Teleparallel dark energy

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Based on:
CQ Geng, CC Lee, E. N. Saridakis, YP Wu PLB 704, 384 (2011)

CQ Geng, CC Lee, E. N. Saridakis JCAP 1201, 002 (2012)

Outline

- Teleparallel Gravity
- Teleparallel Dark Energy model
- Observational Constraints
- Summary

What is the feature of teleparallel gravity?

- An alternative theory of gravity, which is equivalent to General Relativity.
- This is a curvatureless gravity theory, and the gravitational effect comes from torsion instead of curvature.

Teleparallel Gravity
A brief introduction

- The dynamical variable of teleparallel gravity is the vierbein fields $\mathbf{e}_A(x^\mu)$, which form an orthonormal basis for the tangent space at each point x^μ of the manifold: $\mathbf{e}_A \cdot \mathbf{e}_B = \eta_{AB}$, where $\eta_{AB} = diag(1, -1, -1, -1)$.
- Notation:
  Greek indices μ, ν,... : coordinate space-time.
  Latin indices A, B,... : tangent space-time.
- The relationship between metric and vierbein fields is

$$g_{\mu\nu}(x) = \eta_{AB}\, e_\mu^A(x)\, e_\nu^B(x).$$

Teleparallel Gravity
A brief introduction

- Weitzenböck connection: a curvatureless connection

$$\overset{\mathbf{w}}{\Gamma}{}^{\lambda}_{\nu\mu} \equiv e^{\lambda}_{A}\,\partial_{\mu}e^{A}_{\nu}$$

- The torsion tensor is defined as

$$T^{\lambda}_{\mu\nu} \equiv \overset{\mathbf{w}}{\Gamma}{}^{\lambda}_{\nu\mu} - \overset{\mathbf{w}}{\Gamma}{}^{\lambda}_{\mu\nu} = e^{\lambda}_{A}\,(\partial_{\mu}e^{A}_{\nu} - \partial_{\nu}e^{A}_{\mu}).$$

- Under Weitzenböck connection, the Riemann tensor vanishes:

$$R^{\rho}_{\mu\sigma\nu} = \overset{\mathbf{w}}{\Gamma}{}^{\rho}_{\mu\nu,\sigma} - \overset{\mathbf{w}}{\Gamma}{}^{\rho}_{\mu\sigma,\nu} + \overset{\mathbf{w}}{\Gamma}{}^{\rho}_{\delta\sigma}\overset{\mathbf{w}}{\Gamma}{}^{\delta}_{\mu\nu} - \overset{\mathbf{w}}{\Gamma}{}^{\rho}_{\delta\nu}\overset{\mathbf{w}}{\Gamma}{}^{\delta}_{\mu\sigma} = 0.$$

Teleparallel Gravity
A brief introduction

- We can construct the "teleparallel Lagrangian" by using the torsion tensor, $\mathcal{L}_T = T = a_1 T^{\rho\mu\nu} T_{\rho\mu\nu} + a_2 T^{\rho\mu\nu} T_{\nu\mu\rho} + a_3 T_{\rho\mu}{}^{\rho} T_{\nu}{}^{\mu\nu}$.
- It is a good approach of General Relativity when we choose the suitable parameters $a_1 = \frac{1}{4}$, $a_2 = \frac{1}{2}$ and $a_3 = -1$:

$$\tilde{R} = -T - 2\nabla^{\mu} T^{\nu}{}_{\mu\nu},$$

where $\tilde{R}$ is constructed by Levi-Civita connection.

Teleparallel Gravity
A brief introduction

- The action of teleparallel gravity is

$$S = \int d^4 x e \left[\frac{T}{2\kappa^2} + \mathcal{L}_M \right],$$

where $e = det\left(e^A_{\ \mu}\right) = \sqrt{-g}$.

- Varying this action respect to the vierbein fields gives the field equation

$$e^{-1}\partial_\mu (e e^\rho_A S_\rho{}^{\mu\nu}) - e^\lambda_A T^\rho{}_{\mu\lambda} S_\rho{}^{\nu\mu} - \frac{1}{4} e^\nu_A T = \frac{\kappa^2}{2} e^\rho_A \overset{\mathbf{em}}{T}{}_\rho{}^\nu,$$

where $\overset{\mathbf{em}}{T}{}_\rho{}^\nu$ stands for the energy-momentum tensor and $S_\rho{}^{\mu\nu} = \frac{1}{4}(T^{\nu\mu}{}_\rho - T^{\mu\nu}{}_\rho + T_\rho{}^{\mu\nu}) + \frac{1}{2}\left(\delta^\mu_\rho T^{\alpha\nu}{}_\alpha - \delta^\nu_\rho T^{\alpha\mu}{}_\alpha\right)$.

What is teleparallel dark energy model?

- Teleparallel dark energy model is a dark energy model, which can explain the late time accelerating universe.
- This model combines quintessence model with teleparallel gravity.
- This model differs from quintessence model when we turn on the non-minimal coupling term.

- Quintessence is one of the most popular dark energy model.
- The generalized quintessence model action is given by

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} + \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi + \xi R\phi^2\right) - V(\phi) + \mathcal{L}_M\right]$$

- Under the flat Friedmann-Robertson-Walker (FRW) background $ds^2 = dt^2 - a^2(t)d\vec{x}^2$, the effective energy and pressure density can be defined as

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) + 6\xi H\phi\dot{\phi} + 3\xi H^2\phi^2,$$
$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) + \xi\left(2\dot{H} + 3H^2\right)\phi^2 + 4\xi H\phi\dot{\phi}$$
$$+2\xi\phi\ddot{\phi} + 2\xi\dot{\phi}^2$$

Teleparallel Dark Energy

- Similar to quintessence model, we can construct teleparallel dark energy model, and the action is given by

$$S = \int d^4x e \left[\frac{T}{2\kappa^2} + \frac{1}{2}\left(\partial_\mu \phi \partial^\mu \phi + \xi T \phi^2\right) - V(\phi) + \mathcal{L}_M \right].$$

- Variation of action with respect to the vierbein fields yields the field equation

$$\left(\frac{2}{\kappa^2} + 2\xi\phi^2\right)\left[e^{-1}\partial_\mu(e e_A^\rho S_\rho{}^{\mu\nu}) - e_A^\lambda T^\rho{}_{\mu\lambda} S_\rho{}^{\nu\mu} - \frac{1}{4} e_A^\nu T\right]$$
$$-e_A^\nu \left[\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi)\right] + e_A^\mu \partial^\nu \phi \partial_\mu \phi$$
$$+4\xi e_A^\rho S_\rho{}^{\mu\nu} \phi \left(\partial_\mu \phi\right) = e_A^\rho \overset{\mathbf{em}}{T}{}_\rho{}^\nu .$$

Teleparallel Dark Energy

- Again, the effective energy and pressure density under FRW metric $(e_\mu^A = \mathrm{diag}(1, a, a, a))$ are

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) - 3\xi H^2\phi^2,$$
$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) + 4\xi H\phi\dot{\phi} + \xi\left(3H^2 + 2\dot{H}\right)\phi^2.$$

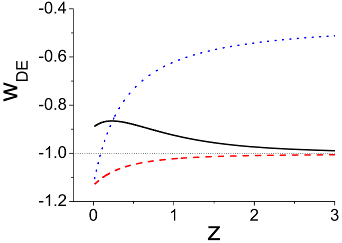
- Variation of action with respect to the scalar field gives us the equation of motion of the scalar field

$$\ddot{\phi} + 3H\dot{\phi} + 6\xi H^2\phi + V'(\phi) = 0.$$

- These equations lead to the continuity equation $\dot{\rho}_\phi + 3H(1 + w_\phi)\rho_\phi = 0$, where w_ϕ is the equation of state of the scalar field, which is defined as $w_\phi \equiv \frac{p_\phi}{\rho_\phi}$.

Teleparallel Dark Energy

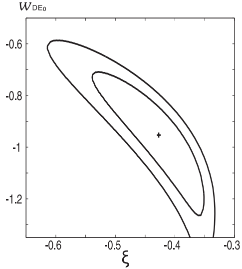
- In the minimal coupling case ($\xi = 0$), the teleparallel dark energy is equivalent to quintessence model
- However, these two models are different theories when we turn on the non-minimal coupling constant ($\xi \neq 0$)
- Teleparallel dark energy model can cross the phantom-divide easily.
- Similar to $f(T)$ theory, this model has the local Lorentz violation problem.

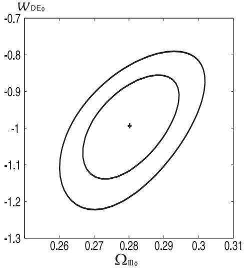


Teleparallel Dark Energy: Observational Constraints

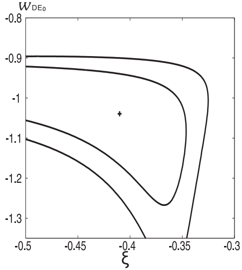
- We would like to test teleparallel dark energy model by using the SNIa, BAO and CMB data. These observational data can tell us whether this is a suitable model for dark energy or not
- We consider three kinds of potential cases:
  Power-Law potential: $V(\phi) = V_0\phi^4$
  Exponential potential: $V(\phi) = V_0 e^{-\kappa\phi}$
  Inverse hyperbolic cosine potential: $V(\phi) = \frac{V_0}{cosh(\kappa\phi)}$

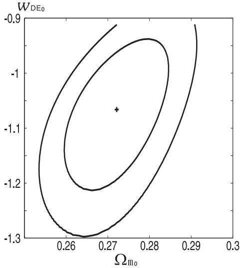
Teleparallel Dark Energy: Observational Constraints

- Potential: $V(\phi) = V_0\phi^4$
- Left: fixing $\Omega_m = 27\%$, the best fit locates at $h \simeq 0.7$, $\xi \simeq -0.42$, $w_\phi \simeq -0.96$ and $\chi^2 \simeq 543.9$
  Right: fixing $\xi = -0.41$, the best fit locates at $h \simeq 0.7$, $\Omega_m \simeq 28.0\%$, $w_\phi \simeq -0.99$ and $\chi^2 \simeq 544.5$
- 

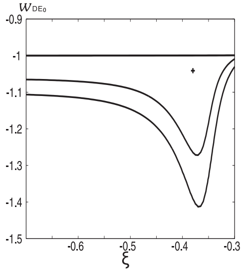


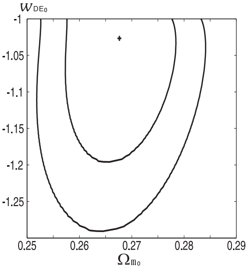
Teleparallel Dark Energy: Observational Constraints

- Potential: $V(\phi) = V_0 e^{-\kappa\phi}$
- Left: fixing $\Omega_m = 27\%$, the best fit locates at $h \simeq 0.7$, $\xi \simeq -0.41$, $w_\phi \simeq -1.04$ and $\chi^2 \simeq 544.3$
  Right: fixing $\xi = -0.41$, the best fit locates at $h \simeq 0.7$, $\Omega_m \simeq 27.1\%$, $w_\phi \simeq -1.07$ and $\chi^2 \simeq 544.6$
- 



Teleparallel Dark Energy: Observational Constraints

- Potential: $V(\phi) = \frac{V_0}{cosh(\kappa\phi)}$
- Left: fixing $\Omega_m = 27\%$, the best fit locates at $h \simeq 0.7$, $\xi \simeq -0.38$, $w_\phi \simeq -1.05$ and $\chi^2 \simeq 544.8$
  Right: fixing $\xi = -0.41$, the best fit locates at $h \simeq 0.7$, $\Omega_m \simeq 26.7\%$, $w_\phi \simeq -1.03$ and $\chi^2 \simeq 545.1$
- 



Summary

- Teleparallel gravity is an alternative gravity theory of the universe.
- Teleparallel dark energy model is equivalent to quintessence model happens at the minimal coupling case ($\xi = 0$), but it has a different behavior when we include a non-minimal coupling term ($\xi \neq 0$).
- We show that the equation of state of teleparallel dark energy model can cross the phantom-divide easily.
- The observational constraints show a good result on this model. This model is suitable for the late-time accelerating universe.