

Jinho Kim

Astronomy program, Department of Physics and Astronomy,
Seoul National University

Pulsar Glitches

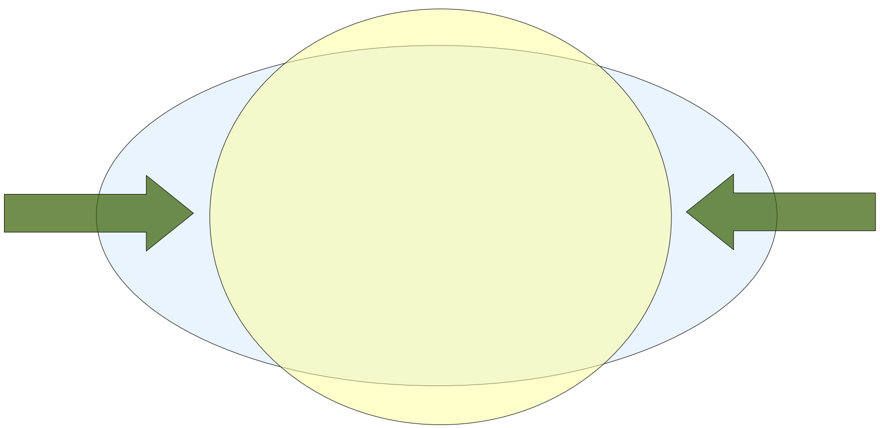
- Typical value of $\delta \nu / \nu$ is between 10^{-6} and 10^{-9}.
- Two possible mechanisms have been proposed
  - Star quake (Ruderman 1969)
  - Angular momentum transfer at the core (superfluid)-crust interface (Packard 1972; Anderson & Itoh 1975)
- Why are they so interesting? Because
  - They can be used to infer the neutron star's interior
  - They can give constraints of neutron star's equation of state
  - They also can excite some modes that can emit periodic gravitational waves.



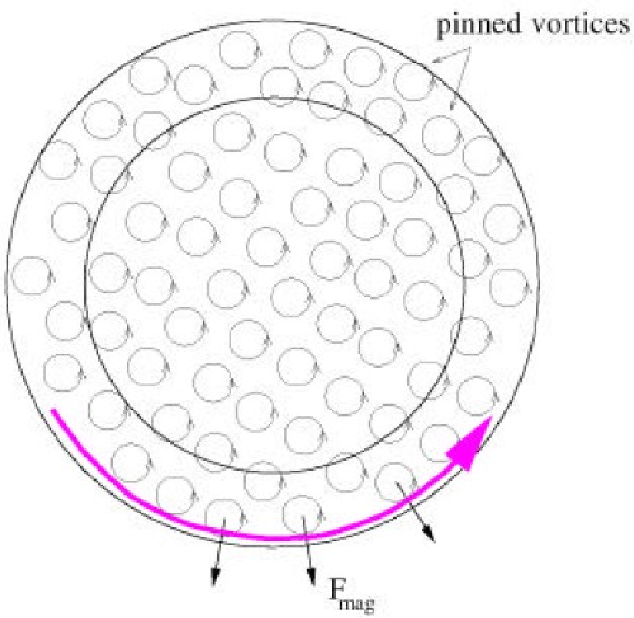
Radharrishnan & Manchester (1969)

Pulsar Glitches

- Typical value of $\delta\nu/\nu$ is between 10^{-6} and 10^{-9}.
- Two possible mechanisms have been proposed
  - Star quake (Ruderman 1969)
  - Angular momentum transfer at the core (superfluid)-crust interface (Packard 1972; Anderson & Itoh 1975)
- They are interesting because
  - They can be used to infer the neutron star's interior
  - They can give constraints of neutron star's equation of state
  - They also can excite some modes that can emit periodic gravitational waves.



Star Quake Model



Vortex Unpinning Model

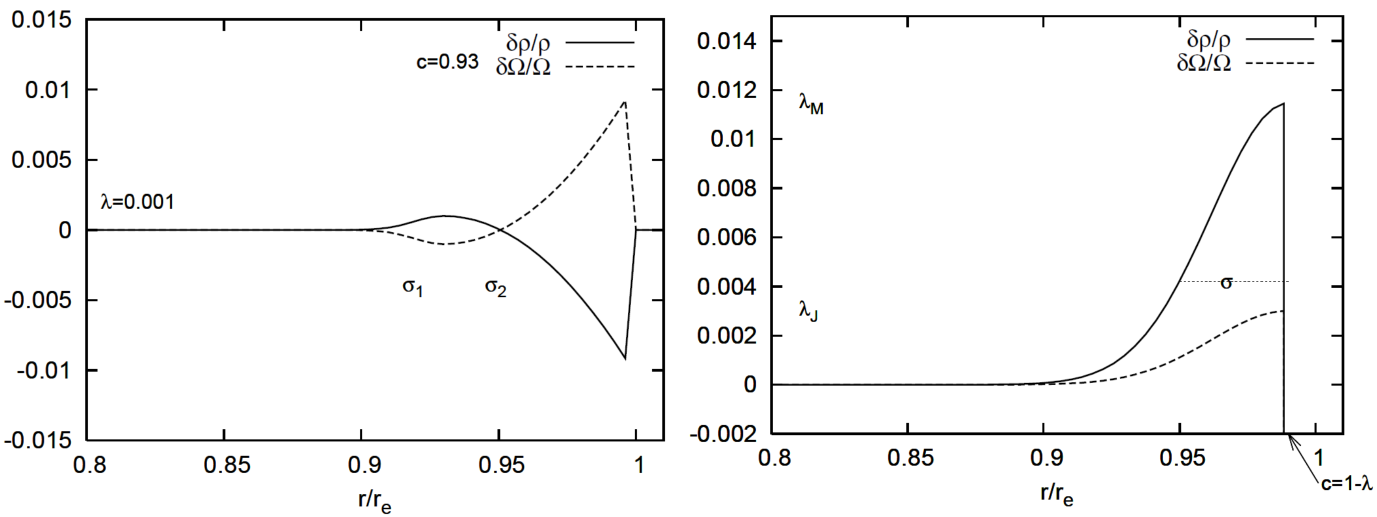
Method

- Time evolution of rotating stars with perturbations which mimic pulsar glitches
- Extraction of the time series of quadrupole moment
- Fourier transformation
- Estimation of GW strain amplitude

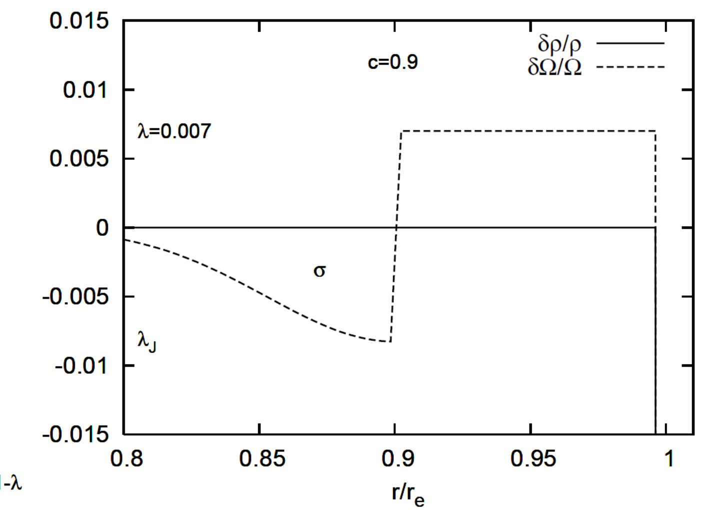
Imposed Perturbations

- We assume that
  - the depth of the neutron star's crust is 10% of its radius.
  - the effects of crust due to the hardness such as fractures are neglected.
- All perturbations should obey two constraints: total mass and total angular momentum conservations i.e.,

$$M_0 = \int \rho_0 W \, dV = \text{constant}, \quad J = \int T^0_{\phi} \, dV = \text{constant}.$$



Star Quake Model



Superfluid Model

Pseudo-Newtonian Approach

- Taking Newtonian limit

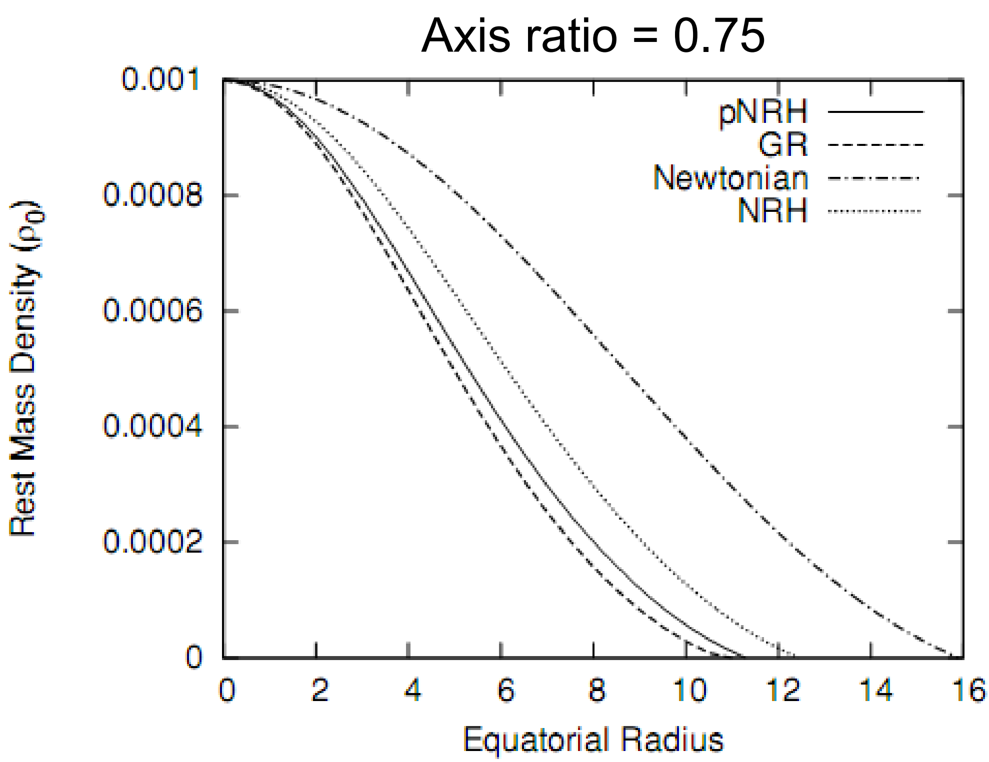
$$ds^2 = -(1+2\Phi)dt^2 + \frac{1}{1+2\Phi}\delta_{ij}dx^i dx^j$$

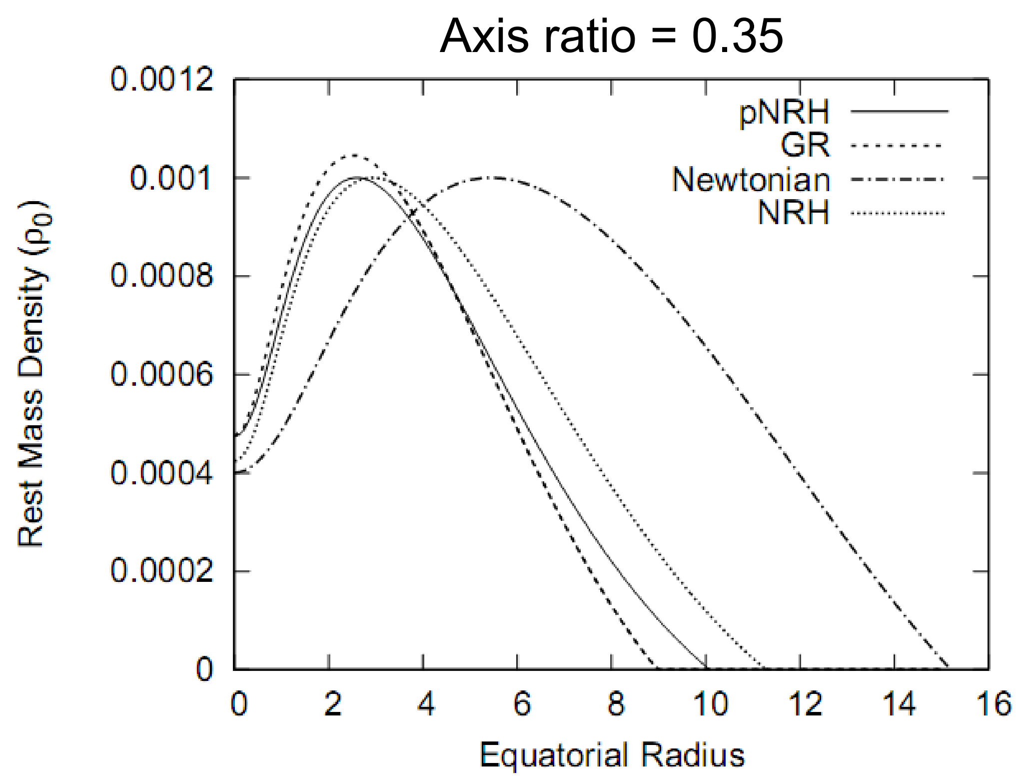
- If the metric is given, hydrodynamics equation can easily be written in standard formulation
- Einstein equation → 2nd order approximation of (v/c) → equation for gravitational potential : Poisson equation $\nabla^2 \Phi = 4\pi \rho_{active}$
  - Note : source term in Poisson equation is 'Active Mass Density' not just baryon density or total mass density
  - Active mass density contains all forms of energy ingredients (baryon number density as well as enthalpy, pressure and velocity)

$$\rho_{\text{active}} = \rho_0 h \frac{1+v^2}{1-v^2} + 2\mathrm{P}$$

Pseudo-Newtonian Approach

- Density profile of the spheroidal and quasi-toroidal shape
  - $P = K \rho_0^{1+1/N}$ (N=1, K=100) and $\rho_{max} = 0.001$





Less than 5% difference!

Mode Analysis & Excited Modes

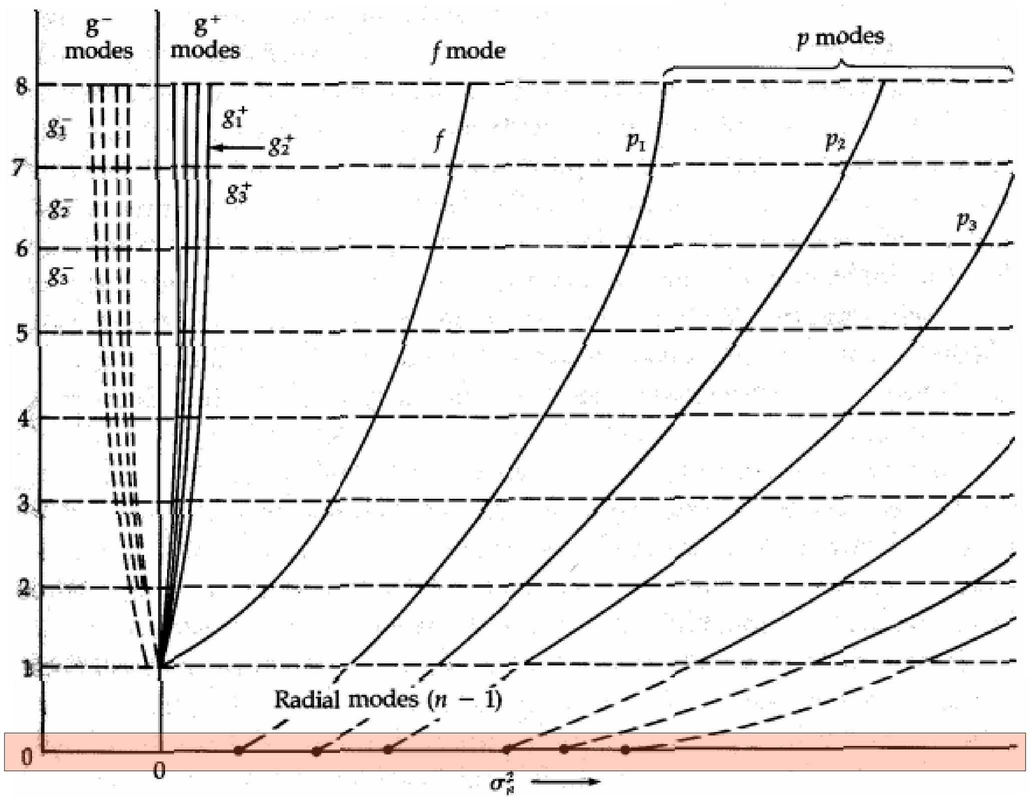
- In order to extract the mode which can produce gravitational wave, we use the time series of quadrupole moment in the simulations.
- The quadrupole moments in our approach are

$$I_{xx} = \int \rho \left(x^2 - \frac{1}{3} r^2 \right) dV \ , \quad I_{zz} = \int \rho \left(z^2 - \frac{1}{3} r^2 \right) dV \ .$$

- To identify specific modes, we compare with the Newtonian and (approximated) general relativistic (Font et. al., 2001; Demmelmeier et. al., 2006; Yoshida & Eriguchi, 2000) ones.

Brief Description of Stellar Pulsation Mode

- Radial Oscillations
  - depend only on δr
  - cannot generate gravitational wave in the spherical star
  - F, H_1, H_2

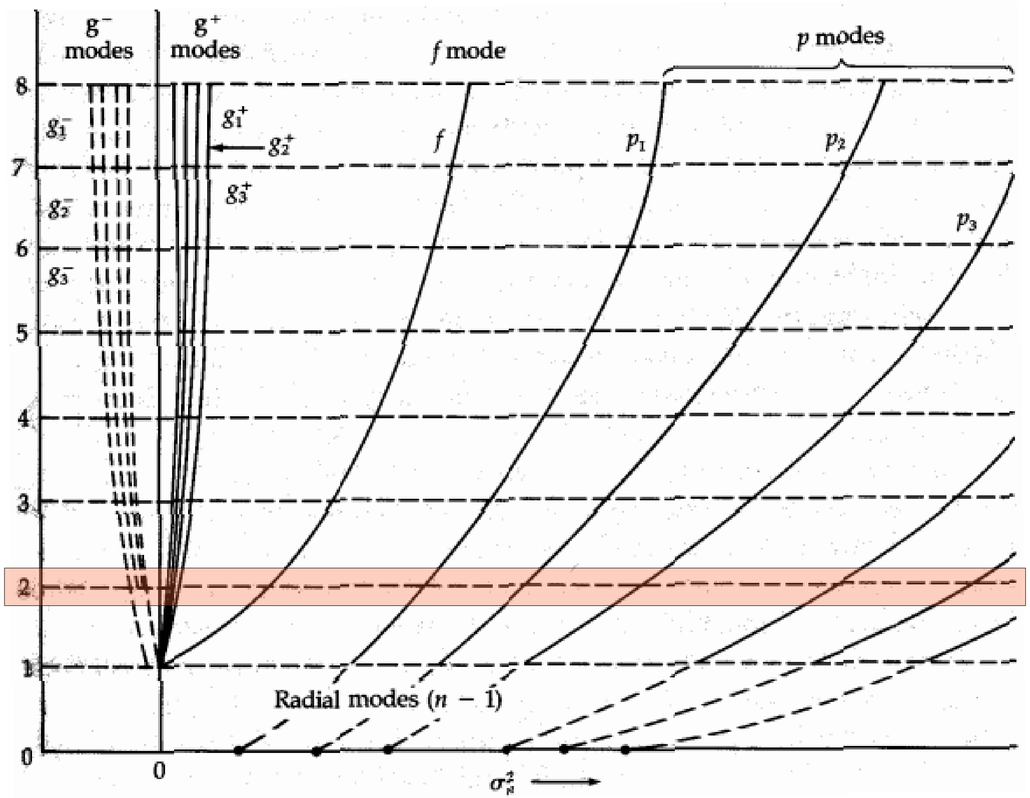


- Non-radial oscillations
  - depend r and angular part
  - are generally separated radial and angular part using spherical harmonics
  - are classified by the restoring forces
    - Inertial mode : Coriolis' force
    - g mode : gravity
    - p mode : pressure force
  - l=2 modes gives strong quadrupole → gravitational wave

•Schematic view of the modes from Cox (1970)

Brief Description of Stellar Pulsation Mode

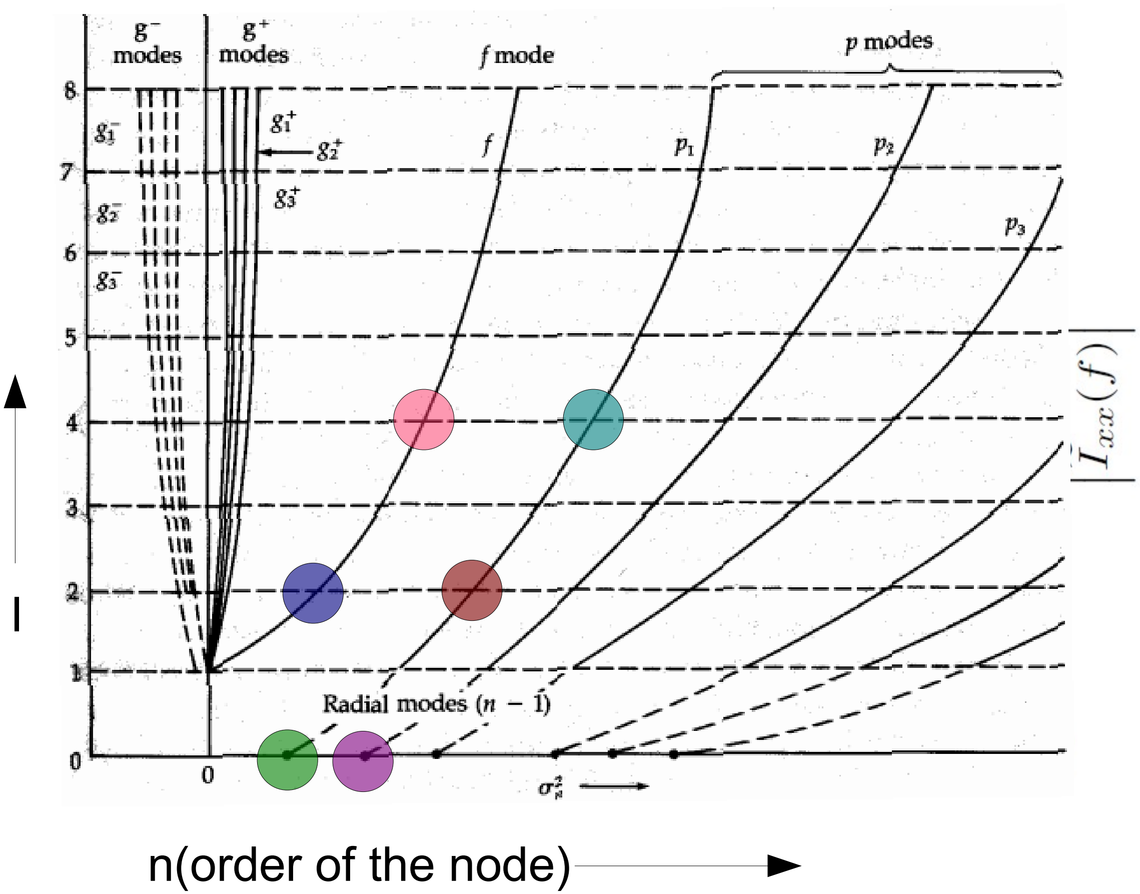
- Radial Oscillations
  - depend only on δr
  - cannot generate gravitational wave in the spherical star
  - $F,\ H_1,\ H_2$



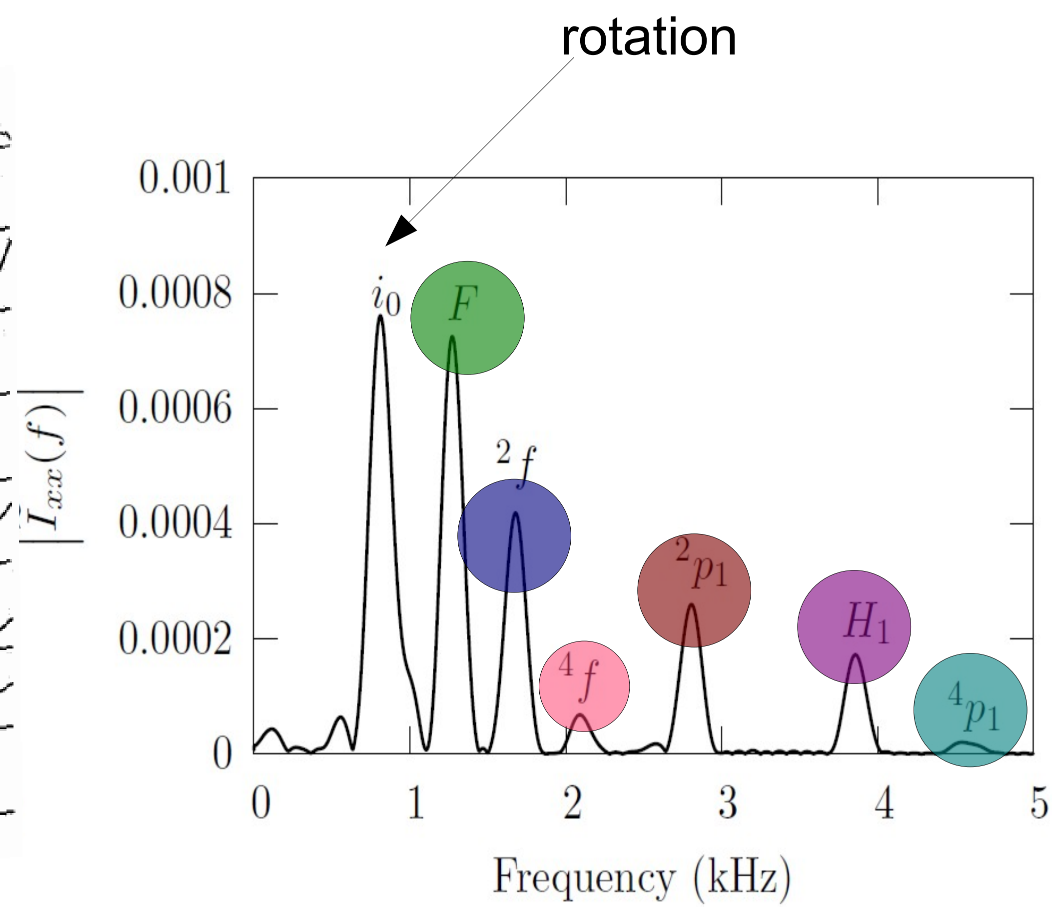
- Non-radial oscillations
  - depend r and angular part
  - are generally separated radial and angular part using spherical harmonics
  - are classified by the restoring forces
    - Inertial mode : Coriolis' force
    - g mode : gravity
    - p mode : pressure force
  - l=2 modes gives strong quadrupole → gravitational wave

•Schematic view of the modes from Cox (1970)

Mode Identification

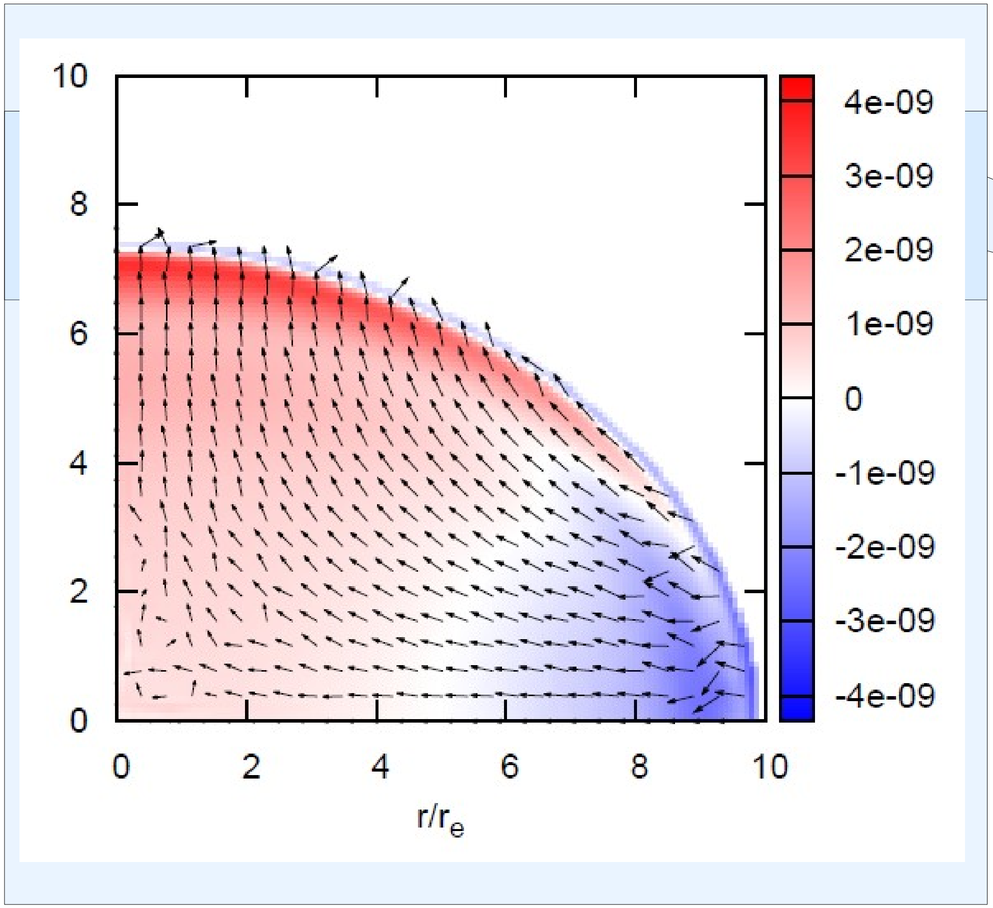


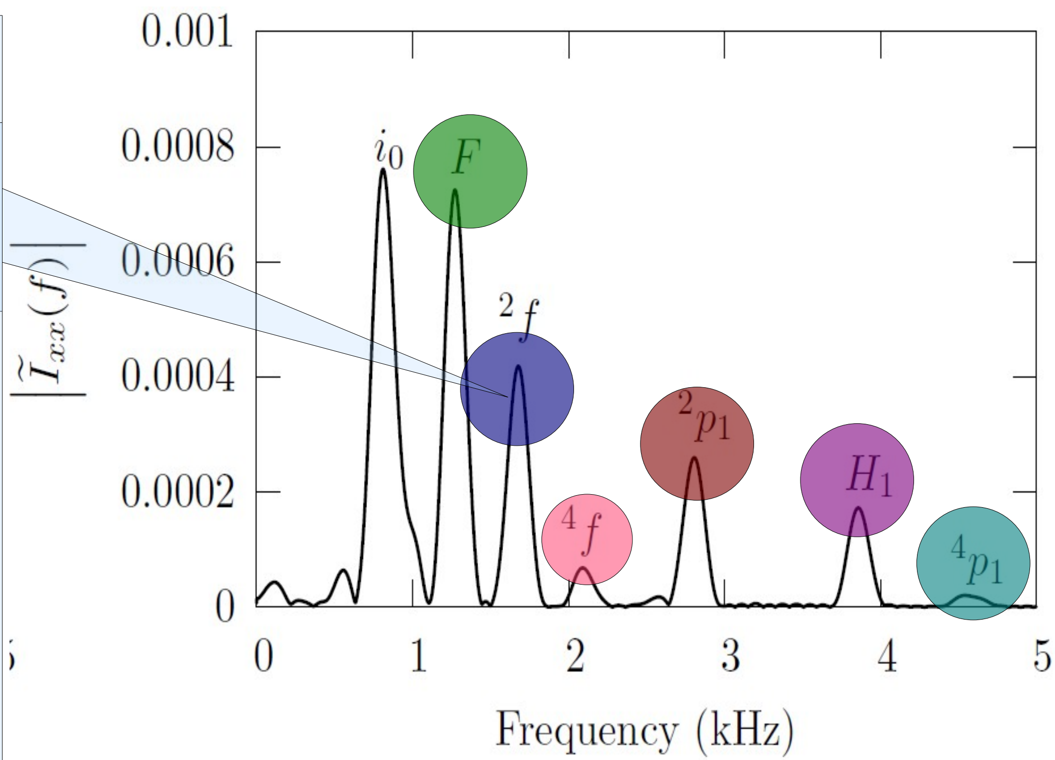
•Schematic view of the modes from Cox (1970)



- Sinusoidal amplitudes excited by perturbation 3 (superfluid model)

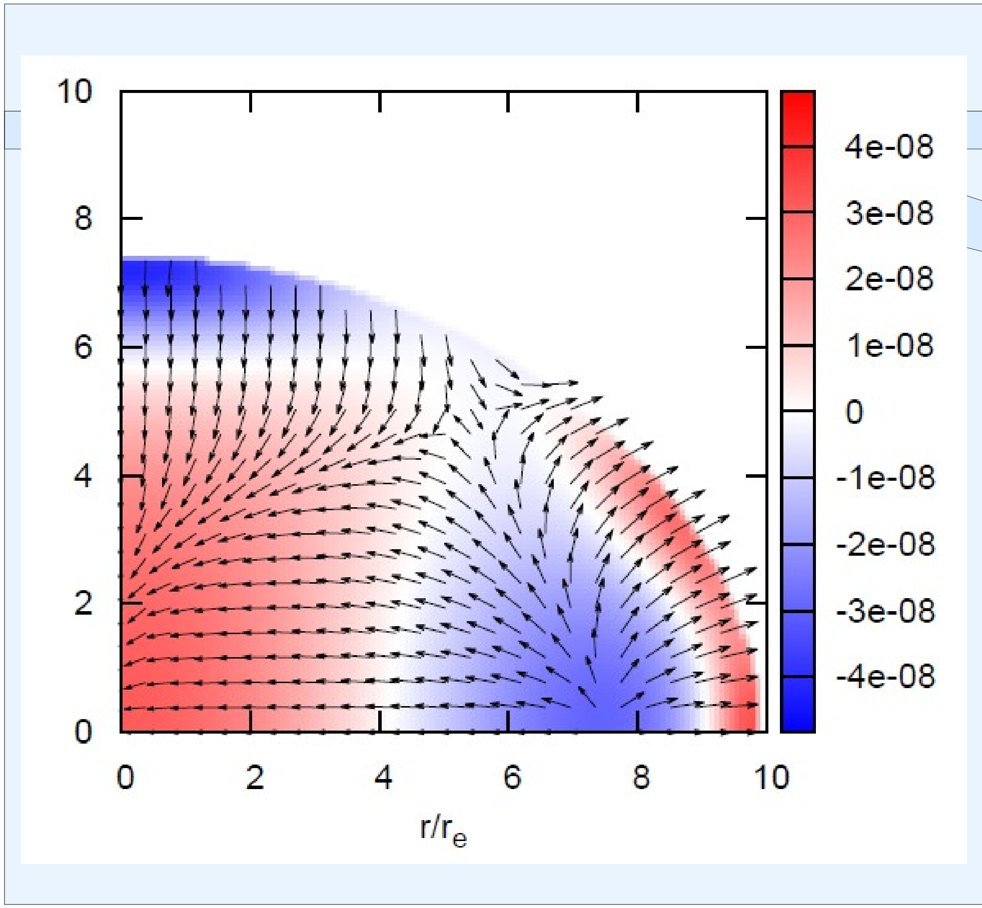
Mode Identification

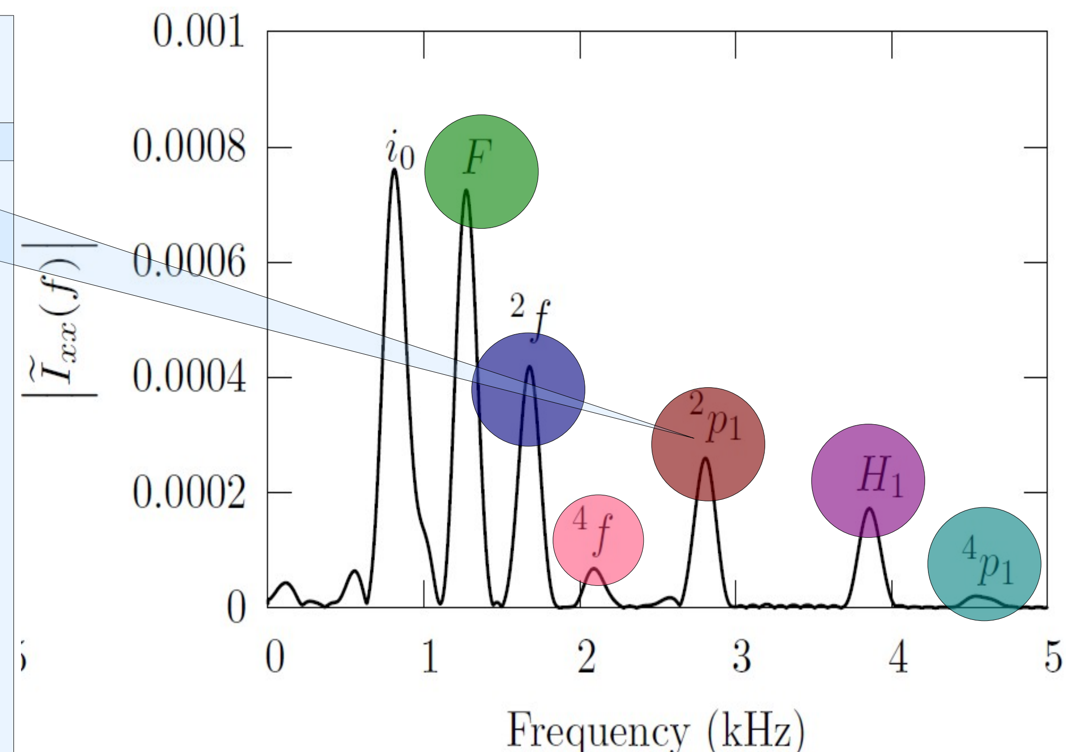




- Sinusoidal amplitudes excited by perturbation 3 (superfluid model)

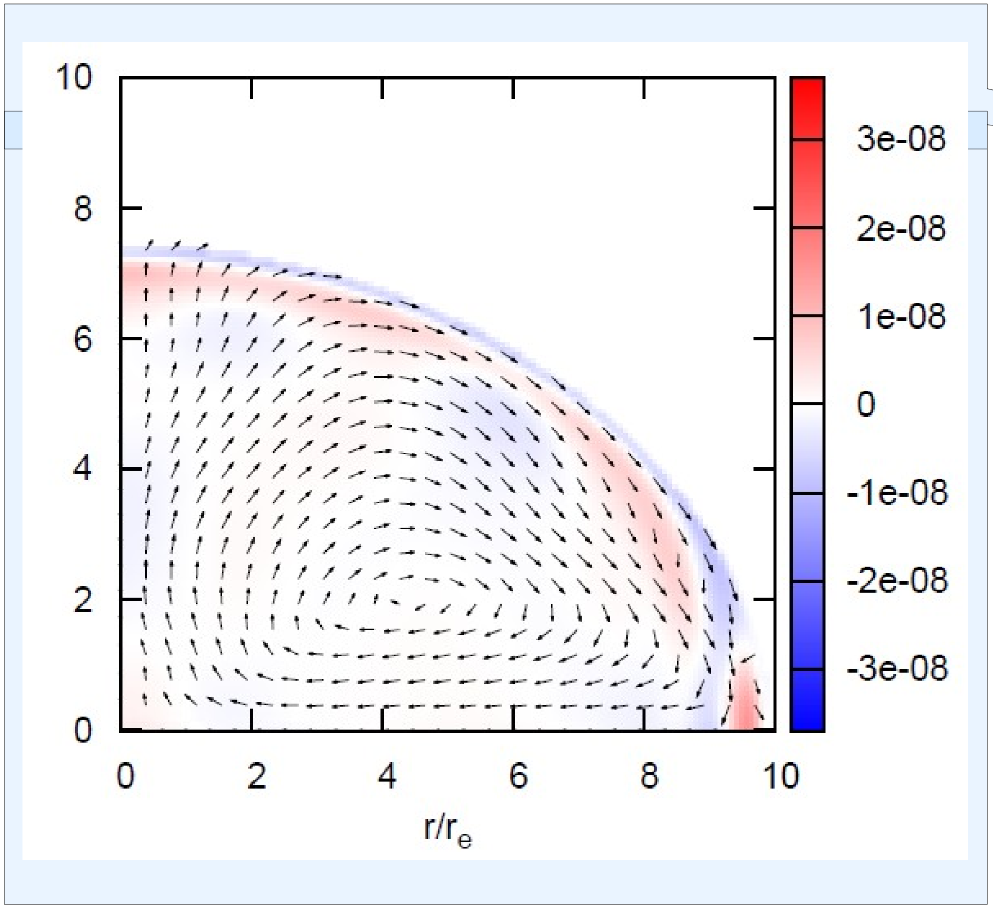
Mode Identification

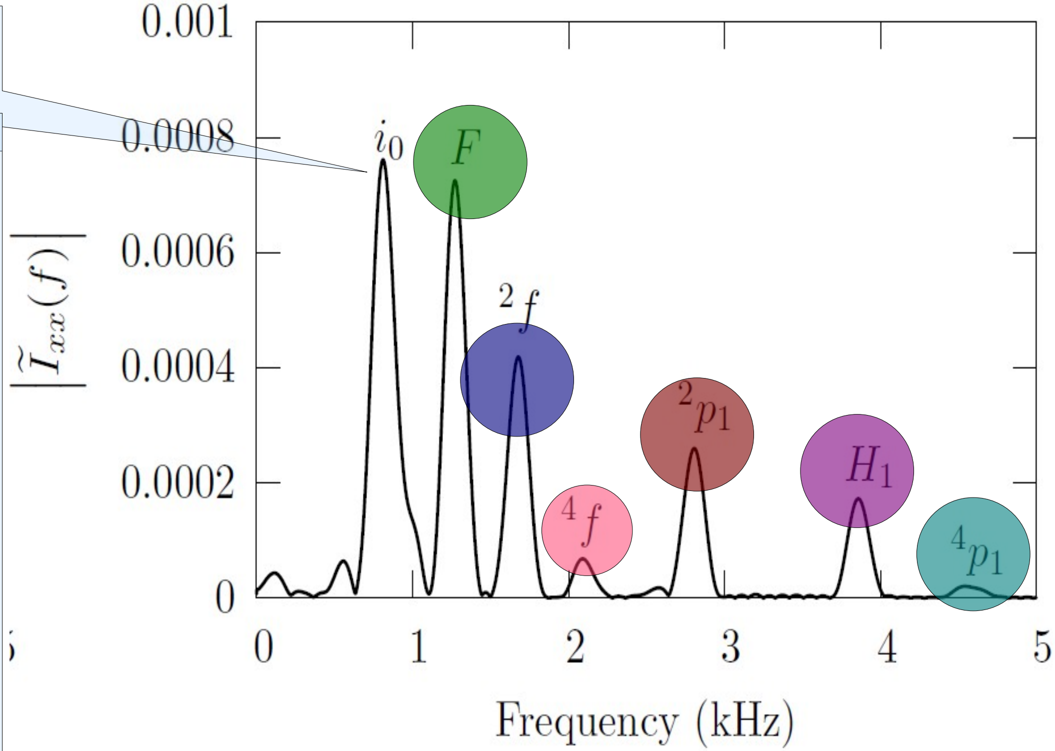




- Sinusoidal amplitudes excited by perturbation 3 (superfluid model)

Mode Identification





- Sinusoidal amplitudes excited by perturbation 3 (superfluid model)

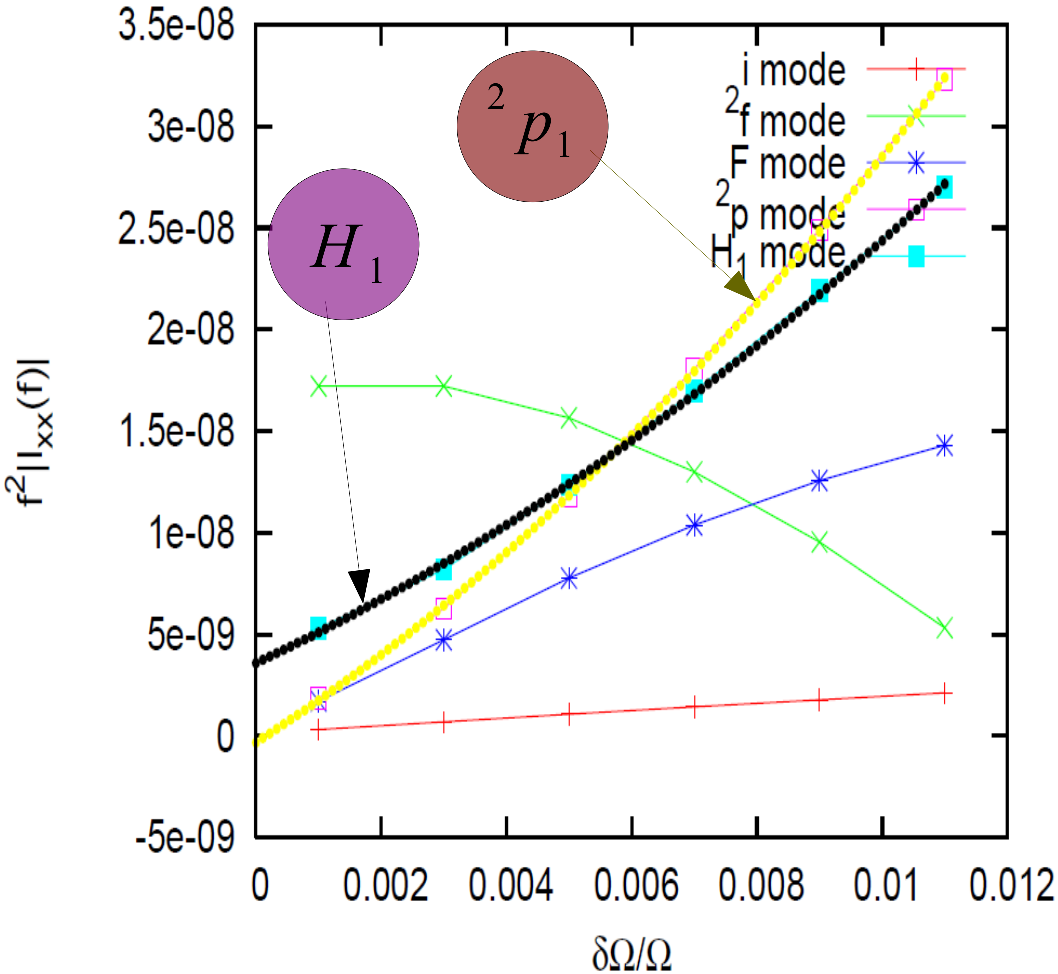
Gravitational Wave From the Glitching Pulsar

- Strain amplitude of gravitational wave at a distance r can be written as

$$h_{xx} \simeq \frac{8\pi^2}{r} f^2 \tilde{I}_{xx}, \quad h_{zz} \simeq \frac{8\pi^2}{r} f^2 \tilde{I}_{zz},$$

where $\tilde{I}$ is amplitude of oscillating quadrupole moment I.

.



- We found that the strongest and second strongest modes are $^2p_1$ and H_1, contrary to the usual assumption of the ^2f mode as the strongest mode.

- The amplitude of inertial mode is not very strong but it may be able to become non-axisymmetric r-mode which can emit stronger gravitational wave.

	Star quake					Superfluid		
		perturbation 1 $(\times 10^{-25})$				perturbation 2 $(\times 10^{-25})$		
	i_0	F	^2f	$^2p_1$	H_1	i_0	$^2p_1$	H_1
AU1	0.00752	-	0.282	2.31	0.373	0.145	3.37	0.975
AU2	0.0731	8.97	6.86	5.03	3.52	1.01	7.49	1.28
AU3	0.367	11.9	15.8	8.64	4.55	3.07	14.2	11.5
AU4	1.39	14.7	24.7	9.00	5.66	8.10	13.9	3.55
BU1	0.00321	3.00	0.128	0.964	0.371	0.0943	2.19	0.746
BU2	0.0130	5.65	0.967	2.30	1.17	0.560	4.86	1.49
BU3	0.0491	6.47	2.39	2.58	1.45	1.61	7.39	3.37
BU4	-	8.28	5.09	2.39	1.30	3.61	8.71	0.0356

Energy of the Each Modes

- We used Newtonian definition of Kinetic energy which is written as $T=\frac{1}{2}\int \rho_0 v^2 \, dV$

Mode	Energy of Mode $(\times 10^{-8})$
i_+	4.39
i_0	38.2
${}^2f$	0.00
F	0.91
${}^2p_1$	0.146
H_1	4.32
${}^2p_2$	2.98
H_2	0.722
${}^4p_1$	0.00
total	51.7

Energy of the each specific mode for the $2M_{sun}$ neutron star with $\delta\Omega/\Omega = 1.1\times10^{-2}$

Total input energy given by perturbation $=41.9\times10^{-8}$

Difference = 19%

↓

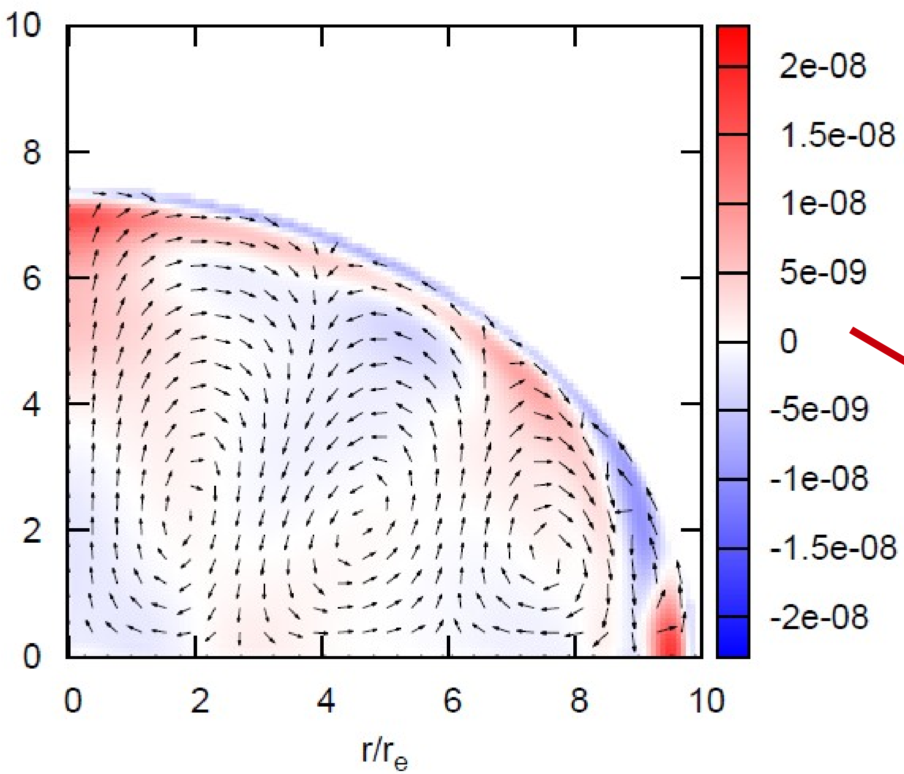
Difference in strain amplitude = 9.1%

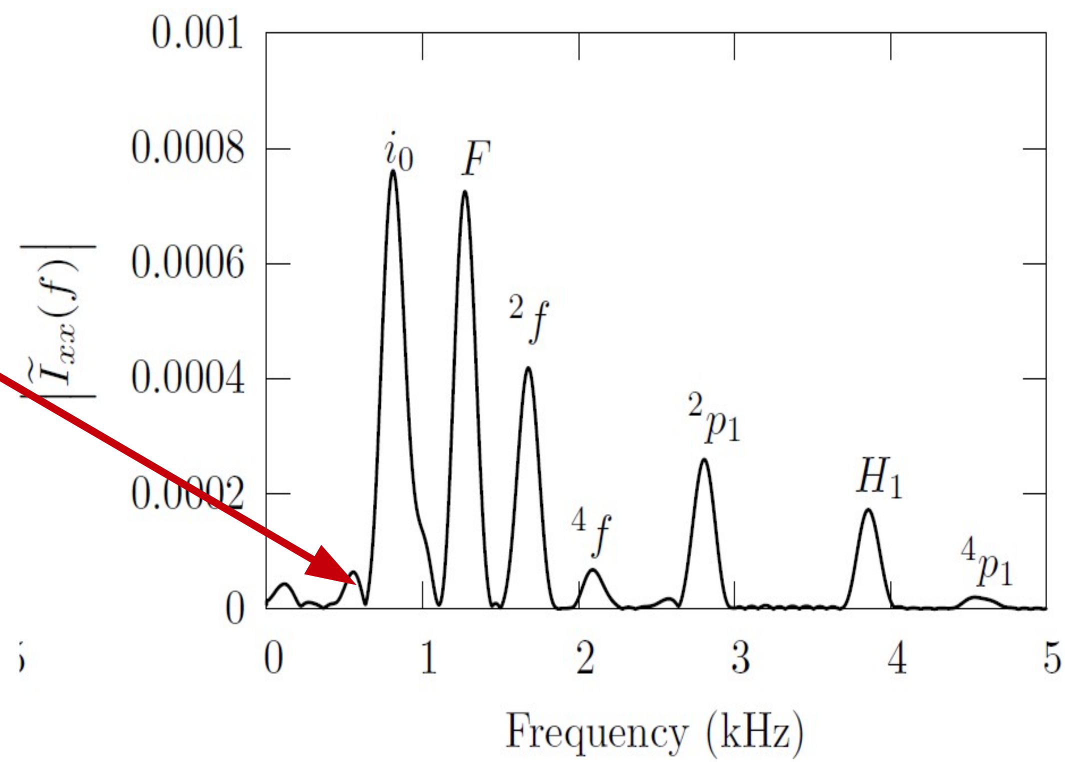
Details of Inertial Modes

- Inertial modes
  - have frequencies proportional to rotating velocity of star.
  - are located at very narrow frequency range.

Details of Inertial Modes

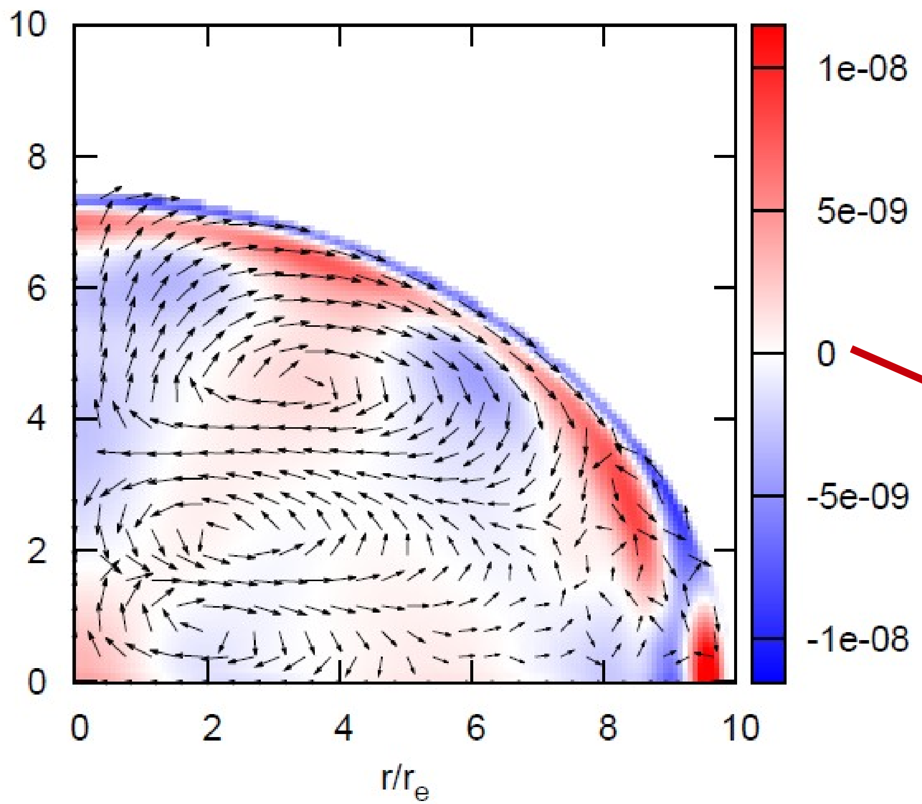
- Inertial modes
  - have frequencies proportional to rotating velocity of star.
  - are located at very narrow frequency range.

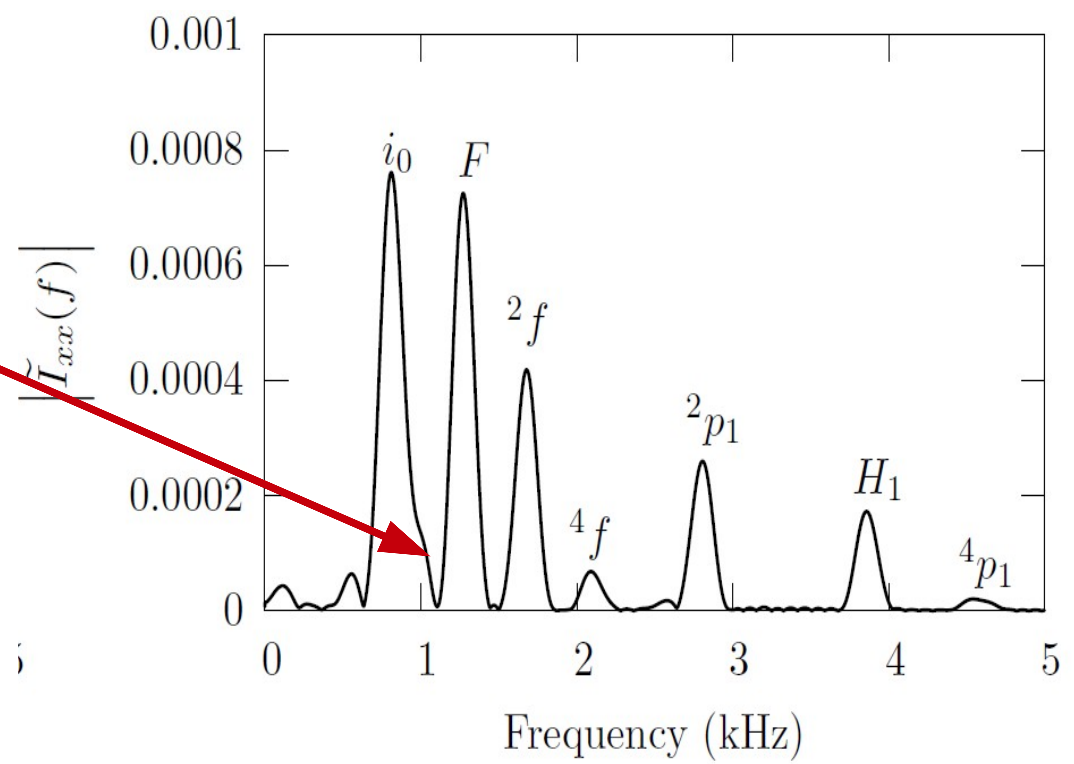




Details of Inertial Modes

- Inertial modes
  - have frequencies proportional to rotating velocity of star.
  - are located at very narrow frequency range.





Details of Inertial Modes

- Inertial modes
  - have frequencies proportional to rotating velocity of star.
  - are located at very narrow frequency range.
  - contain most of kinetic energy : long decay time.

Details of Inertial Modes

- Inertial modes
  - have frequencies proportional to rotating velocity of star.
  - are located at very narrow frequency range.
  - contain most of kinetic energy : long decay time.
  - have a lot to do with r-mode

Summary

- From the hydrodynamical evolution simulations, ${}^2p_1$ and H_1 are found to be the strongest gravitational wave generating modes rather than the f mode.
- The characteristic amplitude of gravitational wave from the 1.4 solar mass pulsar glitch with $\delta\Omega/\Omega = 1\times10^{-5}$ is estimated to be around $h_c \sim 9\times10^{-25}$
- The inertial mode is excited quite effectively in the vortex unpinning model
  - Low frequency → it cannot emit strong gravitational wave
  - It can easily evolve to the non-axisymmetric r-mode which may be a detectable mode.
- This amplitude can be detectable if the sensitivity of gravitational wave detector increase 100 times better than the present one.
  - This can be detected by Einstein telescope.