De Sitter solutions in warped compactifications

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Accelerating solutions play the important roles in recent cosmology.

**Inflation**

✓ explaining the flatness, homogeneity and isotropy of the universe

✓ generating seed of cosmic structures.

**Dark Energy**

✓ 72% of the present cosmic energy, assuming GR is hold on all distance scales.

✓ Modified gravity?
Superstring/ M-theory suggests that our universe is higher-dimensional.

In context of the higher-dimensional gravity, much attention has been paid to *compactifications to de Sitter or accelerating universe*, since it could provide a fairly direct explanation of these issues.

An initial clue was the time-dependence of a hyperbolic internal space.

Townsend & Wohlfarth (03), Ohta (03)

\[ V_{\text{eff}} \approx -R_H > 0 \]

Emparan & Garriga (03)

“Size of internal space”
Since many familiar solutions in the higher-dimensional gravity such as brane solutions have warped structures, it is very interesting to look for the embedding 4D de Sitter universe into the warped spacetime.

\[
\begin{aligned}
&d s_D^2 = A(y)^2 \left( -dt^2 + e^{2Ht} \delta_{ij} dx^i dx^j \right) + \omega_{ab}(Y) dy^a dy^b \\
\text{Warp factor} & \\
\text{n-dimensional de Sitter spacetime} &
\end{aligned}
\]

Among the warped solutions, the time-dependent generalizations of static p-branes could not provide accelerating solutions.

Gibbons, Lu & Pope (05), Maeda, Ohta & Uzawa (09), 1007.1762, 1011.2376, 1109.1415

We have started to find more explicit warped de Sitter solutions.
De Sitter solutions

\[
L = \frac{1}{2\kappa^2} \sqrt{-g} \left( R - \frac{1}{2} (\partial \phi)^2 - \sum_I \frac{1}{2 \cdot p_I!} e^{c_I \phi} F_{(p_I)}^2 \right)
\]

gravity scalar form field strengths

We have to evade the NO-GO restrictions for de Sitter compactifications. Gibbons (84) Maldacena & Nunez (00)

- Introducing an infinite extra dimension
  1103.5326 Gibbons & Hull (01), Neupane (10)

- Introducing 0-form field strength
  1109.4818, 1110.2843
A warped product of an infinite direction, sphere and de Sitter universe.

\[ ds_D^2 = e^{2A(y)} \left[ q_{\mu\nu}(X)dx^\mu dx^\nu + dy^2 + u_{ab}(Z)dz^a dz^b \right] \]

- n-dim external space
- Infinite
- (D-n-1)-dim compact space

Since the bulk volume diverges, now we construct the cosmological brane world model through the cut-and-paste method.

Randall & Sundrum 99
\[ ds_D^2 = e^{2A(y)} \left( q_{\mu\nu}(X) dx^\mu dx^\nu + dy^2 + u_{ab}(Z) dz^a dz^b \right) \]

n-dimensional spacetime

\[ \phi(y) = -\frac{2(D-n-2)}{c_{D-n-1}} A(y) \]

Infinite \( (D-n-1) \)-dimensional Einstein space

Solution

\[ A(y) = \pm \frac{c_{D-n-1} f}{2} \sqrt{\frac{1}{(D-n-2)(D-2)} (y - y_0)} \]

\[ R_{\mu\nu}(X) = \frac{f^2}{4} \left( \frac{c_{D-n-1}^2}{D-n-2} - \frac{2(D-n-2)}{D-2} \right) q_{\mu\nu}(X) \]

\[ R_{ab}(Z) = \frac{f^2}{4} \left[ \left( \frac{c_{D-n-1}^2}{D-n-2} - \frac{2(D-n-2)}{D-2} \right) + 2 \right] u_{ab}(Z) \]
X becomes de Sitter for

\[ c_{D-n-1}^2 > \frac{2(D-n-2)^2}{D-2} \]

Z must be a sphere for de Sitter solutions.

IIA supergravity allows for 7D de Sitter spacetime

\[ n = 7 \quad c_2^2 = \frac{3}{2} > \frac{1}{2} \]

No Kaluza-Klein excitations lighter than Hubble scale.

Physics on the brane is free from the light KK excitations.
Warped de Sitter compactification

\[ D = n + 1 + \sum_{I} L_{I} \]

\[ d s_{D}^{2} = e^{2A(\theta)} \left[ q_{\mu\nu}(X) d x^{\mu} d x^{\nu} + d \theta^{2} + \sum_{I} u_{a_{I}b_{I}}(Z_{I}) d z^{a_{I}} d z^{b_{I}} \right] \]

\( n \)-dim external spacetime

A product of compact spaces

\[ F_{(n)} = f_{n} \Omega(X) \]

\[ F_{(0)} = m \quad 0\text{-form} \]

\[ \phi = -\frac{2}{c_{0}} A \]

We choose coupling constants

\[ c_{n} = (-n + 1)c_{0} \quad c_{I} = (-L_{I} + 1)c_{0} \]

\[ c_{0}^{2} = \frac{2}{(D - 1)(D - 2)} \]
\[ A = A_0 \left[ \cos(\bar{\theta} - \bar{\theta}_0) \right]^{\frac{1}{D-2}} \]

\[ R_{\mu\nu}(X) = \frac{1}{2} \left( - f_n^2 + \frac{K + m^2}{D-1} \right) q_{\mu\nu}(X) \]

\[ K = (n-1)f_n^2 - \sum_I (L_I - 1)\ell_I^2 \]

\[ R_{ab}(Z_I) = \frac{1}{2} \left( \ell_I^2 + \frac{K + m^2}{D-1} \right) u_{ab}(Z_I) \]

- \( X \) becomes de Sitter \( m^2 > (D-n)f_n^2 + \sum_I (L_I - 1)\ell_I^2 \)
  - 0-form is necessary

- \( Z_I \) must be a sphere for de Sitter solutions.

\[ dS_n \times S^1 \times \prod_I S^{L_I} \]

- No de Sitter solution in the massive IIA supergravity with 0-form.

\[ c_0^2 = N_0 + \frac{9}{4} \quad N_0 = -\frac{20}{9} \neq 4 \]

Supergravity solutions only admit AdS without the 0-form.
Summary

We have introduced our recent trials to find the warped de Sitter solutions in the higher-dimensional gravitational theory.

- **Warped de Sitter noncompactifications**
  - Infinite extra dimension evades the NO-GO restriction
  - Cosmology is realized at the boundary brane world.

- **Warped de Sitter spacetime compactifications**
  - The 0-form can evade the NO-GO restrictions.
Thank you
NO-GO theorem
Gibbons (84)   Maldacena & Nunez (00)

Under assumptions

1) There is no higher curvature /derivative correction in gravity action.
   \[ R + \alpha R^2 \] 
   higher-dimensional general relativity

2) All massless fields have kinetic terms with correct sign.
   \[ - (\partial \phi)^2 - F_{A_1 \ldots A_p} F^{A_1 \ldots A_p} \]

3) The scalar potential /cosmological constant is non-positive
   \[ V \leq 0 \]

4) The internal space is compact with a finite volume and no boundary.

⇒ De Sitter solutions are forbidden.
Outline of proof:

a) X is a spatially flat universe
\[ ds^2(X) = -dt^2 + t^{2\lambda} \delta_{ij} dx^i dx^j \]
\[ R(X) = 12 \lambda \left( \lambda - \frac{1}{2} \right) t^{-2} > 0 \]
for any accelerating universe \( \lambda > 1 \)

b) Einstein gravity coupled to matter \( \Rightarrow 1) \)
\[ A^{2(D-2)}(R(X) + A^2 \overline{T}) = \frac{4}{D-2} A^{D-2} \Delta Y A^{D-2} \]
\[ \overline{T} := -T^\mu_\mu + \frac{4}{D-2} T \]

c) Assuming that Y space is compact, integrating Einstein eq. by parts \( \Rightarrow 4) \)
\[ \int_Y d^{D-4} y \sqrt{u} A^{2(D-2)} \left[ R(X) + A^2 \overline{T} \right] = \frac{4}{D-2} \int_Y d^{D-4} y \sqrt{u} A^{D-2} \Delta Y A^{D-2} \]
\[ = - \frac{4}{D-2} \int_Y d^{D-4} y \sqrt{u} \left( u^{ab} \partial_a A^{D-2} \partial_b A^{D-2} \right) < 0 \]
de Sitter /accelerating solution can be obtained if \( \overline{T} < 0 \)
(d) Scalar potential (or cosmological constant) \( \Rightarrow \) 3)

\[
T_{MN} = -V g_{MN} \quad \longrightarrow \quad \overline{T} = -\frac{8}{D-2} V
\]

\[ V \leq 0 \quad \longrightarrow \quad \overline{T} \geq 0 \]

No de Sitter solutions for \( V \leq 0 \)

(e) Massless p-form field strength \( \Rightarrow 2) \)

\[
T_{MN} = \frac{1}{2p!} \left( pF_{MA_2\ldots A_p} F^{A_2\ldots A_p}_N - \frac{1}{2} g_{MN} F^2 \right)
\]

\[ \longrightarrow \quad \overline{T} = \frac{1}{2(p-1)!} \left( -F_{\mu A_2\ldots A_p} F^{\mu A_2\ldots A_p} + \frac{4(p-1)}{(D-2)p} F^2 \right) \]

\( \checkmark \) If the field strength has only components along Y-space,

\[ \overline{T} = \frac{2}{p(p-2)!(D-2)} F^2 > 0 \]

\( \checkmark \) If it has components only along X-spacetime

\[ \overline{T} = -\frac{2(D-p-1)}{(D-2) \cdot p!} F^2 \geq 0 \]

No de Sitter solutions
Evading NO-GO assumptions

➢ adding higher curvature terms ⇒ 1)
  Ishihara (86), Maeda & Ohta (04)
  \[ R^2 \times R^4 \]

➢ introducing noncompact extra dimension ⇒ 4)
  • hyperbolic internal space
    Gibbons & Hull (01)
    • Warped product of \( dS_4 \times R \times S^d \) in the pure gravity
      Neupane (10)
    • Warped product of \( dS_4 \times R \times S^d \) with matter fields
      1103.5326

➢ introducing positive energy source ⇒ 3) \( V > 0 \)
  • localized objects in the internal space
    Danielson, Haque, Shiu, & Riet (09)  Wrase & Zagermann (10)
  • 0-form field strength
    1109.4818, 1110.2843