From Gravity to Fluid

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Outline

1. Motivation and overview
   - Holography (bulk/boundary correspondence)
   - In equilibrium: thermodynamics and phase transition
   - In non-equilibrium: transportation and entropy production
2. From gravity to fluid
   - The gravity/fluid case
   - Non-relativistic long-wavelength expansion on an arbitrary cutoff surface
   - Incompressible Navier-Stokes equations from Petrov-like condition

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Holography: a brief introduction

- Early (rough) ideas of holography
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$$Z_{B^{d+1}}[\bar{\phi}] = \int D\psi \exp(-I_{\mathrm{CFT}}[\bar{\phi},\psi])$$

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Holography: a brief introduction

- More info from superstring theory
  Classical limit $\leftrightarrow$ Large N_c limit
  Weak coupling $\leftrightarrow$ Strong coupling
- Generalization: bulk/boundary correspondence
  AdS/QCD(色即是空), AdS/CMT, HEE, gravity/fluid, ...

$$Z_{\rm bulk}[\bar{\phi}] = \int D\psi \exp(-I_{\rm FT}[\bar{\phi}, \psi])$$

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Basic dictionary: $\phi|_{\rm bdry} \leftrightarrow$ Non-dynamical field $\bar{\phi}$

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The bulk/boundary correspondence

- Under the classical approximation of the bulk gravity,

$$Z_{\rm bulk}[\bar{\phi}] \to \exp(-I_{\rm bulk}[\bar{\phi}]) \Longrightarrow$$

$$\exp(-I_{\rm bulk}[\bar{\phi}]) = \int D\psi \exp(-I_{\rm FT}[\bar{\phi}, \psi])$$

with $I_{\rm bulk}[\bar{\phi}]$ the on-shell action (Hamilton's principal function).

- Variation with respect to $\bar{\phi}$ gives

$$-\frac{\delta I_{\rm bulk}[\bar{\phi}]}{\delta \bar{\phi}(x)} = \langle \mathcal{O}_\phi(x) \rangle_{\rm FT}, \qquad \mathcal{O}_\phi = -\frac{\delta I_{\rm FT}[\bar{\phi}, \psi]}{\delta \bar{\phi}(x)}$$

- Further variations give the correlations of $\mathcal{O}_\phi$ on the boundary.

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The bulk/boundary correspondence

- Important examples (with n^μ the unit normal of the boundary)

Fields	Bulk	Boundary
Electromagnetic	$-n_\mu F^{\mu a}\|_{\mathrm{bdry}}$	Current $\langle J^a \rangle$
Gravitational	Brown-York $t^{ab}\|_{\mathrm{bdry}}$	Stress tensor $\langle T^{ab} \rangle$

- Additional dictionary
  Black holes $\leftrightarrow$ Thermal field theory
  Local Hawking temperature $\leftrightarrow$ Temperature
- Holographic renormalization group (RG) flow
  Position of the boundary $\leftrightarrow$ Energy scale
  Black hole horizon $\leftrightarrow$ IR limit

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A simple example

- On an arbitrary cutoff for the Schwarzschild-AdS black brane

$$ds_{d+1}^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 dx^2, \qquad f(r) = \frac{r^2}{l^2} - \frac{2m}{r^{d-2}}$$

$$ds_d^2 = -f_c dt^2 + r_c^2 dx^2, \qquad f_c := f(r_c)$$

$dE + pdV = TdS$ (the 1st law of thermodynamics)
$E + pV = TS$ (the Gibbs-Duhem relation)

$$\Longrightarrow \begin{cases} dp = sdT \\ \varepsilon + p = Ts \\ d\varepsilon = Tds \end{cases}$$

Thermodynamics

- The Brown-York tensor

$$t^{ab} = \frac{1}{8\pi G}(K\gamma^{ab} - K^{ab})$$

has a form of the (relativistic) ideal fluid:

$$t^{ab} = \varepsilon u^a u^b + p h^{ab}, \qquad h^{ab} = \gamma^{ab} + u^a u^b$$

s: Bekenstein-Hawking entropy density
T: local Hawking temperature
The thermodynamic relations hold.

Generalization

- The Gauss-Bonnet case

$$
\begin{aligned}
I &= \frac{1}{16\pi G}\int d^{d+1}x\sqrt{-g}(R-2\Lambda+\alpha\mathscr{L}_{GB}),\\
\mathscr{L}_{GB} &= R^2-4R_{\mu\nu}R^{\mu\nu}+R_{\mu\nu\sigma\tau}R^{\mu\nu\sigma\tau}
\end{aligned}
$$

with much more complicated Brown-York-like boundary tensor but the same thermodynamic relations.

- The charged case and chemical potential

$$
E+pV=TS+\mu Q, \qquad q=\frac{Q}{r_c^{d-1}}
$$

$$
d\varepsilon=Tds+\mu dq, \qquad \mu=-\frac{d-1}{8\pi G}\frac{Q}{\sqrt{f_c}}\left(\frac{1}{r_c^{d-2}}-\frac{1}{r_h^{d-2}}\right)
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The physical picture

- The physical picture
  Bulk: black holes that eat everything
  Boundary: transportation that smoothes everything

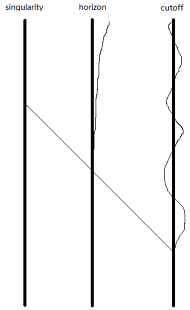

Figure: A sketch map

Transportation

- Linear response theory
- Example 1: Ohm's law

$$J^i = \sigma E^i$$

- Example 2: Newton's law of viscosity

$$T^{xy} = -2\eta\sigma^{xy}$$

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Entropy production

Y. Tian, X.-N. Wu and H.-B. Zhang, in preparation.

- Macroscopic verification of the bulk/boundary correspondence (poor man's way to holography)

Type	Driving force	Entropy production
Heat conduction	Temperature gradient	-
Viscosity	Velocity gradient	Friction heat
Electric condution	Electric field	Joule heat

Table: Transport processes

Entropy production

- The boundary side
  The entropy production rate

$$\Sigma = j_q^i \nabla_i \frac{1}{T} - \frac{1}{T}\Pi^{ij}\sigma_{ij} + \frac{1}{T} j^i E_i$$

- The bulk side
  The entropy variation

$$\delta S = \frac{\delta M}{T_H}$$

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The gravity/fluid case: basics

- Einstein equations:

$$\begin{cases} G^{rb} = 0 \Longrightarrow \nabla_a t^{ab} = 0 & \text{(momentum constraint)} \\ G^{rr} = 0 \Longrightarrow d t^a_b t^b_a = t^2 & \text{(Hamiltonian constraint)} \end{cases}$$

- Stress-energy tensor of a relativistic fluid:

$$t_{ab} = \varepsilon u_a u_b + p h_{ab} - 2\eta \sigma_{ab} + \cdots$$

- Under the non-relativistic limit for $\varepsilon = \text{const}$ (incompressible),

$$\nabla_a t^{ab} = 0 \Longrightarrow \begin{cases} \partial_i v^i = 0 & (b = t) \\ \partial_t v^i + \mathbf{v} \cdot \nabla v^i + \partial_i P - \nu \nabla^2 v^i = 0 & (b = i) \end{cases}$$

(incompressible Navier-Stokes equations)

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Gravitational perturbation for flat cutoff surface

- Long-wavelength expansion for the gravitational perturbation $\leftrightarrow$ Derivative expansion for the dual fluid

- The non-relativistic scaling:

$$\partial_t \sim \varepsilon^2, \qquad \partial_i \sim \varepsilon, \qquad \partial_r \sim 1, \qquad P \sim \varepsilon^2, \qquad v^i \sim \varepsilon$$

How to reduce the gravitational DoF to the dual fluid DoF?

- Dirichlet-type boundary condition on the cutoff surface (γ_{ab} kept fixed)
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The Rindler case

I. Bredberg, C. Keeler, V. Lysov and A. Strominger, [arXiv:1101.2451].

- The ingoing Rindler metric:

$$ds^2_{d+1} = -rd\tau^2 + 2drd\tau + dx^2$$

$$ds^2_d = -r_c d\tau^2 + dx^2$$

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- The perturbed metric solving the bulk Einstein equation up to $\mathcal{O}(\varepsilon^2)$ provided $\partial_i v^i = 0$ (incompressibility).
- The corresponding Brown-York tensor t_{ab} can be computed, which takes a form as the stress-energy tensor of incompressible fluid with viscosity η. Moreover, the regularity condition requires $\frac{\eta}{s} = \frac{1}{4\pi}$ (independent of r_c).
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Other cases

- The AdS black brane case
  R.-G. Cai, L. Li and Y.-L. Zhang, JHEP 1107 (2011) 027 [arXiv:1104.3281].

- The charged AdS black brane case
  C. Niu, Y. Tian, X.-N. Wu and Y. Ling, [arXiv:1107.1430].
  The discussion is also extended to the Gauss-Bonnet case with $\frac{\eta}{s} = \frac{1}{4\pi}\{1-2(d-3)\alpha[d-(d-2)q_h^2]\}$ (independent of r_c but dependent on $q_h = \frac{Q}{r_h^{d-1}}$).
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Lysov-Strominger's basic ideas

V. Lysov and A. Strominger, [arXiv:1104.5502].

- The Brown-York tensor (or extrinsic curvature) is directly taken as fundamental variables.
- The boundary condition for the conformal factor of the intrinsic metric can be Dirichlet-type or Neumann-type.
- Reduction of the DoF by the Petrov-like condition

$$C_{(\ell)i(\ell)j} = \ell^\mu \ell^\nu C_{\mu i \nu j} = 0, \qquad \ell = \frac{\partial_0 - n}{\sqrt{2}}$$

on the boundary with n its unit normal.

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The Rindler case with Dirichlet-type boundary condition

- $ds^2_{d+1} = -rdt^2 + 2drdt + dx^2 \Longrightarrow ds^2_d = -r_c dt^2 + dx^2$
- $ds^2_d = -\frac{d\tau^2}{\lambda^2} + dx^2, \qquad r_c = \lambda^2 \Longrightarrow \tau = \lambda^2 t$
- $\lambda \to 0 \Longrightarrow K = \frac{1}{2\lambda} \to \infty$
- Express everything in terms of t^τ_τ, t^τ_i and t^i_j.
- Expand t^τ_τ, t^τ_i and t^i_j in powers of λ, then the Petrov type I condition gives the incompressible Navier-Stokes equations, upon identifying $t^{\tau(1)}_i = \frac{1}{2}v^i$ and $t^{(1)} = \frac{d-1}{2}P$.

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T. Huang, Y. Ling, W. Pan, Y. Tian and X. Wu, JHEP 1110 (2011) 079 [arXiv:1107.1464].
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- Lysov-Strominger's framework is refined/extended.
- A case of (roughly) mixed boundary condition is illustrated.
- Cases of intrinsically curved boundary are treated, with the incompressible Navier-Stokes equations in curved space obtained.
- The interesting cases of dual fluid on the Schwarzschild(-AdS) horizon and AdS black brane horizon are included.

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Key points for all cases

- The expressions of the Wely tensor components:

$$
\begin{aligned}
C_{abcd} &= {}^{d}R_{abcd} + K_{ad}K_{bc} - K_{ac}K_{bd} + \frac{2\Lambda}{p(p+1)}(\gamma_{ad}\gamma_{bc} - \gamma_{ac}\gamma_{bd}) \\
C_{abnc} &= \nabla_b K_{ac} - \nabla_a K_{bc} \\
C_{nanb} &= -{}^{d}R_{ab} + KK_{ab} - K_{ac}K^{c}_{b} + \frac{2\Lambda}{p+1}\gamma_{ac}
\end{aligned}
$$

- $\nabla_a t^a_b = 0 \Longrightarrow$
$$
\begin{cases}
D_i v^i = 0 & (b=\tau) \\
\partial_\tau v_i + v^k D_k v_i + D_i P - D^2 v_i - R^k_i v_k = 0 & (b=i)
\end{cases}
$$

- $\eta = \frac{1}{16\pi G} \Longrightarrow \frac{\eta}{s} = \frac{1}{4\pi}$

Key points for all cases

- The expressions of the Wely tensor components:

$$\begin{aligned} C_{abcd} &= {}^dR_{abcd} + K_{ad}K_{bc} - K_{ac}K_{bd} + \frac{2\Lambda}{p(p+1)}(\gamma_{ad}\gamma_{bc} - \gamma_{ac}\gamma_{bd}) \\ C_{abnc} &= \nabla_b K_{ac} - \nabla_a K_{bc} \\ C_{nanb} &= -{}^dR_{ab} + KK_{ab} - K_{ac}K^c_b + \frac{2\Lambda}{p+1}\gamma_{ac} \end{aligned}$$

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The end

Thank you!