Self-accelerating universe from nonlinear massive gravity

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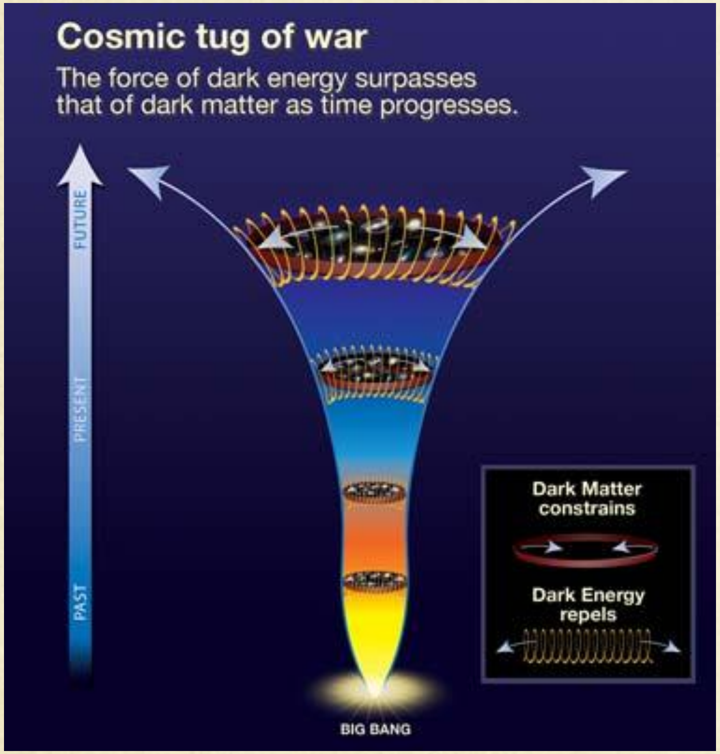
Outline

- Introduction;
  - The nonlinear massive gravity theory
- Self-accelerating solutions in open FRW universe;
  - **The first workable model !**
- Conclusion and Discussion
  - Cosmological perturbations

Part I

Introduction

Introduction



Cosmic acceleration

Introduction

- Can we give graviton a mass?
  - Fierz and Pauli 1939

$$\mathcal{L}_{FP} = f^4 \left(h_{\mu\nu} h_{\mu\nu} - h^2 \right)$$

van Dam–Veltman–Zakharov discontinuity

$$T^{\mu}_{\nu} h^{\nu}_{\mu} = T^{\mu}_{\nu} (\hat{h}^{\nu}_{\mu} + m_g^2 \delta^{\nu}_{\mu} \phi) = T^{\mu}_{\nu} \hat{h}^{\nu}_{\mu} + \frac{1}{M_{\rm Pl}} T \phi^c$$

  - Vainshtein 1972 non–linear interactions
  - Boulware–Deser (BD) ghost 1972

Lack of Hamiltonian constrain and momentum constrain

6 degrees of freedom

Helicity ±2, ±1, 0 $\longrightarrow$ 5 dof ?

6th dof is BD ghost!

Introduction

- Whether there exist a nonlinear model without ghost?
  - N. Arkani–Hamed et al 2002

    decoupling limit $m_g \to 0\,, \qquad M_p \to \infty\,, \qquad \Lambda = \left(m_g^4 M_p\right)^{1/5} \to \text{finite}\,.$
  - C. de Rham and G. Gabadadze 2010

$$\mathcal{L} = M_{\rm Pl}^2\sqrt{-g}R - \frac{M_{\rm Pl}^2 m^2}{4}\sqrt{-g}(U_2(g,H) + U_3(g,H) + U_4(g,H) + U_5(g,H)\cdots),$$

Mass terms

↓

Boundary

where U_i denotes the interaction term at ith order in $H_{\mu\nu}$,

$$U_2(g,H) = H_{\mu\nu}^2 - H^2,$$

$$U_3(g,H) = \underline{c_1}H_{\mu\nu}^3 + \underline{c_2}HH_{\mu\nu}^2 + \underline{c_3}H^3,$$

$$U_4(g,H) = \underline{d_1}H_{\mu\nu}^4 + \underline{d_2}HH_{\mu\nu}^3 + \underline{d_3}H_{\mu\nu}^2H_{\alpha\beta}^2 + \underline{d_4}H^2H_{\mu\nu}^2 + \underline{d_5}H^4,$$

$$U_5(g,H) = \underline{f_1}H_{\mu\nu}^5 + \underline{f_2}HH_{\mu\nu}^4 + \underline{f_3}H^2H_{\mu\nu}^3 + \underline{f_4}H_{\alpha\beta}^2H_{\mu\nu}^3 + \underline{f_5}H(H_{\mu\nu}^2)^2 + \underline{f_6}H^3H_{\mu\nu}^2 + \underline{f_7}H^5.$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{h_{\mu\nu}}{M_{\rm Pl}} = H_{\mu\nu} + \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b,$$

$$H_{\mu\nu} = \frac{h_{\mu\nu}}{M_{\rm Pl}} + \partial_\mu\pi_\nu + \partial_\nu\pi_\mu - \eta_{\alpha\beta}\partial_\mu\pi^\alpha\partial_\nu\pi^\beta.$$

Introduction

- C. de Rham, G. Gabadadze and A. Tolly 2011

$$I_g = M_{Pl}^2 \int d^4x \sqrt{-g} \left[\frac{R}{2} + m_g^2 (\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4) \right]$$

$$\mathcal{L}_2 = \frac{1}{2} \left([\mathcal{K}]^2 - [\mathcal{K}^2] \right) ,$$

$$\mathcal{L}_3 = \frac{1}{6} \left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3] \right) ,$$

$$\mathcal{L}_4 = \frac{1}{24} \left([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4] \right) ,$$

Stukelberg fields

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \left(\sqrt{g^{-1} f} \right)^\mu{}_\nu \qquad f_{\mu\nu} \equiv \eta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b$$

It is often called fiducial metric

- Automatically produce the "appropriate coefficients" to eliminate BD ghost at any order in decoupling limit !
- Free of BD ghost away from the decoupling limit, at fully nolinear level

Hassan, Rosen '11

Part II

Self-accelerating solutions

A.Emir Gumrukcuoglu, Chunshan Lin, Shinji Mukohyama

arXiv:1109.3845

Self-accelerating solutions

- No go result for flat FRW solution (G. D'Amico et al 2011 Aug.)
- However… (A.Gumrukcuoglu, C. Lin, S. Mukohyama: 1109.3845)

It does not extend to open FRW universe

$$g_{\mu\nu}dx^\mu dx^\nu = -N(t)^2 dt^2 + a(t)^2 \Omega_{ij} dx^i dx^j \,,$$
$$\Omega_{ij}dx^i dx^j = dx^2 + dy^2 + dz^2 - \frac{|K|(xdx + ydy + zdz)^2}{1 + |K|(x^2 + y^2 + z^2)} \,,$$

The 4 Stukelberg scalars

$$\begin{aligned}
\phi^0 &= f(t)\sqrt{1 + |K|(x^2 + y^2 + z^2)}, \\
\phi^1 &= \sqrt{|K|}f(t)x, \\
\phi^2 &= \sqrt{|K|}f(t)y, \\
\phi^3 &= \sqrt{|K|}f(t)z.
\end{aligned}$$

motivated by…

Minkowski metric ➡ Open FRW chart

Self-accelerating solutions

- Fiducial metric respect FRW symmetry

$$\phi^0 = f(t)\sqrt{1+|K|(x^2+y^2+z^2)},$$
$$\phi^1 = \sqrt{|K|}f(t)x,$$
$$\phi^2 = \sqrt{|K|}f(t)y,$$
$$\phi^3 = \sqrt{|K|}f(t)z.$$

$$f_{\mu\nu} \equiv \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$$

$$\Rightarrow \quad \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b = -(\dot{f}(t))^2\delta^0_\mu\delta^0_\nu + |K|f(t)^2\Omega_{ij}\delta^i_\mu\delta^j_\nu\,,$$

- (0i) –components of the equation of motion for $g_{\mu\nu}$ are trivially satisfied;

- Evolution equations for cosmic perturbations fully respect homogeneity and isotropy at any order.

Self-accelerating solutions

- Constraint from Stuckelberg scalars:

$$(\dot{a} - \sqrt{|K|}N)\left[\left(3 - \frac{2\sqrt{|K|}f}{a}\right) + \alpha_3\left(3 - \frac{\sqrt{|K|}f}{a}\right)\left(1 - \frac{\sqrt{|K|}f}{a}\right) + \alpha_4\left(1 - \frac{\sqrt{|K|}f}{a}\right)^2\right] = 0.$$

- Branch I

$$\dot{a} = \sqrt{|K|}N,$$

- Branch $\mathrm{II}_{\pm}$

$$f = \frac{a}{\sqrt{|K|}}X_{\pm}, \quad X_{\pm} \equiv \frac{1 + 2\alpha_3 + \alpha_4 \pm \sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}}{\alpha_3 + \alpha_4}.$$

Please notice that these 2 solutions do not exist when K=0.

Self–accelerating solutions

- Freedmann equation

$$3H^2 - \frac{3|K|}{a^2} = \rho_m + c_\pm m_g^2,$$

$$-\frac{2\dot{H}}{N} - \frac{2|K|}{a^2} = \rho_m + p_m,$$

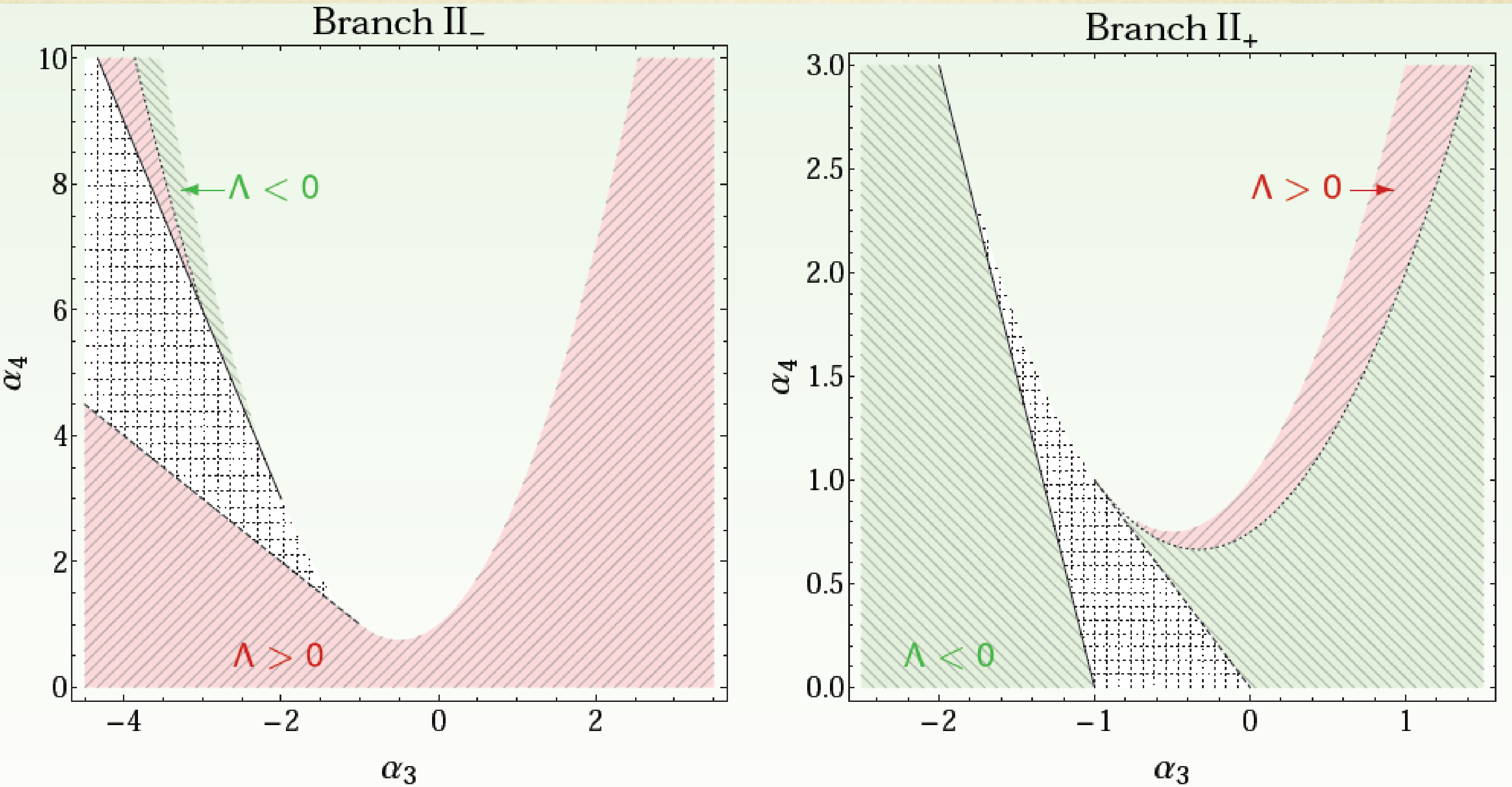
where

$$c_\pm \equiv -\frac{1}{(\alpha_3+\alpha_4)^2}\left[1+\alpha_3 \pm \sqrt{1+\alpha_3+\alpha_3^2-\alpha_4}\right] \times \left[1+\alpha_3^2-2\alpha_4 \pm (1+\alpha_3)\sqrt{1+\alpha_3+\alpha_3^2-\alpha_4}\right].$$

The effective cosmological constant

$$\Lambda_\pm = c_\pm m_g^2.$$

Self-accelerating solutions



$$\Lambda_\pm \equiv -\frac{m_g^2}{(\alpha_3+\alpha_4)^2}\left[(1+\alpha_3)\left(2+\alpha_3+2\,\alpha_3^2-3\,\alpha_4\right)\pm 2\left(1+\alpha_3+\alpha_3^2-\alpha_4\right)^{3/2}\right]$$

Conclusion and discussion

- The nonlinear massive gravity theory
- For Minkowski fiducial metric, only K<0 FRW solution exists

  Extensions of the theory with generic fiducial metric

  Hassan, Rosen, 11
- Cosmological perturbations
  - Scalar sector & vector sector … ?
  - Tensor sector … !

Thank You!