Vainshtein mechanism in a cosmological background in the most general second-order scalar-tensor theory

Rampei Kimura (Hiroshima Univ.)

Asia Pacific School @ YITP

Collaborators : Tsutomu Kobayashi (Kyoto Univ.)
Kazuhiro Yamamoto (Hiroshima Univ.)

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- Vainshtein mechanism in the most general scalar-tensor theory
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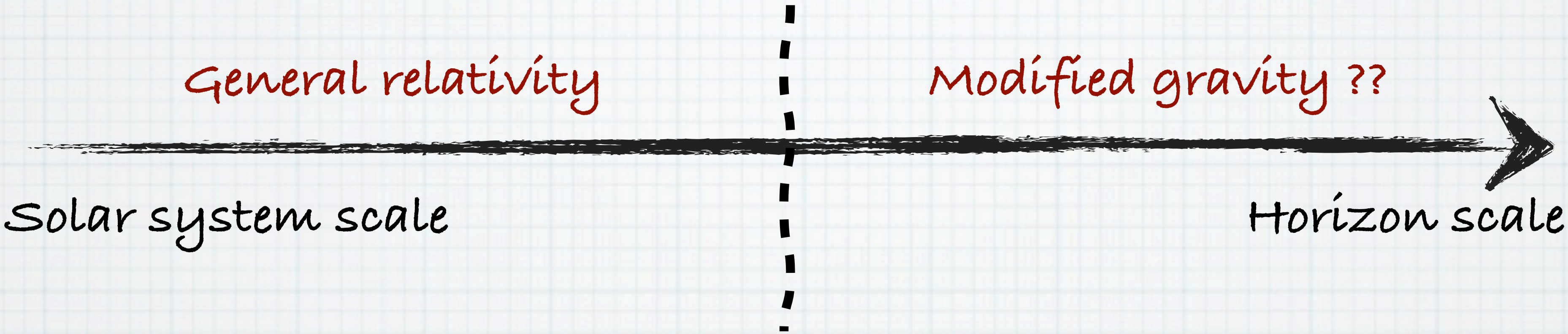
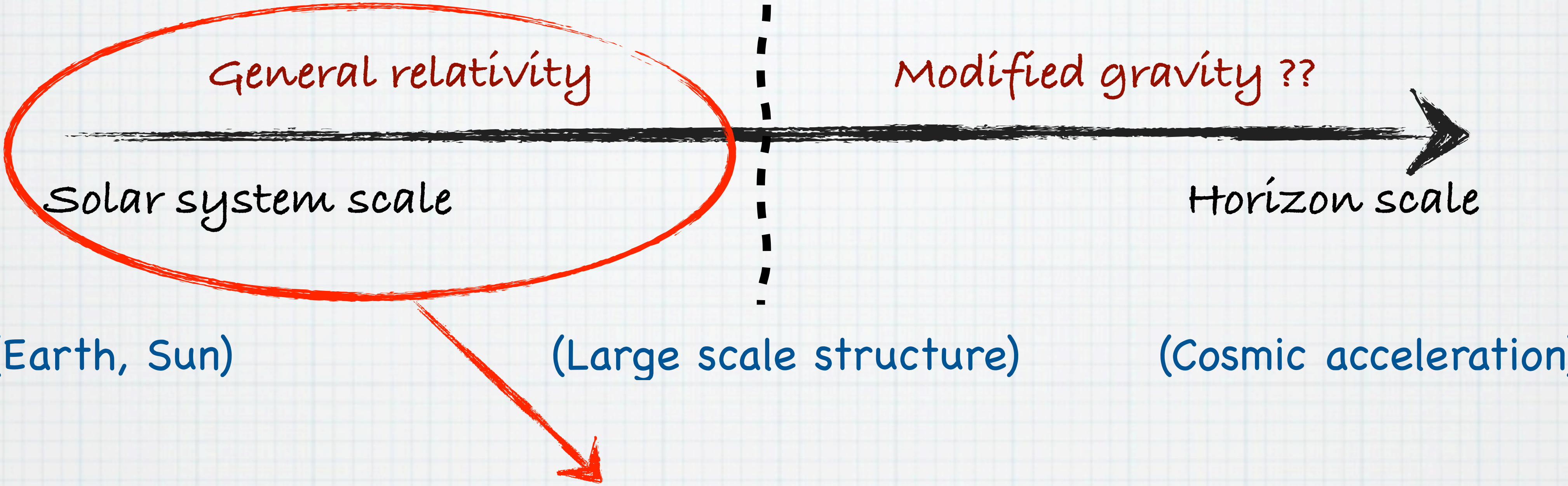
Alternative : Modification of gravity



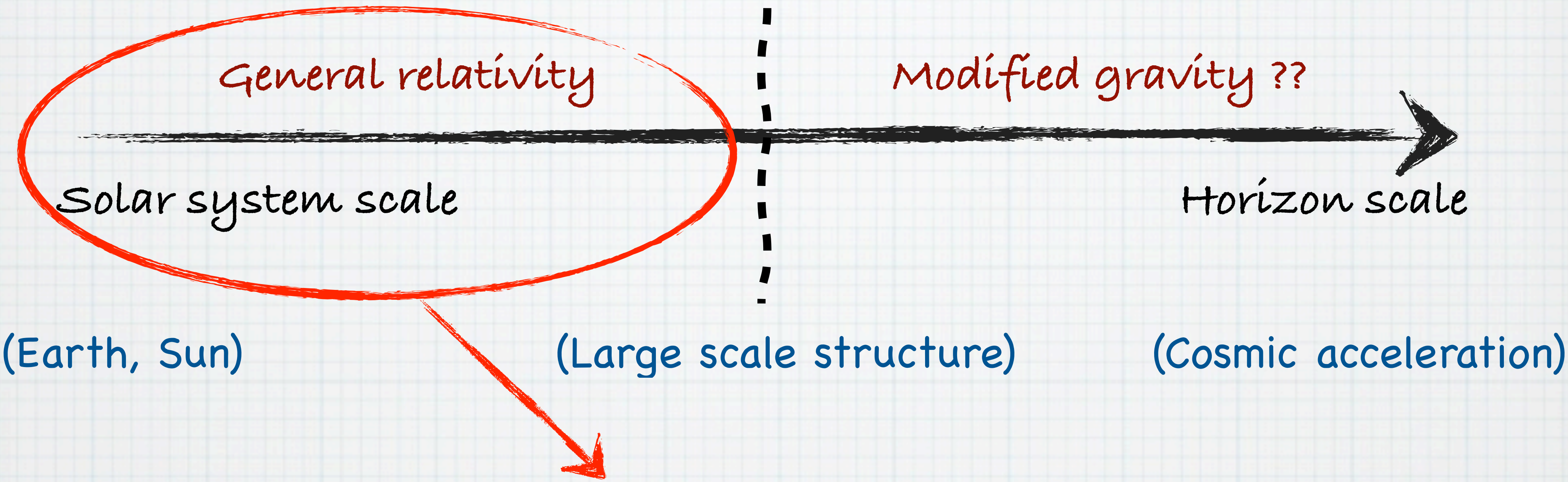
(Earth, Sun) (Large scale structure) (Cosmic acceleration)

Alternative : Modification of gravity



Modified gravity must recover "general relativity behavior" at short distance

Alternative : Modification of gravity



Modified gravity must recover "general relativity behavior" at short distance

→ $\boxed{\text{Screening mechanism}}$

Vainshtein Mechanism

✓ Example (kinetic gravity braiding) (Deffayet et al. '10)

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - \frac{r_c^2}{2M_{\rm Pl}}(\partial\phi)^2\Box\phi + \mathcal{L}_m[\psi, g_{\mu\nu}]$$

$\phi(t,\mathbf{x}) \to \phi(t)[1+\varphi(\mathbf{x})]$

$r_s = GM$: Schwarzshild radius

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self-accelerating solution

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Solar system scale → Horizon scale

“Nonlinear” “Linear”

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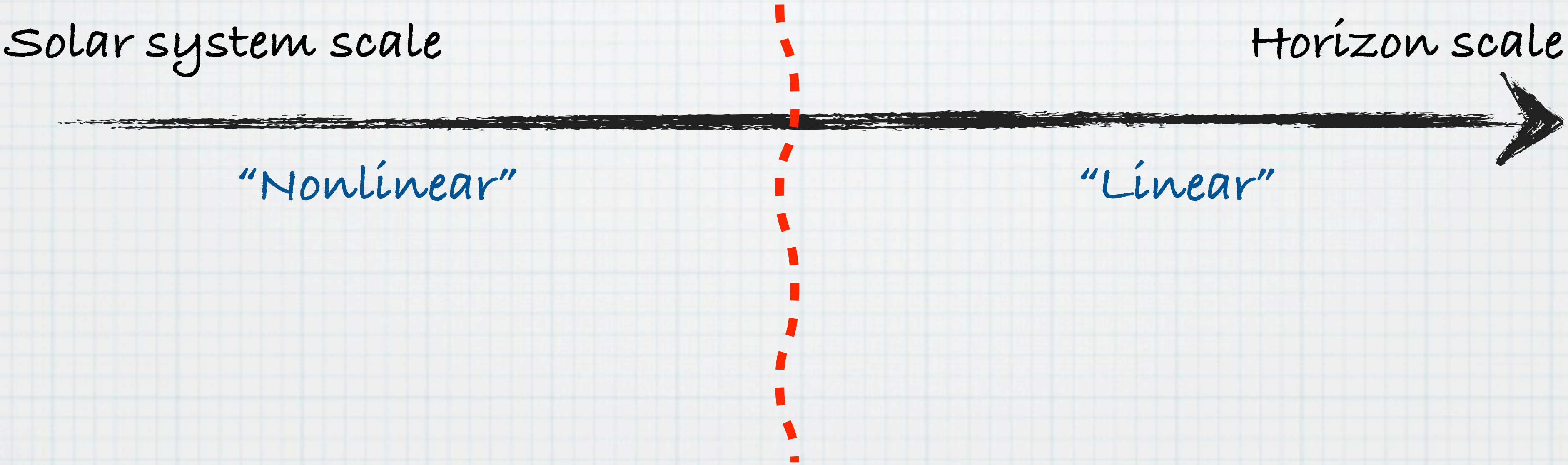
Vainshtein radius $r_V \sim (r_s r_c^2)^{1/3}$



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Horizon scale

"Nonlinear"

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$$\boxed{\frac{d\varphi}{dr} \sim \frac{r_s}{r^2}}$$

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Solar system scale

Horizon scale

“Nonlinear”

$$\frac{d\varphi}{dr} \sim \frac{r_s}{r^2}\left(\frac{r}{r_V}\right)^{3/2} \ll \frac{r_s}{r^2}$$

“Linear”

$$\frac{d\varphi}{dr} \sim \frac{r_s}{r^2}$$

$\phi(t, \mathbf{x}) \to \phi(t)[1 + \varphi(\mathbf{x})]$

$r_s = GM$: Schwarzshild radius

The most general scalar-tensor theory

✓ Horndeski found the most general Lagrangian whose EOM is second-order differential equation for ϕ and $g_{\mu\nu}$ (also known as Generalized galileon)

Deffayet, Gao, Steer (2011)
Kobayashi, Yamaguchi, Yokoyama, Prog. Theor. Phys. 126, 511 (2011)
Horndeski, Int. J. Theor. Phys. 10,363 (1974)

$$\mathcal{L}_2 = K(\phi, X)$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4,X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)]$$

$$\begin{aligned}\mathcal{L}_5 = {} & G_5(\phi, X)G_{\mu\nu}(\nabla^\mu\nabla^\nu\phi) \\ & -\frac{1}{6}G_{5,X}\Big[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) \\ & \qquad + 2(\nabla^\mu\nabla_\alpha\phi)(\nabla^\alpha\nabla_\beta\phi)(\nabla^\beta\nabla_\mu\phi)\Big]\end{aligned}$$

$$X = -(\partial\phi)^2/2, \quad G_{iX} = \partial G_i/\partial X$$

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$$\mathcal{L}_2 = \boxed{K(\phi, X)} \longrightarrow$$

K-essence term $\mathcal{L}_2 \supset (\partial\phi)^2,\ V(\phi)$

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Cubic galileon term

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Einstein-Hilbert term

$$\mathcal{L}_4 \supset (M_{\rm Pl}^2/2)R$$

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Non-minimal derivative coupling

$$\mathcal{L}_5 \supset G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$$

(Germani et al. 2011;
Gubitosi, Linder 2011)

$$X = -(\partial\phi)^2/2, \quad G_{iX} = \partial G_i/\partial X$$

QUESTION :

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Does Vainshtein mechanism work
in the most general second-order
scalar-tensor theory
in a cosmological background???

Formulation

$$Q \equiv H\frac{\delta\phi}{\dot{\phi}}$$

✓ In field equations,

$$\boxed{\epsilon = \Psi, \Phi, \ \text{and } Q} \ll 1$$

$$\mathrm{EOM} \ \supset \ \Big\{ \text{"}\textit{mass terms}\text{"}, \ \text{"}\textit{time derivative terms}\text{"},$$

$$\left(L(t)^2\,\partial^2\epsilon\right)^n, \ \left(L(t)\,\partial\epsilon\right)^m \ldots \Big\}$$

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Neglect

$$\text{EOM} \supset \left\{ \cancel{\text{"}\textit{mass terms}\text{"}},\ \text{"}\textit{time derivative terms}\text{"},\ \left(L(t)^2\,\partial^2\epsilon\right)^n,\ \left(L(t)\,\partial\epsilon\right)^m \ldots \right\}$$

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Quasi-static approximation

$$\partial_t \ll \partial_x$$

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$$L(t) \sim \mathcal{O}(H^{-1})$$

higher-order terms

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higher-order terms

Picking up the terms like

$$\partial^2\epsilon, \ (\partial^2\epsilon)^2, \ (\partial^2\epsilon)^3, \ (\partial^2\epsilon)^4, \ \delta$$

Traceless part of the Einstein Equations

$$\nabla^2\left(\mathcal{F}_T\Psi - \mathcal{G}_T\Phi - A_1 Q\right)$$
$$= \frac{B_1}{2a^2H^2}\mathcal{Q}^{(2)} + \frac{B_3}{a^2H^2}\left(\nabla^2\Phi\nabla^2 Q - \partial_i\partial_j\Phi\partial^i\partial^j Q\right)$$

$$\boxed{\mathcal{Q}^{(2)} \equiv \left(\nabla^2 Q\right)^2 - \left(\partial_i\partial_j Q\right)^2}$$

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Non-linear terms

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Coefficients

Non-linear terms

$$\mathcal{F}_T \equiv 2\left[G_4 - X\left(\ddot{\phi}G_{5X} + G_{5\phi}\right)\right],$$

$$\mathcal{G}_T \equiv 2\left[G_4 - 2XG_{4X} - X\left(H\dot{\phi}G_{5X} - G_{5\phi}\right)\right]$$

✓ The propagation speed of the gravitational waves

$$c_h^2 \equiv \frac{\mathcal{F}_T}{\mathcal{G}_T}$$

Kobayashi, Yamaguchi, Yokoyama, Prog. Theor. Phys. 126, 511 (2011)

$$\boxed{\mathcal{Q}^{(2)} \equiv \left(\nabla^2 Q\right)^2 - \left(\partial_i\partial_j Q\right)^2}$$

00 component of the Einstein equation

$$\mathcal{G}_T \nabla^2 \Psi = \frac{a^2}{2}\rho_{\rm m}\delta - A_2 \nabla^2 Q - \frac{B_2}{2a^2H^2}\mathcal{Q}^{(2)} - \frac{B_3}{a^2H^2}\left(\nabla^2\Psi\nabla^2 Q - \partial_i\partial_j\Psi\partial_i\partial_j Q\right) - \frac{C_1}{3a^4H^4}\mathcal{Q}^{(3)}$$

Scalar field equation

$$A_0\nabla^2 Q - A_1\nabla^2\Psi - A_2\nabla^2\Phi + \frac{B_0}{a^2H^2}\mathcal{Q}^{(2)} - \frac{B_1}{a^2H^2}\left(\nabla^2\Psi\nabla^2 Q - \partial_i\partial_j\Psi\partial^i\partial^j Q\right)$$
$$- \frac{B_2}{a^2H^2}\left(\nabla^2\Phi\nabla^2 Q - \partial_i\partial_j\Phi\partial^i\partial^j Q\right) - \frac{B_3}{a^2H^2}\left(\nabla^2\Phi\nabla^2\Psi - \partial_i\partial_j\Phi\partial^i\partial^j\Psi\right)$$
$$- \frac{C_0}{a^4H^4}\mathcal{Q}^{(3)} - \frac{C_1}{a^4H^4}\mathcal{U}^{(3)} = 0$$

$$\boxed{\begin{aligned}\mathcal{Q}^{(3)} &\equiv \left(\nabla^2 Q\right)^3 - 3\nabla^2 Q\left(\partial_i\partial_j Q\right)^2 + 2\left(\partial_i\partial_j Q\right)^3 \\ \mathcal{U}^{(3)} &\equiv \mathcal{Q}^{(2)}\nabla^2\Phi - 2\nabla^2 Q\partial_i\partial_j Q\partial^i\partial^j\Phi + 2\partial_i\partial_j Q\partial^j\partial^k Q\partial_k\partial^i\Phi\end{aligned}}$$

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Non-linear terms

Scalar field equation

$$A_0\nabla^2 Q - A_1\nabla^2\Psi - A_2\nabla^2\Phi + \frac{B_0}{a^2H^2}\mathcal{Q}^{(2)} - \frac{B_1}{a^2H^2}\left(\nabla^2\Psi\nabla^2 Q - \partial_i\partial_j\Psi\partial^i\partial^j Q\right)$$
$$- \frac{B_2}{a^2H^2}\left(\nabla^2\Phi\nabla^2 Q - \partial_i\partial_j\Phi\partial^i\partial^j Q\right) - \frac{B_3}{a^2H^2}\left(\nabla^2\Phi\nabla^2\Psi - \partial_i\partial_j\Phi\partial^i\partial^j\Psi\right)$$
$$- \frac{C_0}{a^4H^4}\mathcal{Q}^{(3)} - \frac{C_1}{a^4H^4}\mathcal{U}^{(3)} = 0$$

Non-linear terms

$$\mathcal{Q}^{(3)} \equiv \left(\nabla^2 Q\right)^3 - 3\nabla^2 Q\left(\partial_i\partial_j Q\right)^2 + 2\left(\partial_i\partial_j Q\right)^3$$
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Spherically Symmetric Case

✓ EOM for gravity and scalar field can be integrated once,

$$c_h^2 \frac{\Psi'}{r} - \frac{\Phi'}{r} - \alpha_1 \frac{Q'}{r} = \frac{\beta_1}{H^2}\left(\frac{Q'}{r}\right)^2 + 2\frac{\beta_3}{H^2}\frac{\Phi'}{r}\frac{Q'}{r}$$

$$\frac{\Psi'}{r} + \alpha_2 \frac{Q'}{r} = \frac{1}{8\pi\mathcal{G}_T}\frac{\delta M(t,r)}{r^3} - \frac{\beta_2}{H^2}\left(\frac{Q'}{r}\right)^2 - 2\frac{\beta_3}{H^2}\frac{\Psi'}{r}\frac{Q'}{r} - \frac{2}{3}\frac{\gamma_1}{H^4}\left(\frac{Q'}{r}\right)^3$$

$$\alpha_0 \frac{Q'}{r} - \alpha_1 \frac{\Psi'}{r} - \alpha_2 \frac{\Phi'}{r} = 2\left[-\frac{\beta_0}{H^2}\left(\frac{Q'}{r}\right)^2 + \frac{\beta_1}{H^2}\frac{\Psi'}{r}\frac{Q'}{r} + \frac{\beta_2}{H^2}\frac{\Phi'}{r}\frac{Q'}{r} + \frac{\beta_3}{H^2}\frac{\Phi'}{r}\frac{\Psi'}{r} \right.$$
$$\left. + \frac{\gamma_0}{H^4}\left(\frac{Q'}{r}\right)^3 + \frac{\gamma_1}{H^4}\frac{\Phi'}{r}\left(\frac{Q'}{r}\right)^2\right]$$

where

$$c_h^2 \equiv \frac{\mathcal{F}_T}{\mathcal{G}_T}, \quad \alpha_i(t) \equiv \frac{A_i}{\mathcal{G}_T}, \quad \beta_i(t) \equiv \frac{B_i}{\mathcal{G}_T}, \quad \gamma_i(t) \equiv \frac{C_i}{\mathcal{G}_T}.$$

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✓ EOM for gravity and scalar field can be integrated once,

enclosed mass

$$c_h^2 \frac{\Psi'}{r} - \frac{\Phi'}{r} - \alpha_1 \frac{Q'}{r} = \frac{\beta_1}{H^2}\left(\frac{Q'}{r}\right)^2 + 2\frac{\beta_3}{H^2}\frac{\Phi'}{r}\frac{Q'}{r}$$

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$$c_h^2 \frac{\Psi'}{r} - \frac{\Phi'}{r} - \alpha_1 \frac{Q'}{r} = \frac{\beta_1}{H^2}\left(\frac{Q'}{r}\right)^2 + 2\frac{\beta_3}{H^2}\frac{\Phi'}{r}\frac{Q'}{r}$$

$$\frac{\Psi'}{r} + \alpha_2 \frac{Q'}{r} = \frac{1}{8\pi\mathcal{G}_T}\frac{\delta M(t,r)}{r^3} - \frac{\beta_2}{H^2}\left(\frac{Q'}{r}\right)^2 - 2\frac{\beta_3}{H^2}\frac{\Psi'}{r}\frac{Q'}{r} - \frac{2}{3}\frac{\gamma_1}{H^4}\left(\frac{Q'}{r}\right)^3$$

$$\alpha_0 \frac{Q'}{r} - \alpha_1 \frac{\Psi'}{r} - \alpha_2 \frac{\Phi'}{r} = 2\left[-\frac{\beta_0}{H^2}\left(\frac{Q'}{r}\right)^2 + \frac{\beta_1}{H^2}\frac{\Psi'}{r}\frac{Q'}{r} + \frac{\beta_2}{H^2}\frac{\Phi'}{r}\frac{Q'}{r} + \frac{\beta_3}{H^2}\frac{\Phi'}{r}\frac{\Psi'}{r} + \frac{\gamma_0}{H^4}\left(\frac{Q'}{r}\right)^3 + \frac{\gamma_1}{H^4}\frac{\Phi'}{r}\left(\frac{Q'}{r}\right)^2\right]$$

where

$$\boxed{c_h^2 \equiv \frac{\mathcal{F}_T}{\mathcal{G}_T}, \quad \alpha_i(t) \equiv \frac{A_i}{\mathcal{G}_T}, \quad \beta_i(t) \equiv \frac{B_i}{\mathcal{G}_T}, \quad \gamma_i(t) \equiv \frac{C_i}{\mathcal{G}_T}.}$$

Functions of K, G_3, G_4, G_5

Linear Solution

✓ For sufficiently large r,

$$\Phi' = \frac{1}{8\pi\mathcal{G}_T}\frac{c_h^2\alpha_0 - \alpha_1^2}{\alpha_0 + (2\alpha_1 + c_h^2\alpha_2)\alpha_2}\frac{\delta M}{r^2}$$

$$\Psi' = \frac{1}{8\pi\mathcal{G}_T}\frac{\alpha_0 + \alpha_1\alpha_2}{\alpha_0 + (2\alpha_1 + c_h^2\alpha_2)\alpha_2}\frac{\delta M}{r^2}$$

$$Q' = \frac{1}{8\pi\mathcal{G}_T}\frac{\alpha_1 + c_h^2\alpha_2}{\alpha_0 + (2\alpha_1 + c_h^2\alpha_2)\alpha_2}\frac{\delta M}{r^2}$$

Linear Solution

✓ For sufficiently large r,

(time-dependent)
effective gravitational coupling $G_{\rm eff}^{(\rm Linear)} (\neq G_N)$

$$\Phi' = \boxed{\frac{1}{8\pi\mathcal{G}_T}\frac{c_h^2\alpha_0 - \alpha_1^2}{\alpha_0 + (2\alpha_1 + c_h^2\alpha_2)\alpha_2}}\frac{\delta M}{r^2}$$

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Non-linear Solution

Case 1 : $G_{4X} = 0,\ G_5 = 0$

Case 2 : $G_{5X} = 0$

Case 3 : $G_{5X} \neq 0$

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Case 1 : $G_{4X} = 0,\ G_5 = 0$

Quadratic equation for Q′

$$\boxed{a\,Q'^2 + b\,Q' + c = 0}$$

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→ Cubic equation for Q′

$$\boxed{a\,Q'^3 + b\,Q'^2 + c\,Q' + d = 0}$$

Case 3 : $G_{5X} \neq 0$

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Case 3 : $G_{5X} \neq 0$

→ Very complicated ... (sextic equation for Q′)

$$\boxed{a\,Q'^6 + b\,Q'^5 + c\,Q'^4 + d\,Q'^3 + e\,Q'^2 + f\,Q' + g = 0}$$

Case 1 : $G_{4X} = 0, \; G_5 = 0$

✓ Kinetic gravity braiding (with non-minimal coupling) (Deffayet et al. 2010)

$$\mathcal{L} = G_4(\phi)R + K(\phi, X) - G_3(\phi, X)\Box\phi$$

Generalization of the cubic galileon theory

$$\supset (\partial\phi)^2\Box\phi$$

✓ In this case, the propagation speed of the gravitational waves is

$$c_h^2 = 1$$

Case 1 : $\quad G_{4X} = 0,\ G_5 = 0$

✓ In this case, the equation for Q reduces to a quadratic equation

$$\frac{\mathcal{B}(t)}{H^2}\left(\frac{Q'}{r}\right)^2 + \frac{2Q'}{r} = 2\,\mathcal{C}(t)\frac{\mu}{r^3} \qquad \mu \equiv \frac{\delta M}{8\pi\mathcal{G}_T}$$

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✔ Solving for Q'

$$\frac{Q'}{r} = \frac{H^2}{\mathcal{B}}\left(\sqrt{1+\frac{2\mathcal{B}\mathcal{C}\mu}{H^2 r^3}} - 1\right)$$

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- Inside the Vainshtein radius r_V

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Scalar field is screened !!

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time-dependent

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$$\boxed{G_{\cos} = 1/16\pi G_4 = G_N}$$

Time-dependence can be tested by BBN !!

$$\left|1 - \frac{G_N|_{\rm BBN}}{G_N|_{\rm now}}\right| \lesssim 0.1$$

(Uzan 2011)

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✓ PPN parameter

$$\gamma \equiv \frac{\Psi'}{\Phi'} = 1$$

Case 2 : $\quad G_{5X} = 0$

✓ Lagrangian

$$\begin{aligned}\mathcal{L} = &K(\phi, X) - G_3(\phi, X)\Box\phi \\ &+ G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\ &+ \underline{G_5(\phi)}G_{\mu\nu}\nabla^\mu\nabla^\nu\phi\end{aligned}$$

does not depend on the kinetic term X

✓ In this case, the propagation speed of the gravitational waves is

$$c_h^2 = 1 + 2\beta_1 \neq 1$$

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$$(Q')^3 + \mathcal{C}_2 H^2 r (Q')^2 + \left(\frac{\mathcal{C}_1}{2} H^4 r^2 - H^2 \mathcal{C}_\beta \frac{\mu}{r} \right) Q' - \frac{H^4 \mathcal{C}_\alpha \mu}{2} = 0$$

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Functions of K, G_3, G_4, G_5

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Functions of K, G_3, G_4, G_5

- ✓ 3 possible solutions at short distance can be matched to the linear solution

$$Q' \simeq +H\sqrt{\mathcal{C}_\beta \frac{\mu}{r}}, \quad -H\sqrt{\mathcal{C}_\beta \frac{\mu}{r}}, \quad -\frac{\mathcal{C}_\alpha}{\mathcal{C}_\beta}\frac{H^2 r}{2}$$

Case 2 : $G_{5X} = 0$

✓ 3 solutions at short distances

$$Q' \simeq \boxed{\pm H \sqrt{\mathcal{C}_\beta \frac{\mu}{r}},} \quad \cancel{-\frac{\mathcal{C}_\alpha}{\mathcal{C}_\beta} \frac{H^2 r}{2}}$$

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✓ Metric perturbations

$$\Psi' = \Phi' \simeq \frac{C_\Psi(t)}{8\pi\mathcal{G}_T(t)}\frac{\delta M}{r^2}$$

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Newton's constant $G_N(t)$

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$$3H^2 = 8\pi G_{\cos}(\rho_m + \rho_\phi)$$

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Time-dependence of G_N can be tested by BBN

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propagation speed of gravitational waves

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propagation speed of gravitational waves

Solar-system tests $\boxed{|1-\gamma| < 2.3 \times 10^{-5}}$ (Will 2005)

Case 3 : $G_{5X} \neq 0$

✓ At sufficiently small scales,

$$\Psi'(r),\ \Phi'(r) \ \propto \ \frac{1}{r^2}$$

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$$\Downarrow$$

The Vainshtein mechanism no longer works in the presence of G_{5X} !!

Summary

- Vainshtein screening successfully operates in the most general second-order scalar-tensor theory, but
  - Newton's constant $G=G(t)$
  - constrained from PPN and BBN
  - inverse-square law can not be reproduced at small scales if $G_{5X} \neq 0$