EMERGENT LORENTZ SYMMETRY AND DOUBLY SPECIAL RELATIVITY

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Introduction and Outline

Theme of EMERGENT SYMMETRIES:

* **Gauge symmetries as emergent symmetries** ('t Hooft, B.L.Hu, etc.)
- after all a **symmetry at large** may be not a **symmetry at small** : Planck scale symmetries may be different from the usual ones

* **An Example: LORENTZ SYMMETRY**

Lorenz Invariance has shaped the Physics of XX Century
*Relativity, Dirac Equation, QFT, Standard Model,...**

But... is it an Exact symmetry for any Arbitrary Boost???

Lorenz group has an infinite volume

Reasons for Lorenz Invariance violations:

- UV divergences, Landau poles,...
- Various Quantum Gravity approaches:
- Space-time foam
- Loop Quantum Gravity
- GUP → Minimum length

* DSR program incorporates seriously the idea of a minimum length.

Using Relativistic Quantum Mechanics (RQM), [formulated via path integrals] →

+o+ Lorenz Symmetry broken at short space-temporal scales,

+o+ yet emerges as an exact symmetry at large scales.

Relativistic Path Integral written as superposition of non relativistic path integrals:

* * On short spatial scale (**L** << λ_c Compton length) a single particle follows a non-relativistic Brownian motion (Wiener process) with a fluctuating Newtonian mass. The particle moves as if it randomly propagates (in the sense of Brownian motion) through a granular or ``polycrystalline" medium.

** On large spatial scale (**L** >> λ_c particle's Compton length), the particle evolves according to a genuine relativistic motion, with a sharp value of the mass coinciding with the Einstein rest mass.

Extension to Doubly Special Relativistic (DSR) models.

* DSR in a Nutshell: In DSR we have 2 invariant scales: the speed of light c, and a length *l*, assumed typically of order of the Planck length.

* In the present framework, the scale *l* is naturally identified with the minimal grain size of the polycrystalline medium. From the structure of paths in the Feynman summation, as in SR, one can compute correlation lengths and canonical commutation relations .

Emergent Special Relativity (single free particle – no interaction)

$$\int_{\boldsymbol{x}(0)=\boldsymbol{x}'}^{\boldsymbol{x}(t)=\boldsymbol{x}} \mathcal{D}x \frac{\mathcal{D}p}{(2\pi)^D} \exp\left\{\int_0^t \mathrm{d}\tau \left[\mathrm{i}p\cdot\dot{x} - c\sqrt{p^2+m^2c^2}\right]\right\}$$

$$= \int_0^\infty \mathrm{d}\tilde{m}\, f_{\frac{1}{2}}(\tilde{m}, tc^2, tc^2m^2) \int_{\boldsymbol{x}(0)=\boldsymbol{x}'}^{\boldsymbol{x}(t)=\boldsymbol{x}} \mathcal{D}x \frac{\mathcal{D}p}{(2\pi)^D} \exp\left\{\int_0^t \mathrm{d}\tau \left[\mathrm{i}p\cdot\dot{x} - \frac{p^2}{2\tilde{m}} - mc^2\right]\right\}$$

where

$$f_{\frac{1}{2}}(\tilde{m}, tc^2, tc^2m^2) = \sqrt{\frac{c^2t}{2\pi\tilde{m}}}\; \mathrm{e}^{-tc^2(\tilde{m}-m)^2/2\tilde{m}}$$

This is a special case of a more general formula derived in the framework of probabilities formulated as superposition of path integrals (= so called "superstatistics") .

Commutators

$$[\hat{x}_j, \hat{p}_i]_{\mathrm{SR}} = \mathrm{i}\left(\delta_{ij} + \frac{\hat{p}_i\hat{p}_j}{m^2c^2}\right)$$

*The time interval **t** is understood as the time after which the observation (position measurement) is performed.*

*During the time interval **t** the system remains unperturbed.*

Physical interpretation

The structure of the previous identity implies that $\tilde{m}$ can be interpreted as a Newtonian mass which takes on continuous values distributed according to $f_{\frac{1}{2}}(\tilde{m}, tc^2, tc^2m^2)$

with $\langle\tilde{m}\rangle = m + 1/tc^2$ and $\mathrm{var}(\tilde{m}) = m/tc^2 + 2/t^2c^4$

Heuristic interpretation:

Single-particle relativistic th. can be viewed as a single-particle non-relativistic th. whose Newtonian mass $\tilde{m}$ represents a fluctuation par. approaching on average the Einsteinian rest mass m in the large t limit.

In the long run all mass fluctuations are washed out and only a sharp, time-independent, effective Lorentz invariant mass is perceived.
Time scale of this process from the form of $\tilde{m}$:

$$t \sim 1/mc^2$$

This is the Compton time **t_C.**

Further physical implication

when $t >> 1/mc^2$ then $\tilde{m}$ rapidly converges to the relativistic value m:
the motion becomes genuinely relativistic at large times.

For $t >> 1/mc^2 \rightarrow m > 1/tc^2$, which means $mc^2 t > 1$.
That is , for large t, the **relativistic Heisenberg inequality** for the energy/time variables is **satisfied**,

$$\Delta E \Delta t \geq 1$$

For $t << 1/mc^2$,
the fluctuations of Newtonian mass $\tilde{m}$ around the average *m* are **huge**.
The motion seems to happen **inside a specific space region (a "space-grain")**, and in each space-grain the motion is a classical, i.e. non relativistic, Brownian motion controlled by the Newtonian Hamiltonian $p^2 / (2\tilde{m})$.
In fact there, the **relativistic** Heisenberg uncertainty relation is clearly violated,
$mc^2 t < 1$ (remind that *m* is the Einstein relativistic mass).

However, if we compute the **non-relativistic** Heisenberg relation, using the Newtonian mass $\tilde{m}$ and the non-relativistic **kinetic energy** $E \approx \tilde{m} v^2$, we find that Heisenberg relation is not violated. → the motion is **NON RELATIVISTIC !**

•**Fluctuations** of the Newtonian mass happen AS IF the particle evolves in an ``inhomogeneous'' or a **``polycrystalline''** medium.

• **Granularity** → → corrections in the **local dispersion relation**

→ → **alterations in the local effective mass**.

The following picture emerges:

* **On the short-distance scale** (fast-time process, time scale ≈ $1/mc^2$ ≈ Compton time t_C), a non-relativistic particle propagates through a single grain with a local mass ***m̃***, in a classical Brownian motion.

* **For time scales >> t_C** (large-distance scale), each space grain encountered by the particle endows it with a mass m̃, with probability given by the function f_1/2 .

The smearing distribution **f_1/2(…) → δ(m̃ – m)** for **t → ∞**

So, at large times, the particle acquires a sharp mass equal to Einstein's mass (i.e., Lorentz invariant).

We may also observe that the averaged (or coarse-grained) velocity over the correlation time $t = 1/mc^2$ equals the speed of light c. In fact

$$\langle |v| \rangle_{t=1/mc^2} = \left. \frac{\langle |p| \rangle}{\langle \tilde{m} \rangle} \right|_{t=1/mc^2}$$

$$= \frac{1}{2m} \int_0^{\infty} \mathrm{d}\tilde{m}\, f_{\frac{1}{2}}(\tilde{m}, 1/m, m) \sqrt{\left(\frac{8m\tilde{m}c^2}{\pi} \right)} = c\,.$$

For paths L >> λc (t >> t_C) the particle net velocity is < C (as it **should be** for massive relativistic particles!)

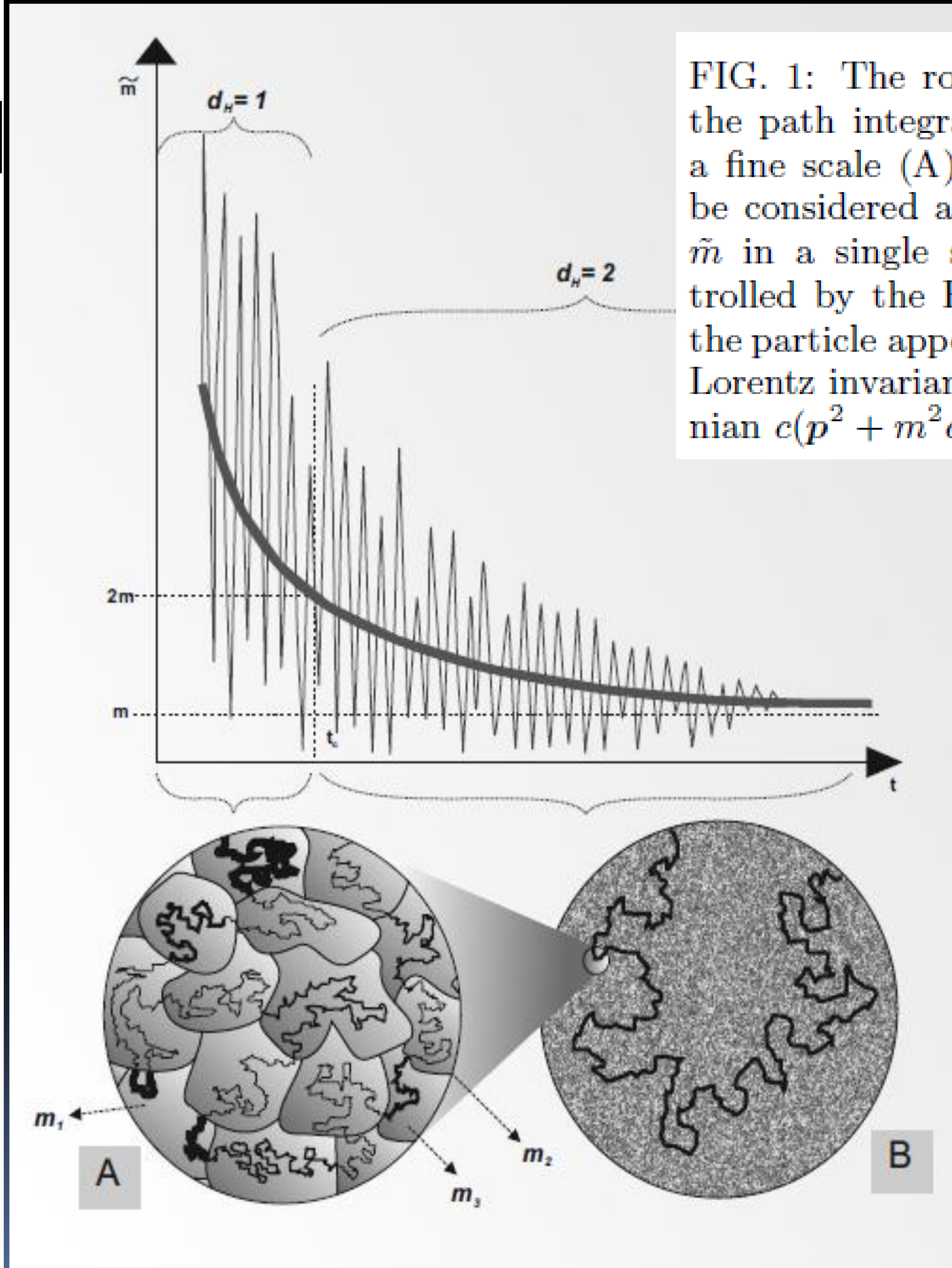

FIG. 1: The roughness of the representative trajectories in the path integral depends on a spatial/temporal scale. On a fine scale (A), where $t \ll t_c$ (or $\ell \ll \lambda_C$) a particle can be considered as propagating with a sharp Newtonian mass $\tilde{m}$ in a single spatial grain with a Brownian motion controlled by the Hamiltonian $p^2/2\tilde{m}$. On a coarser scale (B) the particle appears to follow a Brownian process with a sharp Lorentz invariant mass m, driven by the relativistic Hamiltonian $c(p^2 + m^2c^2)^{1/2}$.

The Hausdorff dimensions of representative trajectories in Path Integral:

* L < λc Compton → d_H=1 (super-diffusive process).

* L > λc Compton → d_H=2 (Brownian diffusion).

Emergent Doubly Special Relativity

DSR tries to implement a second invariant, besides the speed of light, into the transformations among inertial reference frames .
This new invariant comes directly from the research in Quantum Gravity, and it is assumed to be an observer-independent length-scale --- the Planck length l_P, or its inverse, i.e., the Planck energy $E_P = c / l_P$.

Connection between DSR and Special Relativity

When microstructure of space-time is considered,
then Special Relativity or DSR seem to emerge from particular choices of such microstructure itself, and from classical Hamiltonian mechanics.

DSR 1 Deformed Dispersion Relation

$$\frac{\eta^{ab} p_a p_b}{(1 - \ell_p p_0)^2} = m^2 c^2$$

Physical Hamiltonian ***H = c p₀***, generator of the temporal translations in respect to coordinate time ***t***.

$$\bar{H} = c \, \frac{-m^2 c^2 \ell \mp \sqrt{p^2 (1 - m^2 c^2 \ell^2) + m^2 c^2}}{1 - m^2 c^2 \ell^2}$$

SUPERSTATISTICS IDENTITY

$$\int_{\boldsymbol{x}(0)=\boldsymbol{x}'}^{\boldsymbol{x}(t)=\boldsymbol{x}} \mathcal{D}x \frac{\mathcal{D}p}{(2\pi)^D} \exp\left\{ \int_0^t d\tau \left[ip\cdot\dot{x} \;+\; c\frac{\left(m^2c^2\ell - \sqrt{p^2(1-m^2c^2\ell^2)+m^2c^2}\right)}{(1-m^2c^2\ell^2)} \right] \right\}$$

$$= \int_0^\infty d\tilde{m}\, f_{\frac{1}{2}}(\tilde{m}, tc^2\lambda, tc^2m^2\lambda) \int_{\boldsymbol{x}(0)=\boldsymbol{x}'}^{\boldsymbol{x}(t)=\boldsymbol{x}} \mathcal{D}x \frac{\mathcal{D}p}{(2\pi)^D} \exp\left\{ \int_0^t d\tau \left[ip\cdot\dot{x} - \frac{p^2}{2\tilde{m}} - E_0 \right] \right\},$$

where $\lambda = \frac{1}{1-m^2c^2\ell^2}$ Deformation parameter $E_o = \frac{mc^2}{1+mc\ell}$ Particle rest energy

Averaged Newtonian mass $\langle\tilde{m}\rangle = m + 1/(tc^2\lambda)$

The fluctuating Newtonian mass $\tilde{m}$ converges rapidly, for long times t, to the **DSR1** invariant rest mass m. The rate of convergence is controlled also by the parameter λ.

Since

$$\lambda = 1/(1 - E^2/E_p^2).$$

then $\tilde{m}$ can converge rapidly to the DSR1 value m, even at short times, provided that the particle's energy E be close to the Planck energy E_p. The correlation distance (= typical size of a space grain) is now given by 1/(m c λ), and since λ > 1, then **1/(m c λ) < 1/(m c)** always.

Commutators

$$[\hat{x}_i, \hat{p}_j]_{\mathrm{DSR1}} = \mathrm{i}\left(\delta_{ij} + \frac{\kappa^2 - m^2c^2}{\kappa^2 m^2 c^2}\,\hat{p}_i\hat{p}_j\right)$$

with

$$\kappa = 1/\ell.$$

DSR 2 Deformed Dispersion Relation

$$\frac{p_0^2 - \mathrm{p}^2}{1-(\ell_p p_0)^2} = m^2 c^2$$

Physical Hamiltonian $H = c\, p_0$.

$$\bar{H} = \pm \frac{\sqrt{p^2c^2 + m^2c^4}}{\sqrt{1+m^2c^2\ell^2}}$$

SUPERSTATISTICS IDENTITY

$$\int_{\boldsymbol{x}(0)=\boldsymbol{x}'}^{\boldsymbol{x}(t)=\boldsymbol{x}} \mathcal{D}x \frac{\mathcal{D}p}{(2\pi)^D} \exp\left\{ \int_0^t d\tau \left[ip\cdot\dot{x} - \frac{\sqrt{p^2c^2+m^2c^4}}{\sqrt{1+m^2c^2\ell^2}} \right] \right\}$$

$$= \int_0^\infty d\tilde{m}\, f_{\frac{1}{2}}(\tilde{m}, tc^2\zeta^2, tc^2m^2) \int_{\boldsymbol{x}(0)=\boldsymbol{x}'}^{\boldsymbol{x}(t)=\boldsymbol{x}} \mathcal{D}x \frac{\mathcal{D}p}{(2\pi)^D} \exp\left\{ \int_0^t d\tau \left[ip\cdot\dot{x} - \frac{p^2}{2\tilde{m}} - \bar{E}_0 \right] \right\}$$

where $E_o = \frac{mc^2}{\sqrt{1+m^2c^2\ell^2}}$ Particle rest energy

$\zeta = 1/\sqrt{1+m^2c^2\ell^2}$ Deformation parameter

Averaged Newtonian mass $\langle \tilde{m} \rangle = \frac{m}{\zeta} + \frac{1}{tc^2\zeta^2}$

The DSR 2 model **does not have** the desired property that its fluctuating mass converges to a Lorentz mass in the large *t* limit.
In addition, since $\zeta \in (0,1)$, the fluctuations at short times cannot be suppressed and one cannot hope to have a relativistic system with a sharp Einstein mass at the Planck energy.

Commutators

$$[\hat{x}_j, \hat{p}_i]_{\mathrm{DSR2}} = \mathrm{i}\left(\delta_{ij} + \frac{\hat{p}_i \hat{p}_j}{m^2 c^2}\right)$$

These CCR coincides with the SR commutator.
This fact is not surprising, since CCR's directly reflect the roughness of the representative paths and we know that the fractal dimension of the DSR2 system coincides with that of SR.

REMARKS

- The presented concept of **statistical emergence** of SR and DSR offers insights into the Planck-scale structure of space-time.
- The existence of a discrete polycrystalline substrate could be welcomed in various quantum gravity constructions.
- The discrete structure of space and time (predicted by many Quantum Gravity models) can cure classical singularities.

  (See e.g. Loop Quantum Cosmology, space-time foam).

Open Questions

- **Extension to Interacting Systems**
- Deeper understanding of a **dynamical origin** of our smearing functions f_1/2
- A **small departure** from the standard form of **$f_{1/2}$** leads **from** Lorenz symmetry **to** DSRs symmetries
- Lorenz symmetry is not fundamental: is controlled by the specific form of **$f_{1/2}$** **(grain distribution)**
- Extension to curved space-times
- Connection with Horava-Lifshitz gravity (space and time are not equivalent at the fundamental level)

SUMMARY

- Both SR and DSR systems can arise by statistically coarse-graining underlying non relativistic (Wiener) process, making the latter more fundamental and the former emergent.

- The coarse-graining arises from a superposition of **two** stochastic processes.

- On a short spatial scale (shorter than particle's Compton wavelength) the particle moves according to a Brownian, non-relativistic, motion. Its Newtonian mass fluctuates according to an inverse Gaussian distribution.

- The averaged (or coarse-grained) velocity over the Compton time is the light velocity **c**.

- On a time scale larger than the Compton time, the particle behaves as a genuine relativistic particle, with a sharp mass equal to Einstein (i.e., Lorentz invariant) mass. In this case a massive particle moves with a net velocity **smaller** than **c**.