A Cosmological Scenario without Initial Singularity

——Bouncing Cosmology

Taotao Qiu
LeCosPA Center, National Taiwan University
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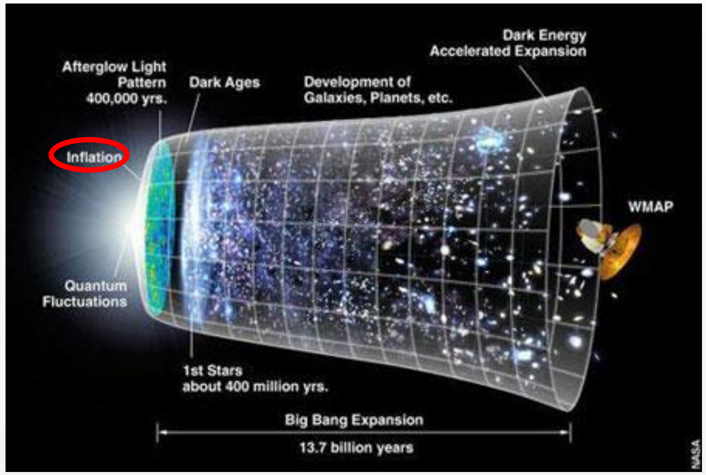
Outline

- Preliminary: Bouncing Scenario As an Alternative of Inflation
- Perturbations of Bouncing Cosmology vs. Inflationary Cosmology
- Matter Bounce

Inflation, And Its Alternatives

Inflation:



- *fast expansion*
- *slow roll*
- *flat spectrum*

……

Inflation can solve many Big-Bang-caused puzzles but suffers initial singularity problem

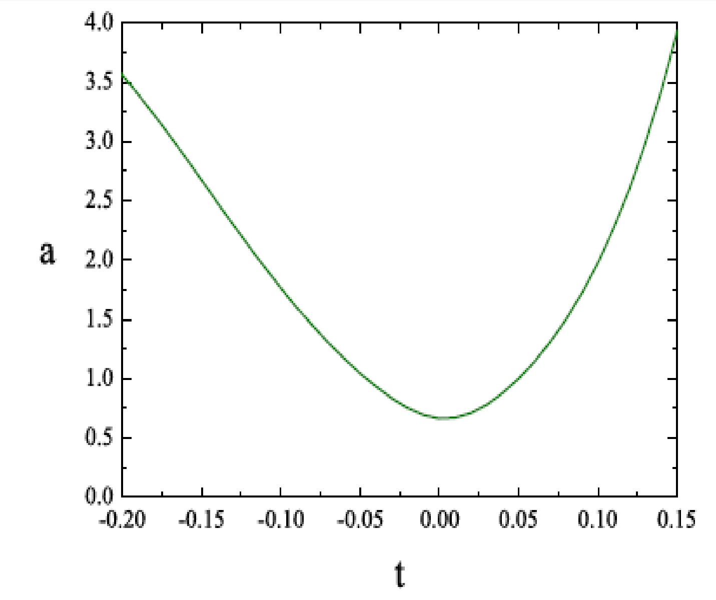
- S.W. Hawking, G.F.R. Ellis, Cambridge University Press, Cambridge, 1973.
- Borde and Vilenkin, Phys.Rev.Lett.72,3305 (1994).
- J. Martin, and R.Brandenberger, Phys.Rev.D63:123501 (2001).

Alternatives of inflation:

- Pre-big bang Scenario
- Ekpyrotic Scenario
- String gas/Hagedorn Scenario
- Non-local SFT Scenario
- Bouncing Scenario

(Non-singular) Bounce Cosmology

Basic Picture:



Contraction → IR size with Low energy scale → Expansion

Singularity problem avoided!

Y. Cai, T. Qiu, Y. Piao, M. Li and X. Zhang, JHEP 0710:071, 2007

Conditions for Bounce to Happen

From the naïve picture, we can see:

Contraction: $H < 0$ Expansion: $H > 0$

Bouncing Point: $H = 0$ or $\rho = 0$ Nearby: $\dot{H} > 0$

From Friedmann Equation:

$$\dot{H} = -4\pi G(\rho + p) \Rightarrow w < -1$$ *null energy condition violation!*

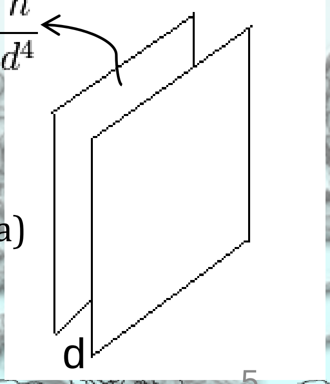
~~NEC~~ → instability for perfect fluid

→ maybe not if special effects introduced, e.g. nonlocal effects, see

Bouncing Galileon Cosmologies.
T. Qiu, J. Evslin, Y. Cai, M. Li, X. Zhang, JCAP 1110:036, 2011.

Not Necessarily Unphysical!

$$\varepsilon = \frac{-\pi^2}{720}\frac{\hbar}{d^4}$$

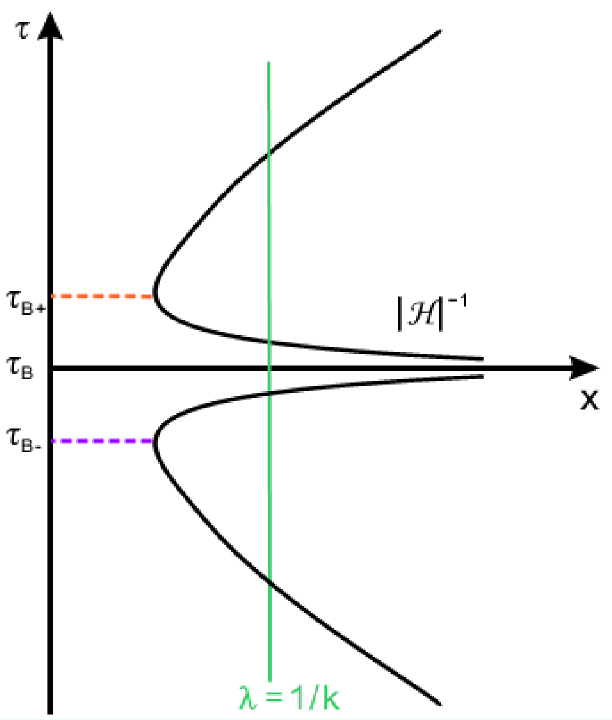


Casimir Effect
(from Wikipedia)

How does Bounce solve other cosmological problems?

Horizon problem: the horizon in the far past in contracting phase is very large;

(also provide mechanism for survival of quantum fluctuations, which Seeds for Large Scale Structure. See perturbation theory later on.)



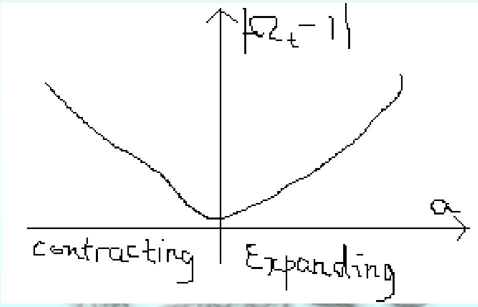
Flatness problem:

$$\Omega_{tot}(t) - 1 = \frac{k}{a^2 H^2}$$

$$|\Omega_{tot}(t) - 1| \propto a^2 \propto T^{-2}$$

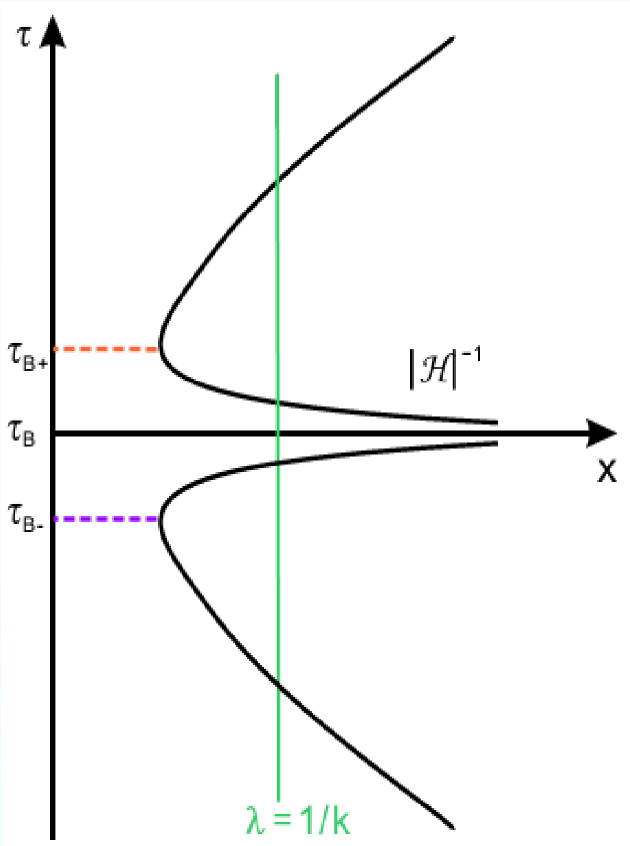
e. g. for radiation domination

avoided if the spatial curvature in the contracting phase when the temperature is comparable to today is not larger than the current value.



How does Bounce solve other cosmological problems?

Trans-Planckian and Unwanted relics problem:



✓If the energy density at the bounce point is given by the Grand Unification scale ($\sim 10^{16} GeV$), then

$$\rho \sim 10^{64} GeV^4 \qquad H \sim 10^{13} GeV^{-1}$$

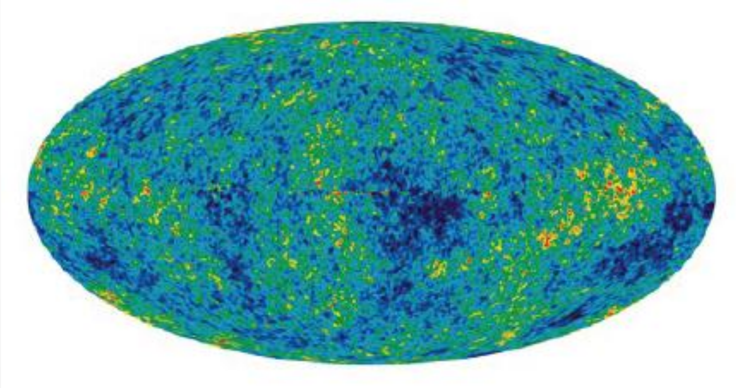
and the wavelength of a perturbation mode is about

$$H^{-1} \sim 1mm.$$

✓Unwanted relics can also be avoided because of the low energy scale

Y. F. Cai, T. t. Qiu, R. Brandenberger and X. m. Zhang, Phys. Rev. D 80, 023511 (2009)

Perturbation theory of a bounce



Why perturbations?

In order to form structures of our universe that can be observed today.

$$\frac{\Delta T}{T} \sim 10^{-5}$$

Variables for testing perturbations:

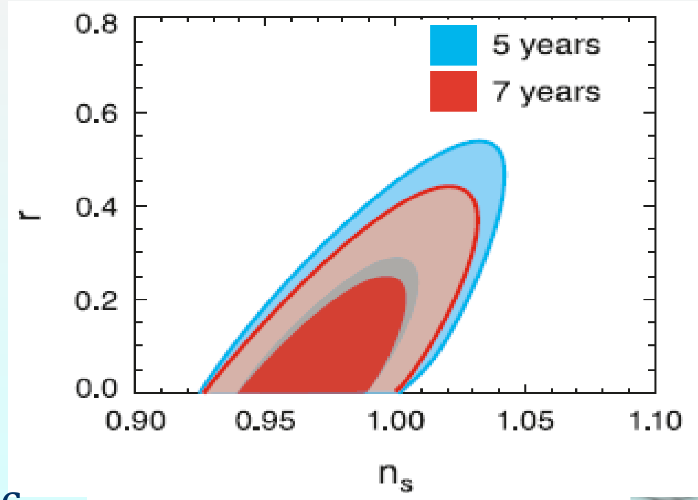
Power spectrum:

$$\mathcal{P}_\zeta(k) \equiv \frac{k^3}{2\pi^2}|\zeta|^2$$

With spectral index:

$$n_s \equiv \frac{d \ln \mathcal{P}_\zeta}{d \ln k} + 1$$

Observationally, nearly scale-invariant power spectrum ($n_s \simeq 1$) is favored by data!



n_s 0.963 ± 0.014

D. Larson *et al.* [WMAP collaboration], arXiv:1001.4635 [astro-ph.CO].

Others: bispectrum, trispectrum, gravitational waves, etc.

Perturbations in Inflationary Cosmology

Perturbed metric in conformal Newtonian gauge:

$$ds^2 = a^2(\eta)[(1+2\Phi)d\eta^2 - (1-2\Psi)dx^i dx^i]$$

Perturbation Equations for metric:

$$\Phi'' - c_s^2 \nabla^2 \Phi + \frac{2\mathcal{H}^3 - 4\mathcal{H}\mathcal{H}' + \mathcal{H}''}{\mathcal{H}^2 - \mathcal{H}'}\Phi' + \frac{\mathcal{H}\mathcal{H}'' - 2\mathcal{H}'^2}{\mathcal{H}^2 - \mathcal{H}'}\Phi = 0$$

Assume: $a(t) \sim t^{\frac{2}{3(1+w)}}$ solution: $\Phi_k = D + S(\pm\eta)^{2\nu}$ where $\nu \equiv -\frac{5+3w}{2(1+3w)}$

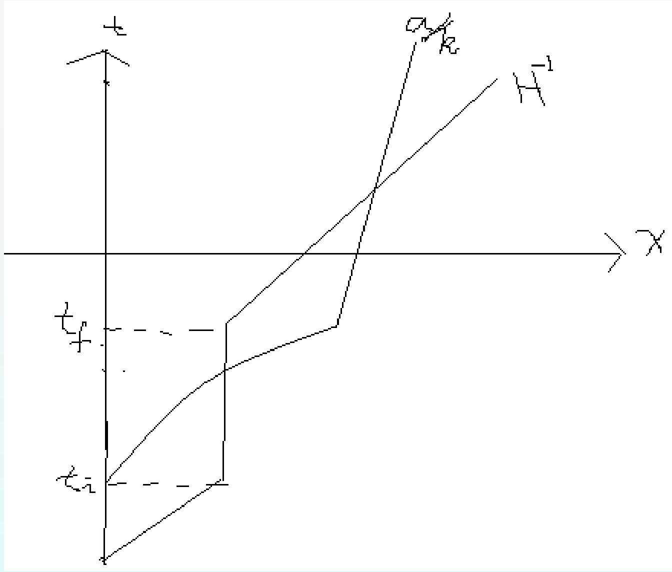
Curvature perturbation: $\zeta_k \equiv \Phi_k + \frac{1}{\epsilon}(\Phi_k + \frac{\Phi'_k}{\mathcal{H}})$

The spectrum: $\mathcal{P}_\zeta = \frac{H^2}{64\pi^3 M_{pl}^2 \epsilon}$ with index: $n_s = 1 - 6\epsilon + 2\xi$

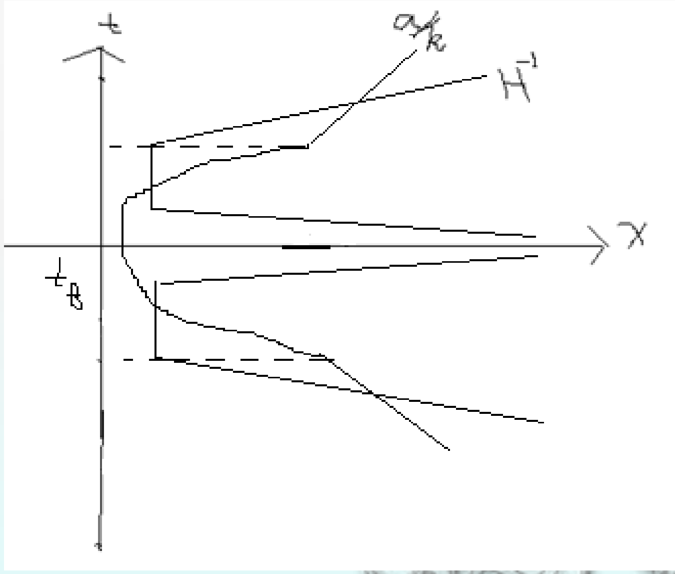
Inflation: $\epsilon, \xi \ll 1$ $n_s \simeq 1$

The differences between perturbations in inflationary and bounce cosmologies

1.There is pre-evolution in contracting time, when horizon was crossed



Inflationary cosmology



bounce cosmology

The differences between perturbations in inflationary and bounce cosmologies

2. Evolutions of different stages are connected via matching conditions

Deruelle-Mukhanov matching conditions → $D_{+} = AD_{-} + Bk^2(S_{+} - S_{-})$

J. c. Hwang and E. T. Vishniac, Astrophys. J. 382, 363 (1991);
N. Deruelle and V. F. Mukhanov, Phys. Rev. D 52, 5549 (1995);
R. Brandenberger and F. Finelli, JHEP 0111, 056 (2001).

3. Thermodynamic generation of the perturbations

J. Magueijo and L. Pogosian, Phys. Rev. D 67, 043518 (2003);

J. Magueijo and P. Singh, Phys. Rev. D 76, 023510 (2007).

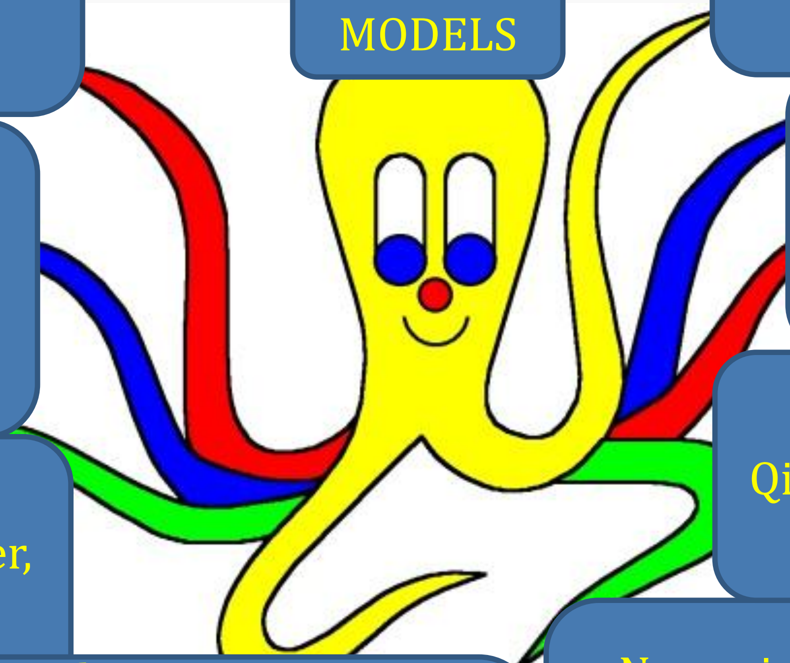
The Zoo of Bounce models

Bounce + Large field inflation
Cai, Qiu, Brandenberger, Piao, Zhang,
JCAP 0803: 013,2008.

Bounce + Small field inflation
Cai, Qiu, Xia, Zhang, .
Phys.Rev.D79: 021303,2009.

Lee-Wick Bounce
Cai, Qiu, Brandenberger, Zhang, Phys.Rev.D80: 023511,2009

BOUNCE MODELS



Holographic Bounce
Cai, Xue, Brandenberger, Zhang, JCAP 0906: 037,2009.

Radiation Bounce
Karouby, Qiu, Brandenberger,
Phys.Rev.D84:043505,2011.

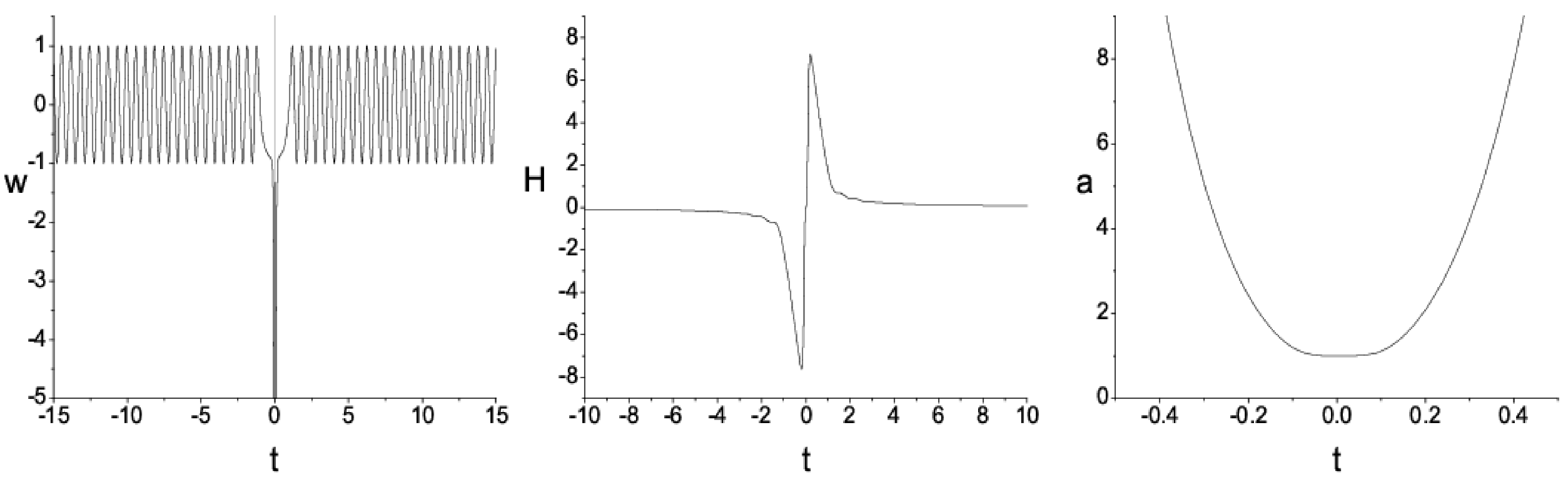
Galileon Bounce
Qiu, Evslin, Cai, Li, Zhang,
JCAP 1110:036,2011.

Non-minimal coupling Bounce
Qiu, Yang, JCAP 1011: 012, 2010;
Qiu, Class.Quant.Grav.27: 215013, 2010.

Others:
Bouncing in Modified Gravity
New Ekpyrotic model
K-Bounce
…………

A Bounce Scenario with Scale-invariant Power Spectrum: Matter Bounce

Background parameters:



Y. F. Cai, T. t. Qiu, R. Brandenberger and X. m. Zhang, Phys. Rev. D 80, 023511 (2009)

Perturbations in Matter Bounce

1. Analytical Analysis:

Perturbed Einstein Equations:

$$\Phi'' - c_s^2\nabla^2\Phi + \frac{2\mathcal{H}^3 - 4\mathcal{H}\mathcal{H}' + \mathcal{H}''}{\mathcal{H}^2 - \mathcal{H}'}\Phi' + \frac{\mathcal{H}\mathcal{H}'' - 2\mathcal{H}'^2}{\mathcal{H}^2 - \mathcal{H}'}\Phi = 0$$

Initial condition: Bunch-Davies vacuum $\Phi_i \sim \frac{1}{\sqrt{2k^3}}e^{-ik\eta}$

In the matter-dominant era: $w = 0$

Solution: $\Phi_k = D + S(\pm\eta)^{2\nu}$ with $\nu = -\frac{5}{2}$

Near the bounce point: $\mathcal{H} \simeq \alpha(\eta - \eta_B)$

Solution: $\Phi_k^b = e^{-y(\eta-\eta_B)^2}\left\{E_k H_l[\sqrt{y}(\eta - \eta_B)] + F_k\,{}_1F_1[-\frac{l}{2}, \frac{1}{2}, y(\eta - \eta_B)^2]\right\}$

where $y = \frac{12}{\pi}\alpha a_B^2$ $l = -\frac{2}{3} + \frac{c_s^2k^2}{2y}$

Perturbations in Matter Bounce

1. Analytical Analysis:

Before bounce:

$$\Phi_k^c = \bar{D}_- + \frac{\bar{S}_-}{(\eta - \tilde{\eta}_{B-})^{2\nu_c}}$$

After bounce:

$$\Phi_k^e = \bar{D}_+ + \frac{\bar{S}_+}{(\eta - \tilde{\eta}_{B+})^{2\nu_e}}$$

Matching condition: *(Deruelle-Mukhanov)*

$$\hat{E}_k\sqrt{y}(\eta_{B-} - \eta_B) = -\left(\frac{1}{3} + 2l\right)\Phi_k^c - \hat{\zeta}_k^c|_{B-},$$
$$\hat{F}_k = \left(\frac{4}{3} + 2l\right)\Phi_k^c + \hat{\zeta}_k^c|_{B-}.$$

$$\hat{E}_k\sqrt{y}(\eta_{B+} - \eta_B) = -\left(\frac{1}{3} + 2l\right)\Phi_k^e - \hat{\zeta}_k^e|_{B+},$$
$$\hat{F}_k = \left(\frac{4}{3} + 2l\right)\Phi_k^e + \hat{\zeta}_k^e|_{B+}.$$

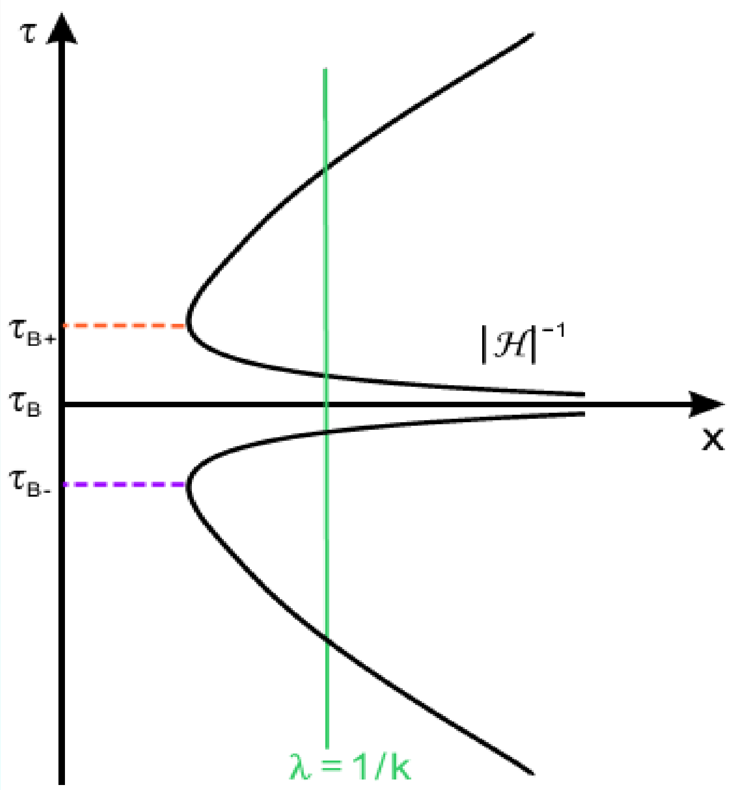
Result: (nearly) scale-invariant power spectrum:

$$\mathcal{P}_\zeta \simeq \frac{\rho_{B-}}{(20\pi)^2 M_p^4}$$

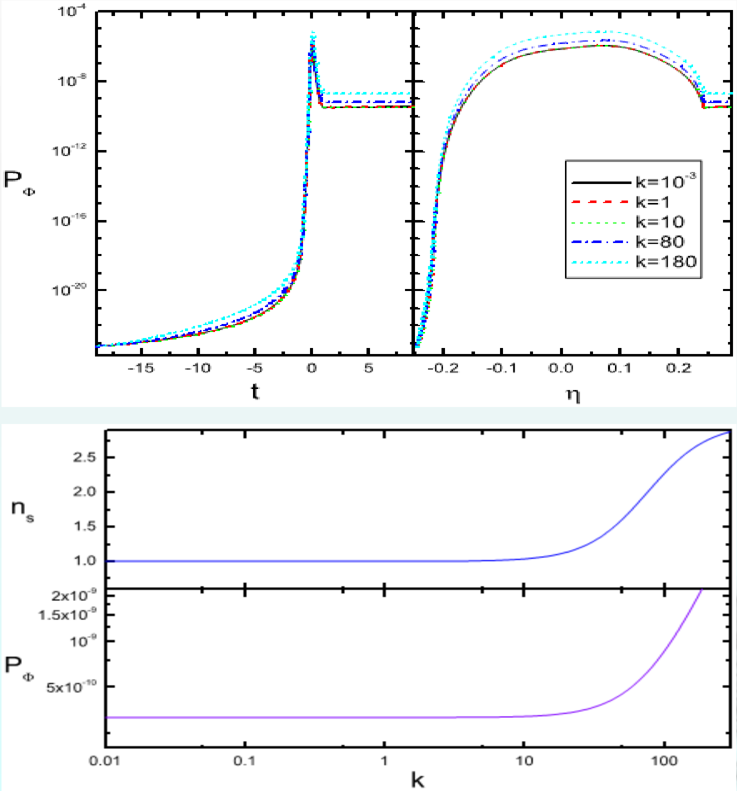
Perturbations in Matter Bounce

2. Numerical Calculation:

Sketch plot of perturbation:



Power spectrum and index:



Summary on bouncing cosmology

- Can solve the singularity problem as well as other problems that are encountered by Big Bang theory;
- Have different evolution mechanisms of perturbations from inflationary cosmology;
- Can give rise to scale-invariant power spectrum of primordial perturbations.

Thank you!!!