De Sitter solutions in warped compactifications

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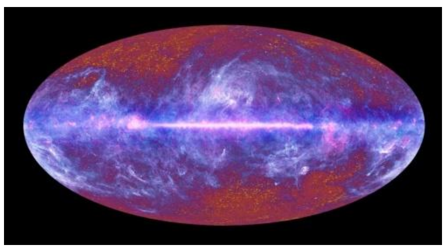
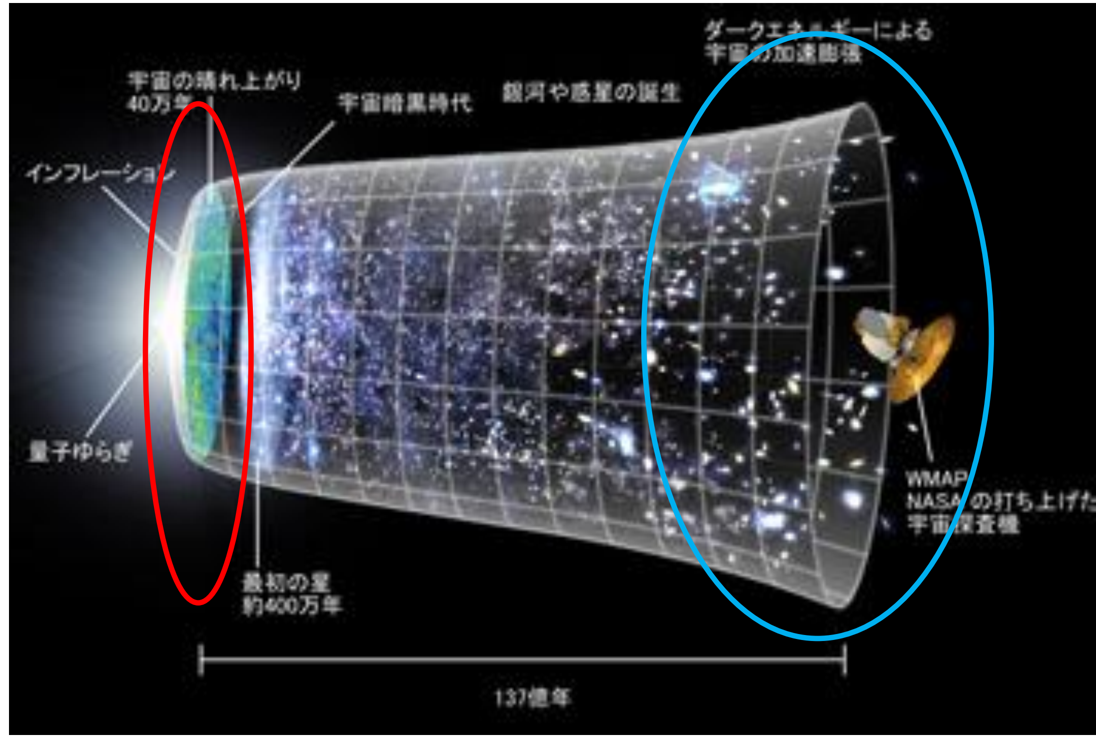
Accelerating solutions play the important roles in recent cosmology.

Inflation

✓explaining the flatness, homogeneity and isotropy of the universe

✓generating seed of cosmic structures.

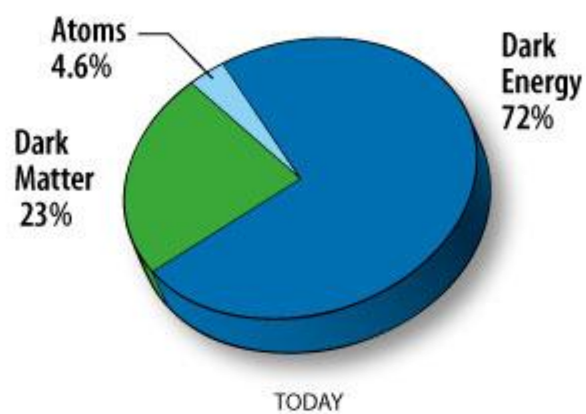




Dark Energy

✓ 72 % of the present cosmic energy, assuming GR is hold on all distance scales.

✓Modified gravity?



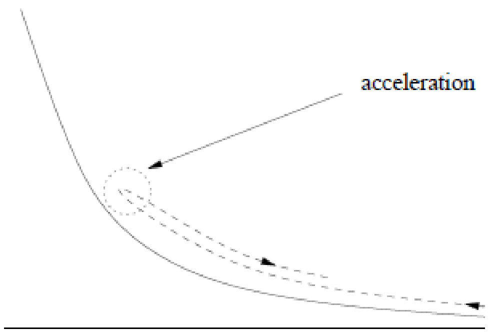
- Superstring/ M-theory suggests that our universe is higher-dimensional.

- In context of the higher-dimensional gravity, much attention has been paid to *compactifications to de Sitter or accelerating universe*, since it could provide a fairly direct explanation of these issues.

- An initial clue was the time-dependence of a hyperbolic internal space.

Townsend & Wohlfarth (03), Ohta (03)

$$V_{eff} \approx -R_H > 0$$



Emparan & Garriga (03)

"Size of internal space"

- Since many familiar solutions in the higher-dimensional gravity such as brane solutions have warped structures, it is very interesting to look for the embedding 4D de Sitter universe into the warped spacetime.

$$ds_D^2 = A(y)^2\left(-dt^2 + e^{2Ht}\delta_{ij}dx^i dx^j\right) + w_{ab}(Y)dy^a dy^b$$

Warp factor

n-dimensional de Sitter spacetime

(D-4)-dimensional internal space

- Among the warped solutions, the time-dependent generalizations of static p-branes could not provide accelerating solutions.

Gibbons, Lu & Pope (05), Maeda, Ohta & Uzawa (09) , 1007.1762,1011.2376, 1109. 1415

We have started to find more explicit warped de Sitter solutions.

De Sitter solutions

1103.5326, 1109.4818,1110.2843

$$L = \frac{1}{2\kappa^2}\sqrt{-g}\left(R - \frac{1}{2}(\partial\phi)^2 - \sum_I \frac{1}{2\cdot p_I!} e^{c_I\phi} F_{(p_I)}^2 \right)$$

gravity scalar form field strengths

We have to evade the NO-GO restrictions for de Sitter compactifications. Gibbons (84) Maldacena & Nunez (00)

- Introducing an infinite extra dimension

  1103.5326 Gibbons & Hull (01), Neupane (10)

- Introducing 0-form field strength

  1109.4818, 1110.2843

Warped de Sitter *non*compactification

1103.5326

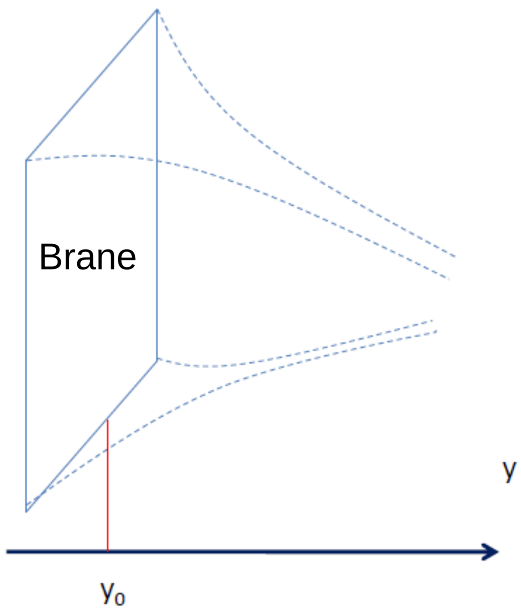
- A warped product of an infinite direction, sphere and de Sitter universe.

$$ds_D^2 = e^{2A(y)}\left[q_{\mu\nu}(X)dx^\mu dx^\nu + dy^2 + u_{ab}(Z)dz^a dz^b\right]$$

n-dim external space — Infinite — (D-n-1)-dim compact space

- Since the bulk volume diverges, now we construct the cosmological brane world model through the cut-and-paste method.

Randall & Sundrum 99



$$ds_D^2 = e^{2A(y)}\left(q_{\mu\nu}(X)dx^\mu dx^\nu + dy^2 + u_{ab}(Z)dz^a dz^b\right)$$

n-dim spacetime

Infinite

(D-n-1)-dim Einstein space

$$\phi(y) = -\frac{2(D-n-2)}{c_{D-n-1}}A(y)$$

$$F_{(D-n-1)} = f\,\sqrt{u}dz^1 \wedge \cdots \wedge dz^{D-n-1}$$

Solution

$$A(y) = \pm\frac{c_{D-n-1}f}{2}\sqrt{\frac{1}{(D-n-2)(D-2)}}(y-y_0)$$

$$R_{\mu\nu}(X) = \frac{f^2}{4}\left(\frac{c_{D-n-1}^2}{D-n-2} - \frac{2(D-n-2)}{D-2}\right)q_{\mu\nu}(X)$$

$$R_{ab}(Z) = \frac{f^2}{4}\left[\left(\frac{c_{D-n-1}^2}{D-n-2} - \frac{2(D-n-2)}{D-2}\right) + 2\right]u_{ab}(Z)$$

- X becomes de Sitter for $$c_{D-n-1}^2 > \frac{2(D-n-2)^2}{D-2}$$

- Z must be a sphere for de Sitter solutions.

- IIA supergravity allows for 7D de Sitter spacetime

$$n = 7 \qquad c_2^2 = \frac{3}{2} > \frac{1}{2}$$

- No Kaluza-Klein excitations lighter than Hubble scale.

Physics on the brane is free from the light KK excitations.

Warped de Sitter compactification

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$$D = n + 1 + \sum_I L_I$$

$$ds_D^2 = e^{2A(\theta)}\left[q_{\mu\nu}(X) dx^\mu dx^\nu + \underset{S^1}{d\theta^2} + \sum_I u_{a_I b_I}(Z_I) dz^{a_I} dz^{b_I} \right]$$

n-dim external spacetime

A product of compact spaces

$$F_{(n)} = f_n \Omega(X)$$

$$F_{(L_I)} = \ell_{\mathbf{I}}\, \omega_I$$

$$F_{(0)} = m$$ 0-form

$$\phi = -\frac{2}{c_0} A$$

- We choose coupling constants

$$c_n = (-n+1) c_0 \qquad c_I = (-L_I + 1) c_0 \qquad \boxed{c_0^2 = \frac{2}{(D-1)(D-2)}}$$

$$A = A_0 \left[\cos\left(\bar{\theta} - \bar{\theta}_0\right)\right]^{\frac{1}{D-2}}$$

$$R_{\mu\nu}(X) = \frac{1}{2}\left(-f_n^2 + \frac{K+m^2}{D-1}\right) q_{\mu\nu}(X)$$

$$K = (n-1) f_n^2 - \sum_I (L_I - 1) \ell_I^2$$

$$R_{ab}(Z_I) = \frac{1}{2}\left(\ell_I^2 + \frac{K+m^2}{D-1}\right) u_{ab}(Z_I)$$

$F_{(n)}$ $F_{(LI)}$

- X becomes de Sitter $m^2 > (D-n) f_n^2 + \sum_I (L_I - 1) \ell_I^2$

0-form is necessary

- Z_I must be a sphere for de Sitter solutions.

$$dS_n \times S^1 \times \prod_I S^{L_I}$$

- No de Sitter solution in the massive IIA supergravity with 0-form.

$$c_0^2 = N_0 + \frac{9}{4} \qquad N_0 = -\frac{20}{9} \neq 4$$

Supergravity solutions only admit AdS without the 0-form.

Summary

We have introduced our recent trials to find the warped de Sitter solutions in the higher-dimensional gravitational theory.

- **Warped de Sitter noncompactifications**
  - ✓ Infinite extra dimension evades the NO-GO restriction
  - ✓ Cosmology is realized at the boundary brane world.
- **Warped de Sitter spacetime compactifications**
  - ✓ The 0-form can evade the NO-GO restrictions.

Thank you

NO-GO theorem

Gibbons (84) Maldacena & Nunez (00)

Under assumptions

1) There is no higher curvature /derivative correction in gravity action.

$$R + \alpha R^2$$ higher-dimensional general relativity

2) All massless fields have kinetic terms with correct sign.

$$-(\partial\phi)^2 \qquad -F_{A_1\cdots A_p}F^{A_1\cdots A_p}$$

3) The scalar potential /cosmological constant is non-positive

$$V \leq 0$$

4) The internal space is compact with a finite volume and no boundary.

⇒De Sitter solutions are forbidden.

Outline of proof:

a) X is a spatially flat universe

$$ds^2(X) = -dt^2 + t^{2\lambda}\delta_{ij}dx^i dx^j$$

$$R(X) = 12\lambda\left(\lambda - \frac{1}{2}\right)t^{-2} > 0$$ for any accelerating universe $\lambda > 1$

b) Einstein gravity coupled to matter ⇒ 1)

$$A^{2(D-2)}\left(R(X) + A^2\bar{T}\right) = \frac{4}{D-2}A^{D-2}\Delta_Y A^{D-2} \qquad \bar{T} := -T^{\mu}_{\mu} + \frac{4}{D-2}T$$

c) Assuming that Y space is *compact*, integrating Einstein eq. by parts
⇒ 4)

$$\oint_Y d^{D-4}y\sqrt{u}A^{2(D-2)}\left[R(X) + A^2\bar{T}\right] = \frac{4}{D-2}\oint_Y d^{D-4}y\sqrt{u}A^{D-2}\Delta_Y A^{D-2}$$

$$= -\frac{4}{D-2}\oint_Y d^{D-4}y\sqrt{u}\left(u^{ab}\partial_a A^{D-2}\partial_b A^{D-2}\right) < 0$$

de Sitter /accelerating solution can be obtained if $\bar{T} < 0$

(d) Scalar potential (or cosmological constant) ⇒ 3)

$$T_{MN} = -V g_{MN} \longrightarrow \bar{T} = -\frac{8}{D-2} V$$

$$V \leq 0 \longrightarrow \bar{T} \geq 0$$

No de Sitter solutions for $V \leq 0$

(e) Massless p-form field strength ⇒ 2)

$$T_{MN} = \frac{1}{2p!}\left(p F_{MA_2 \cdots A_p} F_N{}^{A_2 \cdots A_p} - \frac{1}{2} g_{MN} F^2 \right)$$

$$\longrightarrow \bar{T} = \frac{1}{2(p-1)!}\left(-F_{\mu A_2 \cdots A_p} F^{\mu A_2 \cdots A_p} + \frac{4(p-1)}{(D-2)p} F^2 \right)$$

✓If the field strength has only components along Y-space,

$$\bar{T} = \frac{2}{p(p-2)!(D-2)} F^2 > 0$$

✓If it has components only along X-spacetime

$$\bar{T} = -\frac{2(D-p-1)}{(D-2) \cdot p!} F^2 \geq 0$$

No de Sitter solutions

Evading NO-GO assumptions

- ➢ adding higher curvature terms ⇒ 1)

  Ishihara (86), Maeda & Ohta (04)

  R^2 R^4

- ➢ introducing noncompact extra dimension ⇒ 4)
  - hyperbolic internal space

    Gibbons & Hull (01)
  - Warped product of $dS_4 \times R \times S^d$ in the pure gravity

    Neupane (10)
  - Warped product of $dS_4 \times R \times S^d$ with matter fields

    1103.5326

- ➢ introducing positive energy source ⇒ 3) $V > 0$
  - localized objects in the internal space

    Danielson, Haque, Shiu, &Riet (09) Wrase & Zagermann (10)
  - 0-form field strength

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