APS2012, Kyoto, 2nd March 2012

Non-minimal k-inflation

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Nomination for shortest title at APS2012

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My non-Gaussianity

- Prof. Moss (Ph.D.)
- Prof. Sasaki (Post Doc)
- Prof. Kubota (HETOU)
- Kubota-Moss-Sasaki scale invariant spectrum ⇒ I should know everything they know
    - Non-Gaussianity ⇒ I don’t know and need students like Misumi and Okuda

Het Camp 2011

- Misumi and Okuda are nowhere to be seen?

Introduction

- k-inflation can be motivated from effective field theory + curvature coupling terms, $\xi\varphi^2 R$
- Jordan or Einstein frame?
    - Single field models give full agreement between two frames including non-Gaussianity
- Qiu and Yang, *Non-Gaussianities of single field inflation with non-minimal coupling*, Phys. Rev. D **83** (2011) 084022.
- non-minimal coupled DBI (ξ=0, 1/6) [see Easson et al., Phys. Rev. D **80** (2009); ibid. **81**(2010)]

Model

Non-minimal k-inflation

$$S = \int d^4x\sqrt{-g}\left[f(\varphi)R + 2P(\varphi, X)\right]$$

$$P(\varphi, X) = K(\varphi)X + L(\varphi)X^2 + \cdots \qquad X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$$

- For example in a two field Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\rho)^2 + \frac{\rho}{M}(\partial\phi)^2 + \frac{1}{2}M^2\rho^2 + V(\phi) - \frac{1}{2}\xi R(\phi^2 + \rho^2)$$

- Then below some $H \ll M$ we can integrate out ρ to get

$$\mathcal{L}_{eff} = \frac{1}{2}(\partial\phi)^2 + \frac{(\partial\phi)^4}{M^4} + \ldots + V(\phi) - \frac{1}{2}\xi R\phi^2$$

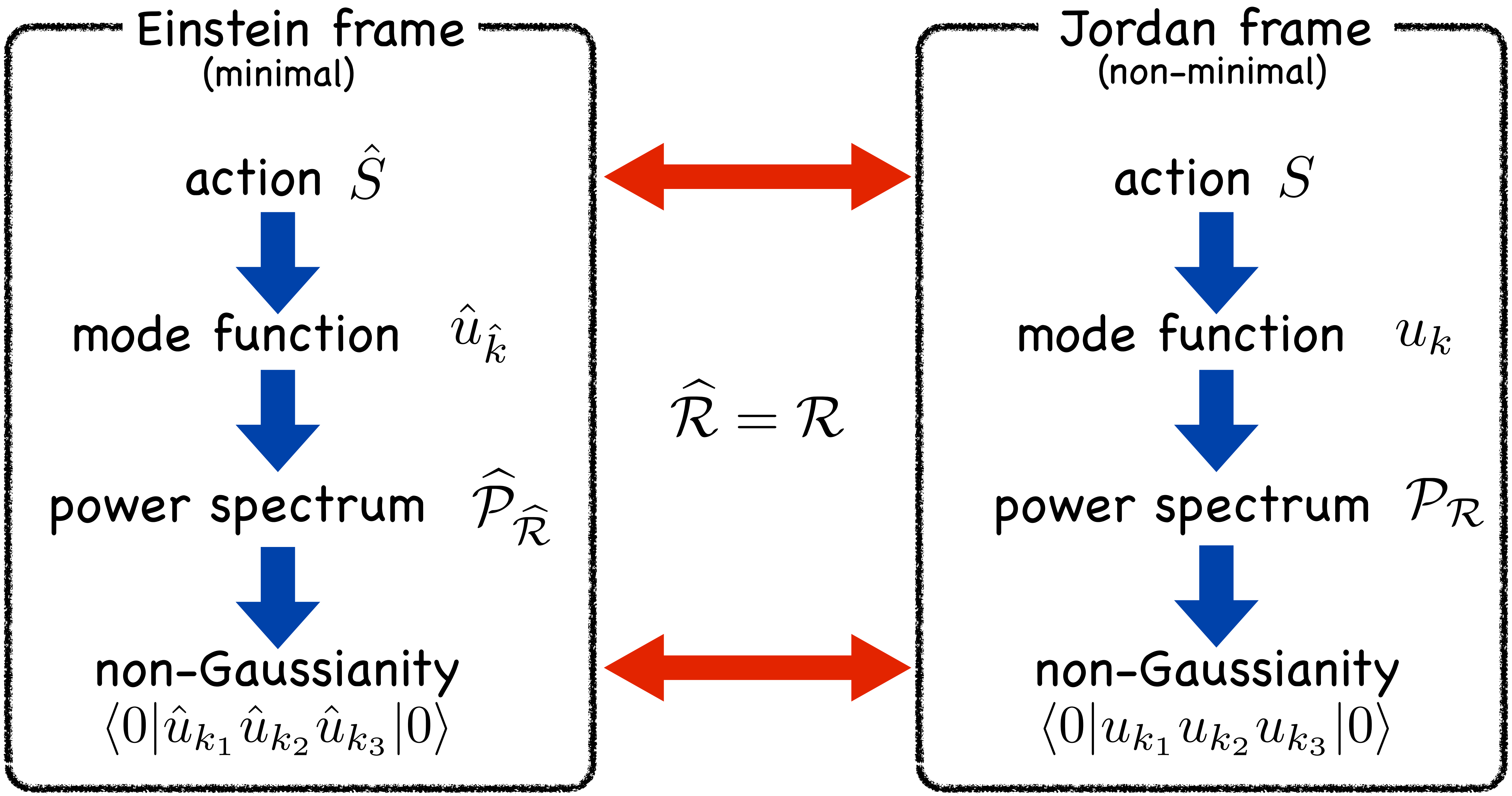
where $M \rightarrow M - \xi R/2$

- Other examples might be DBI with more general $\xi R\varphi^2$ term

Motivation

- Obtain constraints on inflation models via observational data
  - k-inflation with non-minimal coupling
    - Most general single field theory with up to 1st order derivatives in ϕ (cf. Horndeski/G-inflation at 2nd order)
  - Can we get constraints on conformal coupling ξ?
- Due to variable sound speed, k-inflation can generate large non-Gaussianity (e.g., equilateral limit): $f_{NL} \propto \frac{1}{c_s^2}$

Comment on **E** and **J** frames



*T. Kubota, N. Misumi, W. Naylor and N. Okuda, JCAP **02** (2012) 034, arXiv: 1112.5233 [gr-qc]*

Standard k-inflation

(Armendariz-Picon, Damour, Mukhanov, 1999)

$$S = \int d^4x \sqrt{-g} \Big(\frac{1}{2\kappa^2} R + P(\phi, X) \Big)$$

$$X = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

$$P(\phi, X) = K(\phi) X + X^2 + \cdots$$

$$T_{\mu\nu} = \frac{\partial P}{\partial X} \nabla_\mu \phi \nabla_\nu \phi - P g_{\mu\nu} \quad \longrightarrow \quad E(\phi, X) = 2XP_{,X} - P$$

- Consider FLRW universe described by line element

$$ds^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j$$

- Friedmann equation and the continuity equation are

$$3H^2 = E$$

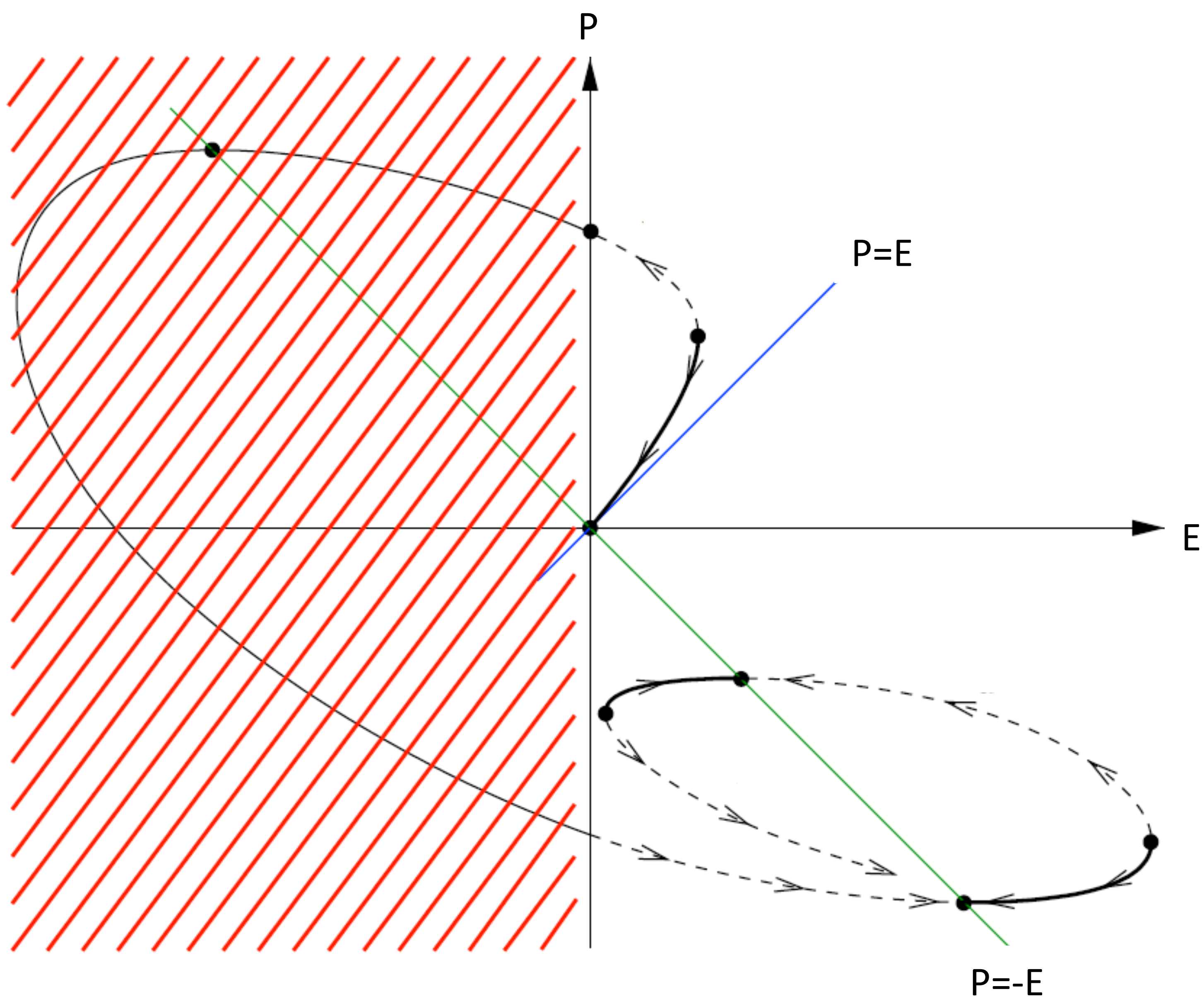
$$\dot{E} = -3\sqrt{E}(E + P)$$

master equation

- K(ϕ) changing from <0 to >0 essentially leads to inflationary attractor solutions

Attractors in k-inflation

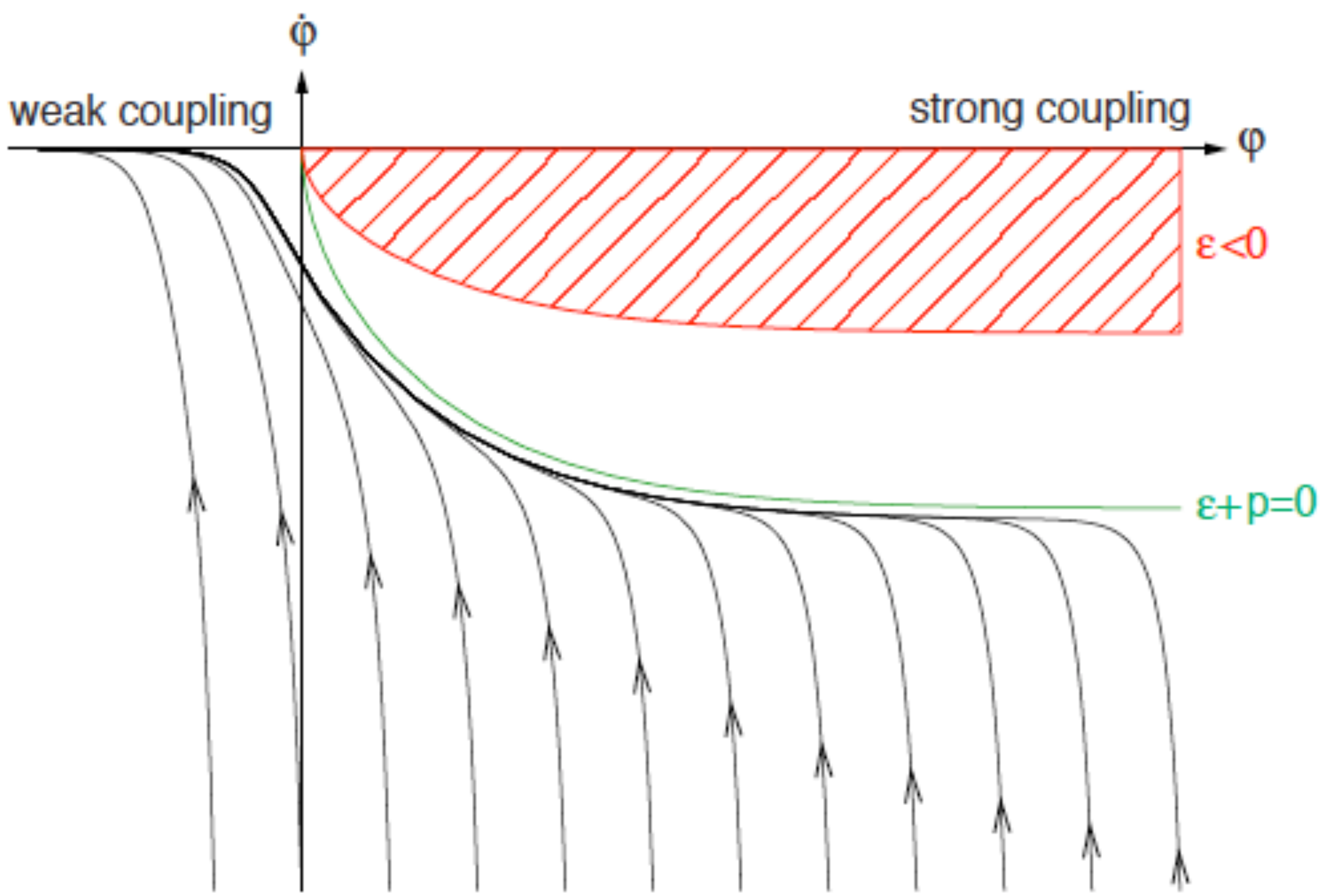
In k-inflation, the universe can expand exponentially using only master equation $\dot{E} = -3\sqrt{E}(E+P)$



Solid lines are stable attractors

(Armendariz-Picon, et al., PLB **458** 1999)

k-inflation phase diagram



We obtain similar attractor/slow-roll solutions in the non-minimal case (see later if time permits)

(Armendariz-Picon, et al., PLB **458** 1999)

Non-minimal k-inflation

$$S = \int d^4x \sqrt{-g}\Big(\frac{1}{2\kappa^2}R - \frac{1}{2}\xi\phi^2 R + P(\phi, X)\Big)$$

non-minimal coupling

Conformal transformation

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \qquad \Omega^2 \equiv |1 - \xi\kappa^2\phi^2|$$

$$X = -\frac{1}{2}\Omega^2 \hat{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = \Omega^2\hat{X}$$

$$\hat{S} = \int d^4x \sqrt{-\hat{g}}\Big(\frac{1}{2\kappa^2}\hat{R} + \hat{P}(\phi, X)\Big)$$

$$\hat{P}(\phi, X) = \hat{K}(\phi)\hat{X} + \hat{L}(\phi)\hat{X}^2$$

via field redefinition

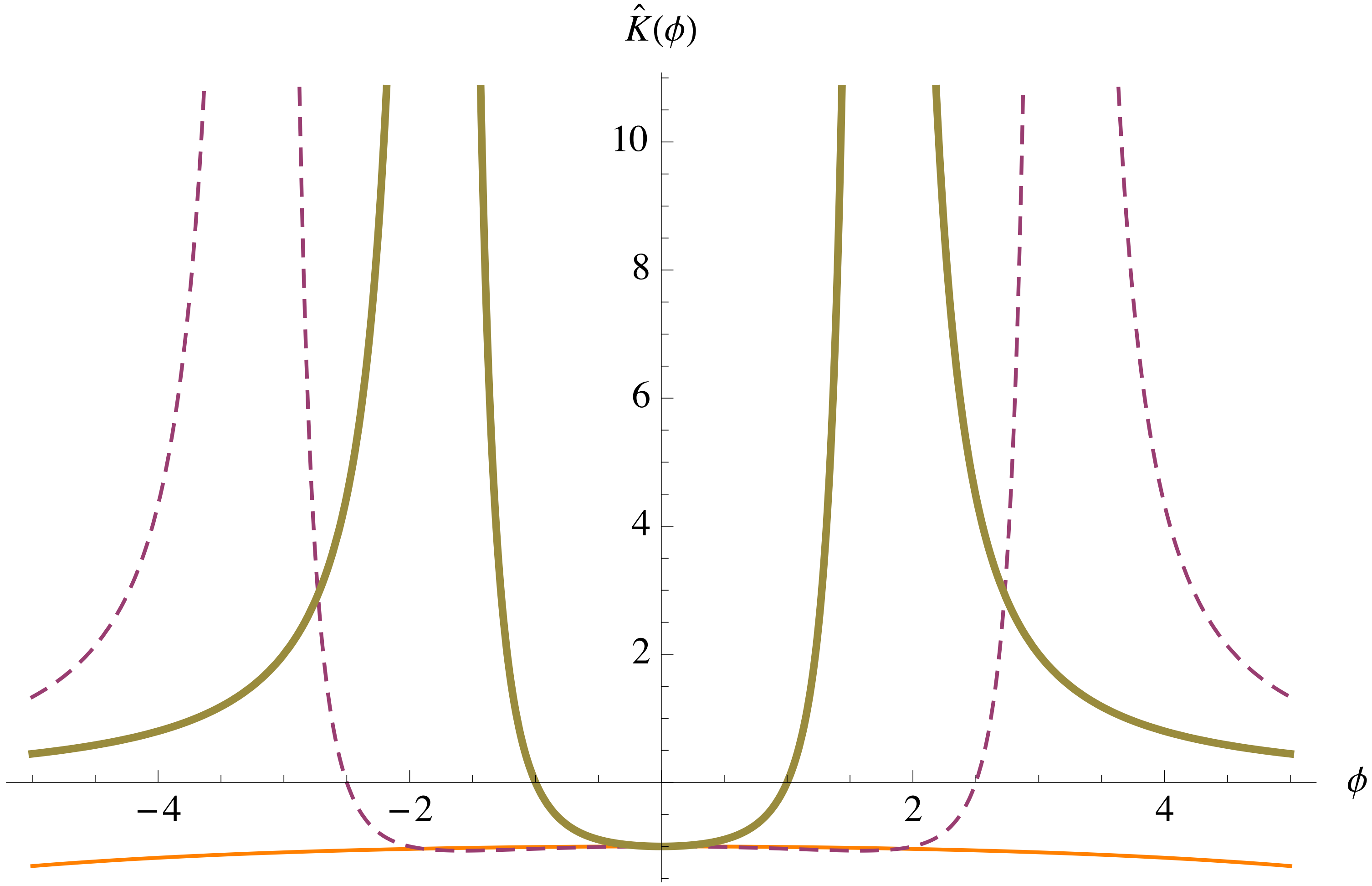
$$\hat{K}(\phi) = \frac{K(\phi) - \xi(K(\phi) - 6\xi)\kappa^2\phi^2}{(1 - \xi\kappa^2\phi^2)^2}$$

$$\hat{L}(\phi) = L(\phi) = 1$$

$\xi < 0$ obtained by substituting $\xi \to -|\xi|$

E.g., setting $K(\varphi)=-1$

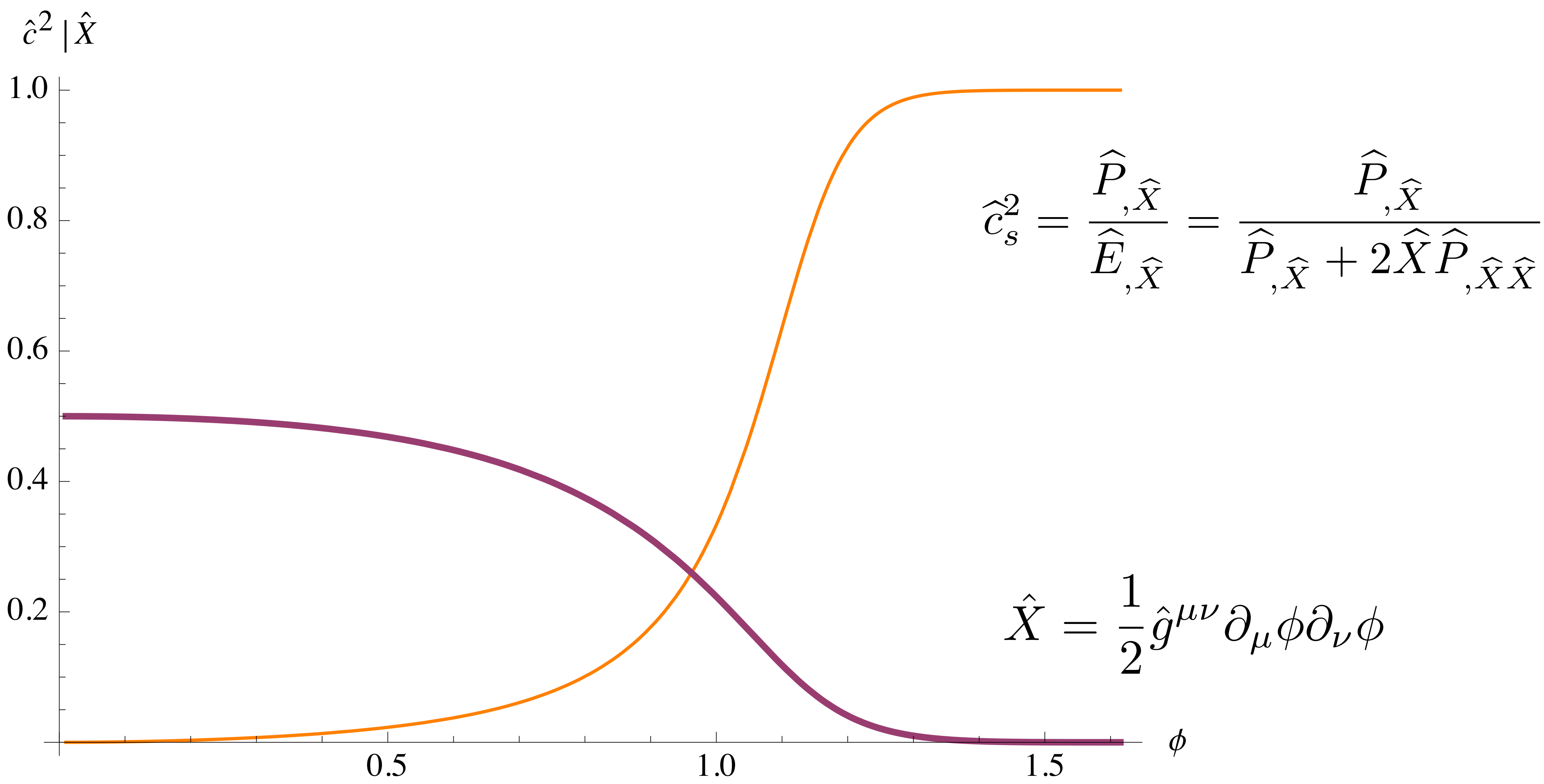
- Actually even for $K(\varphi)<0$ effect of $\xi>0$ coupling leads to a sign change in $\hat{K}(\varphi)$



- $\xi = 1/100, 1/10$ and $1/3$

$K(\varphi)$=-1

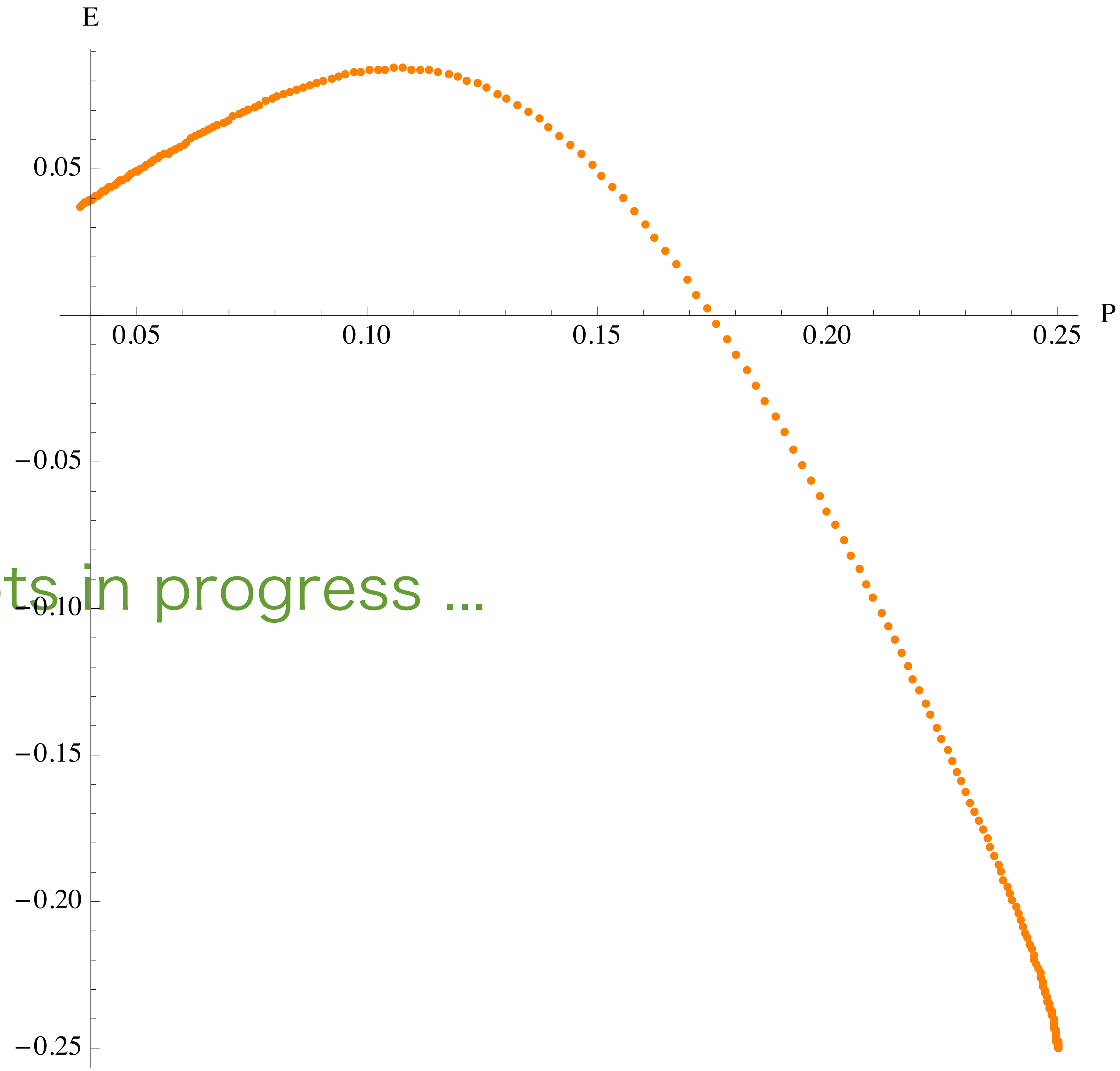
- Speed of sound c_s stays small until dϕ/dt (∝X) drops



- Note that level of non-Gaussianity $f_{NL} \propto \frac{1}{c_s^2}$

Example attractor solution

- We see an attractor for E=p and E=-p (here for ξ =1/3)



- More plots in progress ...

Power spectrum & spectral index

$$\hat{S}^{(2)} = \int d\hat{t} d^3x \left[\hat{a}^3 \frac{\hat{\epsilon}}{\hat{c}_s^2} \dot{\hat{\mathcal{R}}}^2 - \hat{a}\hat{\epsilon}(\partial\mathcal{R})^2 \right]$$

- Slow roll parameters: $\widehat{\epsilon} = -\frac{\dot{\widehat{H}}}{\widehat{H}^2} = \frac{\widehat{X}\widehat{P}_{,\widehat{X}}}{\widehat{H}^2}\ , \quad \widehat{\eta} = \frac{\dot{\widehat{\epsilon}}}{\widehat{\epsilon}\widehat{H}}\ , \quad \widehat{s} = \frac{\dot{\widehat{c}}_s}{\widehat{c}_s\widehat{H}}$

new variables

$$v = z\widehat{\mathcal{R}}\ , \qquad z^2 = \frac{2\widehat{a}^2\widehat{\epsilon}}{\widehat{c}_s^2}$$

$$\ddot{v} - c_s^2\nabla^2 v - \frac{\ddot{z}}{z}v = 0$$

$$\ddot{v}_{\widehat{k}} + \left(c_s^2\widehat{k}^2 - \frac{\ddot{z}}{z}\right)v_{\widehat{k}} = 0$$

(Note that $\hat{c}_s$ is a function of time ⇒ $d\hat{c}_s/dt << 1$)

- Standard quantization leads to:

$$P_{\widehat{k}}^{\widehat{\mathcal{R}}} = \frac{1}{36\pi^2}\frac{\widehat{E}^2}{\widehat{E}+\widehat{P}} = \frac{1}{8\pi^2}\frac{\widehat{H}^2}{\widehat{c}_s\widehat{\epsilon}}\ , \qquad \widehat{n}_s - 1 = \frac{d\ln P_{\widehat{k}}^{\widehat{\mathcal{R}}}}{d\ln\widehat{k}} = -2\widehat{\epsilon} - \widehat{\eta} - \widehat{s}$$

plots currently in progress (including e-foldings)...

Appendix: Conformal properties

$$\hat{a} = \Omega a \qquad d\hat{\tau} = d\tau \qquad d\hat{t} = \Omega dt$$

$$R = \Omega^2 \Big[\hat{R} + 3 \Big(\frac{(\Omega^2)'}{\Omega^2} \Big)^2 \hat{X} \Big] \qquad \Omega^2 \equiv |1 - \xi \kappa^2 \phi^2|$$

$$\hat{K} = \frac{K}{\Omega^2} + \frac{3}{2} \Big(\frac{(\Omega^2)'}{\Omega^2} \Big)^2 \qquad \hat{P} = \frac{P}{\Omega^4} + \frac{3}{4} \Big(\frac{\dot{\Omega}^2}{\Omega^2} \Big)^2$$

$$\widehat{\mathcal{R}} = \mathcal{R} \qquad \hat{H} = \frac{1}{\Omega} \Big(H + \frac{1}{2} \frac{\dot{\Omega}^2}{\Omega^2} \Big)$$

$$\hat{\dot{H}} = \frac{1}{\Omega^2} \Big[\dot{H} - \frac{1}{2} H \frac{\dot{\Omega}^2}{\Omega^2} - \frac{3}{4} \Big(\frac{\dot{\Omega}^2}{\Omega^2} \Big) + \frac{1}{2} \frac{\ddot{\Omega}^2}{\Omega^2} \Big]$$

Final Comments

- In k-inflation with non-minimal coupling, is there a characteristic signal, different from other models?
- Classical stability (backreaction) needs further investigation, even for DBI non-minimal models (next slide)
- Non-canonical models lead to non-Gaussianity & non-minimal coupling broadens allowed parameter space (to be confirmed)
    - Shape of non-Gaussianity is important and has contribution coming from ξ (later slide, time permitting)
- Preheating in non-minimal K-inflation interesting?

Classical stability?

- In **Jordan** frame easy to see $G_{\rm eff} = \dfrac{G}{1-\phi^2/\phi_c^2}$, where
  $\varphi_c^{\,2} = (\kappa^2\xi)^{-1} = M_{pl}^{\,2}/(8\pi\xi)$

- However another instability can be found from EOM:

$$P_X\Box\phi + P_\phi + (P_{XX}\nabla^\mu X + P_{X\phi}\nabla^\mu\phi)\nabla_\mu\phi = \xi\phi R \qquad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{M_{\rm pl}^2}T_{\mu\nu}$$

$$T_{\mu\nu} = P_X\nabla_\mu\phi\nabla_\nu\phi + g_{\mu\nu}P + \xi\Big[g_{\mu\nu}\Box(\phi^2) - \nabla_\mu\nabla_\nu(\phi^2) + (R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R)\phi^2$$

Taking trace of $T_{\mu\nu}$ and subbing back into EOM ⇒

$$R = \frac{\tilde\phi_c^2}{M_{\rm pl}^2(\phi^2-\tilde\phi_c^2)}\Big[\Big(P_X + 6\xi - 6\xi\phi\frac{P_{X\phi}}{P_X}\Big)\nabla^\mu\phi\nabla_\mu\phi - 6\xi\phi\frac{P_{XX}}{P_X}\nabla^\mu X\nabla_\mu\phi + 4P - 6\xi\phi\frac{P_\phi}{P_X}\Big]$$

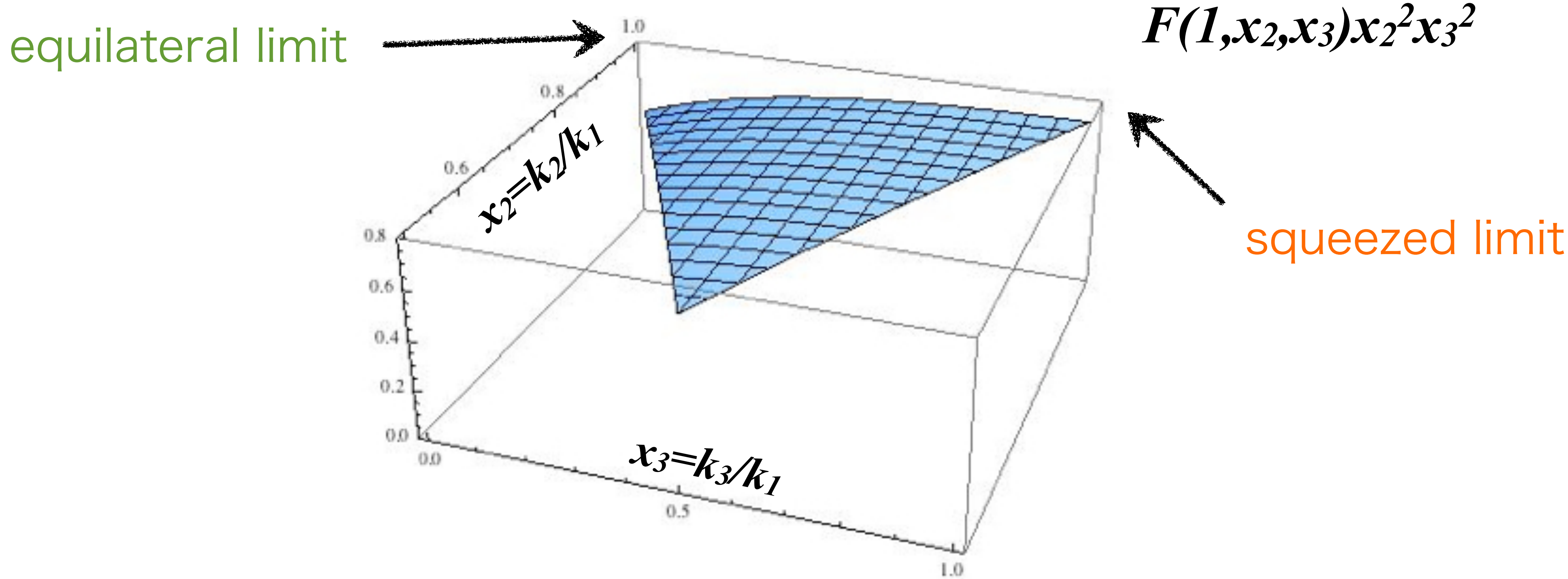
where $\tilde\phi_c^2 = \dfrac{M_{\rm pl}^2}{\xi(1-6\xi/P_X)}$ (***P_x=1***, standard result)

- $\mathscr{H}$ constraints ⇒ $\phi > \tilde\phi_c$ only for anistropic spacetimes; *cf.*

  Futamase et al., Phys.Rev. D ***39*** *(1989) 405-411*

k-inflation with non-minimal coupling

- Shape of non-Gaussianity from ξ-contribution

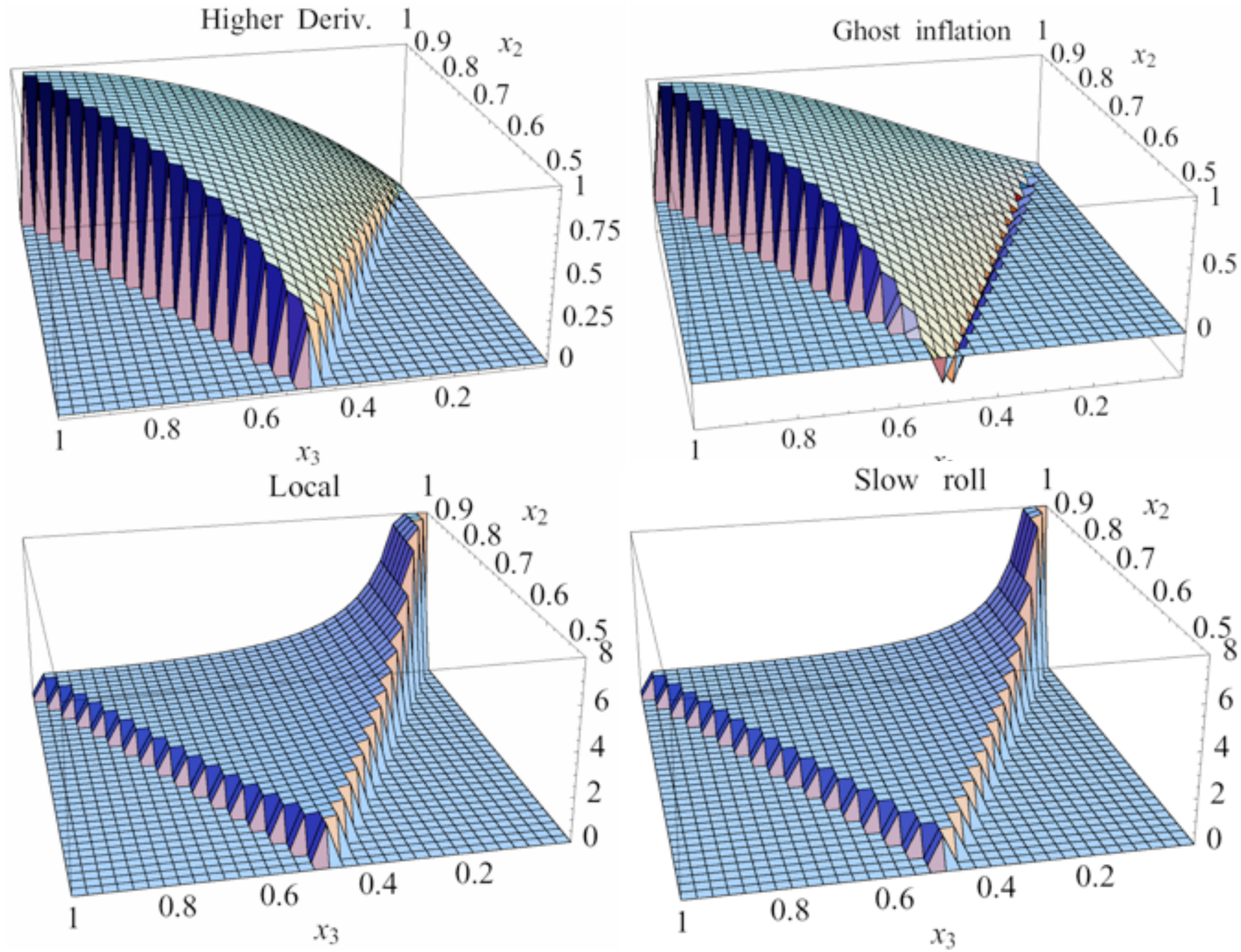


- In all limits non-minimal part has nonzero value

$$\frac{F(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)}{\boldsymbol{k}_1\boldsymbol{k}_2\boldsymbol{k}_3} \propto \frac{3\boldsymbol{k}_1\boldsymbol{k}_2\boldsymbol{k}_3}{2\boldsymbol{k}^3} + \frac{1}{\boldsymbol{k}^2}\sum_{i>j}\boldsymbol{k}_i\boldsymbol{k}_j$$

- Result from N. Misumi's Master's thesis (cf. Qiu & Yang)

Appendix: Other shapes



$F(1,x_2,x_3)x_2^2x_3^2$

$x_3=k_3/k_1$,
$x_2=k_2/k_1$

(Babich et al. 2004)

The shape of non-Gaussianity depends on model of inflation; however, current data not sensitive to shape; only overall amplitude f_{NL}

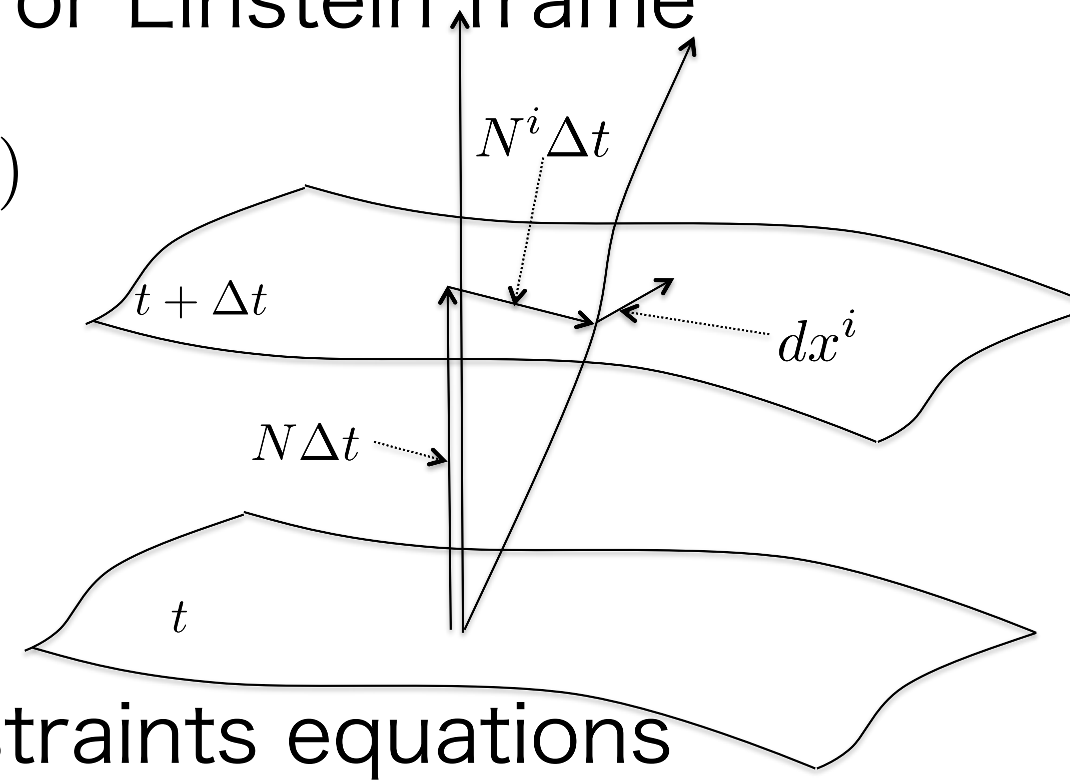
Appendix: ADM Decomposition

- In what follows drop the hat and for Einstein frame

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$S = \frac{1}{2}\int d^4x\sqrt{h}N[R^{(3)} + 2P + N^{-2}(E_{ij}E^{ij} - E^2)]$$

$$E_{ij} = \frac{1}{2}(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i)$$



- Hamiltonian and momentum constraints equations are given by

$$R^{(3)} + 2P - 2N^{-2}P_{,X}(\dot{\phi} - N^i\partial_i\phi)^2 - N^{-2}(E_{ij}E^{ij} - E^2) = 0$$

$$\nabla_j(N^{-1}E^j{}_i) - \nabla_i(N^{-1}E) = P_{,X}N^{-1}\partial_i\phi(\dot{\phi} - N^i\partial_i\phi)$$

- Solve constraints equations to first order only for N and N^i (Chen et al. JCAP 01); in unitary gauge

$$\delta\phi = 0\ , \qquad h_{ij} = a^2 e^{2\mathcal{R}}\delta_{ij}$$

Appendix: Invariance of curvature perturbation R

- In the Einstein frame fluctuations around FLRW imply

$$d\widehat{s}^2 = -(d\widehat{t})^2 + \widehat{a}(\widehat{t})^2 e^{2\widehat{\mathcal{R}}}\left(\delta_{ij} + \widehat{\gamma}_{ij}\right), \qquad \left(\partial^i \widehat{\gamma}_{ij} = 0,\ \delta^{ij}\widehat{\gamma}_{ij} = 0\right).$$

which via a conformal transformation leads to

$$\begin{aligned} ds^2 &= -(dt)^2 + a(t)^2 e^{2\mathcal{R}}\left(\delta_{ij} + \gamma_{ij}\right), \qquad \left(\partial^i \gamma_{ij} = 0,\ \delta^{ij}\gamma_{ij} = 0\right), \\ &= \frac{1}{\Omega^2}\left\{-(d\widehat{t})^2 + \widehat{a}(\widehat{t})^2 e^{2\mathcal{R}}\left(\delta_{ij} + \gamma_{ij}\right)\right\} \end{aligned}$$

- Comparison between the metrics in each frame implies:

$$\widehat{\mathcal{R}} = \mathcal{R}, \qquad \widehat{\gamma}_{ij} = \gamma_{ij}$$

- Thus, scalar curvature (R) and tensor perturbations are conformally invariant

- More details see: Chiba & Yamaguchi JCAP 0810 (slow roll), and Gong et al. JCAP 1109 (Tautology & δN formalism)