GRAVITATIONAL LENSING IN A DARK MATTER FREE BRANEWORLD MODEL

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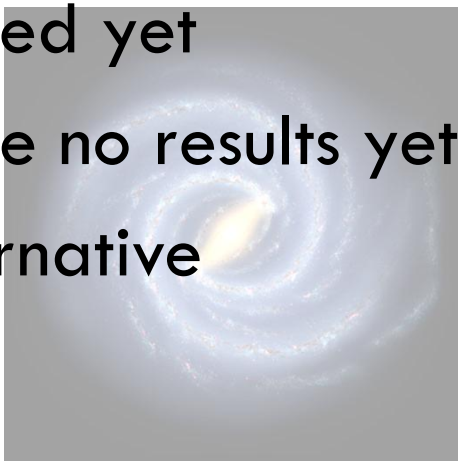
Outline of presentation

- Some background information
- Our model setting
- Gravitational Lensing in braneworld models
- The geometry of extra dimension

Status of dark matter problem

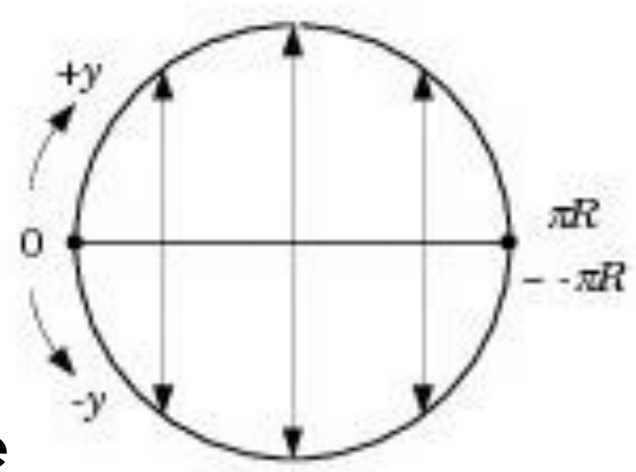
1. Evidences of dark matter always inferred from motion under gravitational interaction
2. No simple theoretical framework for dark matter

- Supersymmetry (SUSY) offers a beautiful solution
- Signal of Higgs candidate support SUSY
- Detection of sparticle has not succeeded yet
- Dark matter direct detections also give no results yet
- Behavior of sparticle is not clear, alternative approach cannot be ignored.



Randall-Sundrum models

- 2-brane model
- 2 brane located at end point of a orbifold, AdS5 volume in between



- 1-brane model – one of the brane located at infinity
- Showing a new way to hide extra dimension

Braneworld as modified 4D Einstein theory

- The matter on membrane infer discontinuity of extrinsic curvature (Israel, 1966)
- Project 5D Einstein equation to 4D (Shiromizu, Maeda, Sasaki, 2000)

$$R_{MN} + \frac{1}{2} g_{MN} R = k_5^2 T_{MN} \text{ in 5D}$$

$$\Downarrow$$

$$R_{\mu\nu} + \frac{1}{2} g_{\mu\nu} R = -\Lambda g_{\mu\nu} + \frac{k_5^4 \lambda}{6} T_{\mu\nu} + k_5^4 S_{\mu\nu} - E_{\mu\nu}$$

$$S_{\mu\nu} = -\frac{1}{4} T_{\mu\alpha} T_\nu^\alpha + \frac{1}{12} T T_{\mu\nu} + \frac{1}{8} g_{\mu\nu} T_{\alpha\beta} T^{\alpha\beta} - \frac{1}{24} g_{\mu\nu} T^2$$

The dark radiation $E_{\mu\nu}$

- A traceless tensor from projection of 5D Weyl tensor

$$E_{\mu\nu} = {}^{(5)} C_{ABCD} n^C n^D g_\mu^A g_\nu^B$$

- Weyl tensor = component of curvature that is not governed by Einstein equation
- It could be "sourced" by the brane via

$$\nabla^\mu S_{\mu\nu} = k_5^4 \nabla^\mu E_{\mu\nu}$$

- It is shown to be small so that Randall Sundrum model converge to Newtonian gravity
- Is it possible to sufficiently change the gravity? (e.g. by giving up the AdS5 geometry)

Model setting – A phenomenological approach

- Assume a corrected Schwarzschild metric

$$ds^2 = -\left(1 - \frac{r_b}{r} + B_t(r)\right) dt^2 + \frac{dr^2}{1 - \frac{r_b}{r} + B_r(r)} + r^2 d\Omega$$

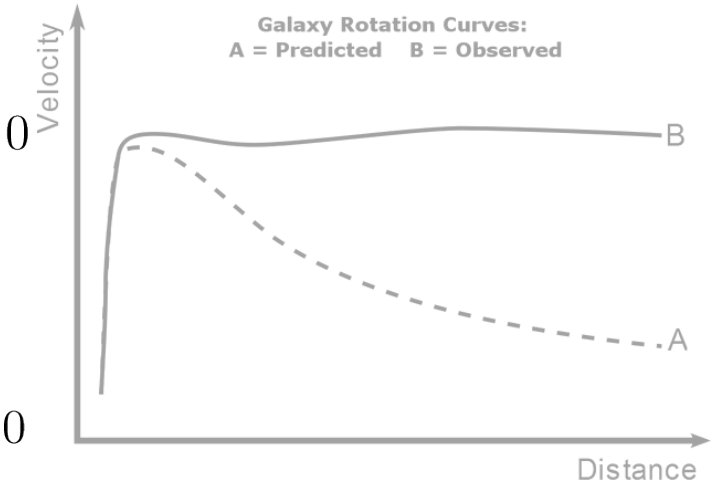
- Bt can be fixed by orbital velocity profile in rotation curves

$$v_{tg}^2 = \frac{r g_{tt}'}{2 g_{tt}}$$

- Obtain the differential equation of Br by traceless property of dark radiation, e. g.

$$B_t' - \frac{2v_{tg}^2}{r}\left(1 - \frac{r_b}{r} + B_t\right) + \frac{r_b}{r^2} = 0$$

$$(2 + v_{tg}^2) B_r' + \left(\frac{2}{r} + 4 v_{tg} v_{tg}' + \frac{2v_{tg}^2}{r} + \frac{2v_{tg}^4}{r}\right) B_r$$
$$+ v_{tg}^4 \left(\frac{2}{r} - \frac{2r_b}{r^2}\right) + v_{tg}^2 \left(\frac{2}{r} - \frac{r_b}{r^2}\right) + 4 v_{tg} v_{tg}' \left(1 - \frac{r_b}{r}\right) = 0$$



Constant velocity limit

- We consider a limit of flat rotation curve.
- In dark matter picture, the motion is supported by a Halo with r^{-2} density.
- In braneworld, the metric correction is given by

$$B_t(r) \to -1 + \frac{r_b}{r} + C_t r^{2v_\infty^2}$$

$$B_r(r) \to r^{-(v_\infty^2+1)} \left[C_r - r^{\frac{v_\infty^2}{2}} \left(v_\infty^2 r - r_b \right) \right]$$

- Test if these metric corrections contradict what dark matter predict

Motion in weak field low velocity limit

- For weak field and slow velocity, acceleration can be approximated by

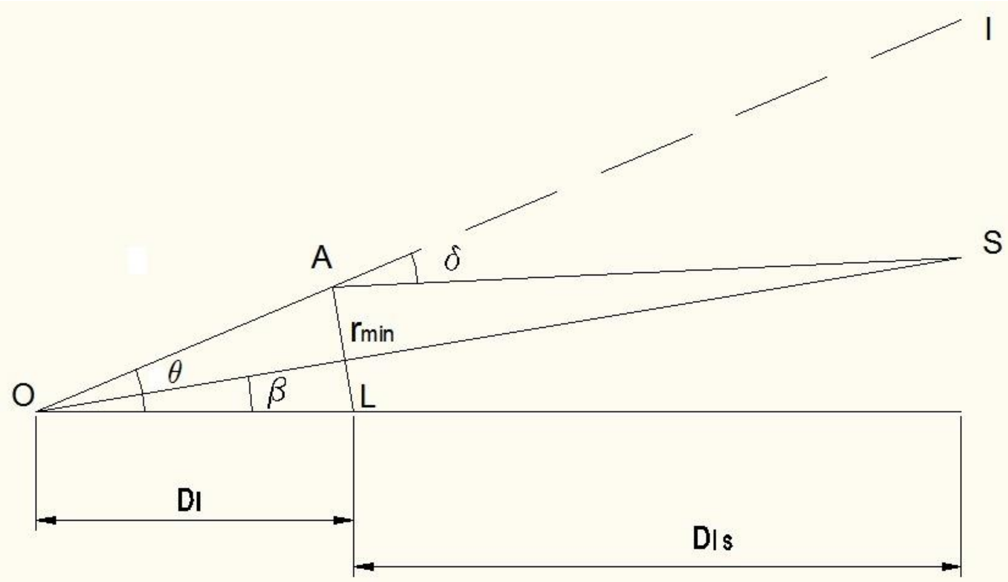
$$\frac{d^2x^\mu}{dt^2} = -\Gamma^\mu_{00}$$

- For radial in-falling motion

$$a = \frac{1}{2r}\left(1 - \frac{r_b}{r} + B_r\right)\left(\frac{r_b}{r} + rB_t'\right)$$
$$\approx \frac{v_\infty^2}{r} + \frac{r_b^2}{2r^3} - \frac{r_b B_r}{2r^2} \approx \frac{v_\infty^2}{r}$$

- Consistent with dark matter prediction
- Gravitational deflection of light?

A Simple lensing model



- Compare angle between asymptotes

$$\delta(r_{\min}) = 2\int_{r_{\min}}^{\infty} \mathcal{I} - \pi$$

$$\mathcal{I}(r) = \frac{1}{r}\left\{\frac{g_{rr}(r)}{\left[g_{tt}(r_{\min})/g_{tt}(r)\right](r/r_{\min})^2 - 1}\right\}^{1/2}$$

Series expansion of small parameters

- Small parameters $r_b/r, v_\infty^2$

$$\delta = -\int_{r_{\min}}^{\infty} \frac{r_{\min}^2 - r^2 + 2\ln(\frac{r_{\min}}{r})r^2}{(r^2 - r_{\min}^2)r} \sqrt{\frac{r_{\min}^2}{r^2 - r_{\min}^2}} v_\infty^2 dr$$

- Integrated to

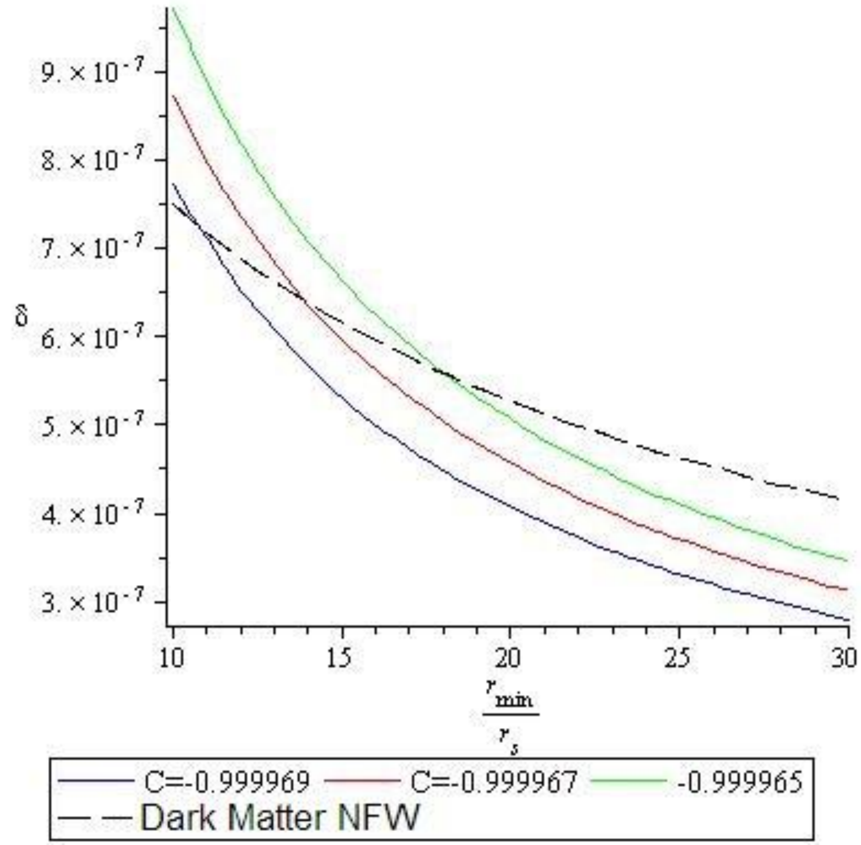
$$\delta = \frac{3\pi v_\infty^2}{2}$$

- For dark matter that produce the same rotation curve

$$\delta = 4v_\infty^2$$

Large distance behavior beyond constant velocity

- NFW model – dark matter distribution that motivated by gravitating n-body simulation
- If the rotation curve is well described by a Newtonian motion in NFW density halo
- It is general for Braneworld predict different deflection against dark matter



Energy and Pressure of $E_{\mu\nu}$

- Alternative approach – obtain theoretical form of dark radiation (Gergely, et al., 2011)
- Dark radiation view as energy U and pressure P of "some" fluid
- Guess equation of state based on Schwarzschild case in 2+1+1 decomposition

$$P = (a-2)\,U + \frac{B}{r^2}$$

LSB metric and Visible mass matching

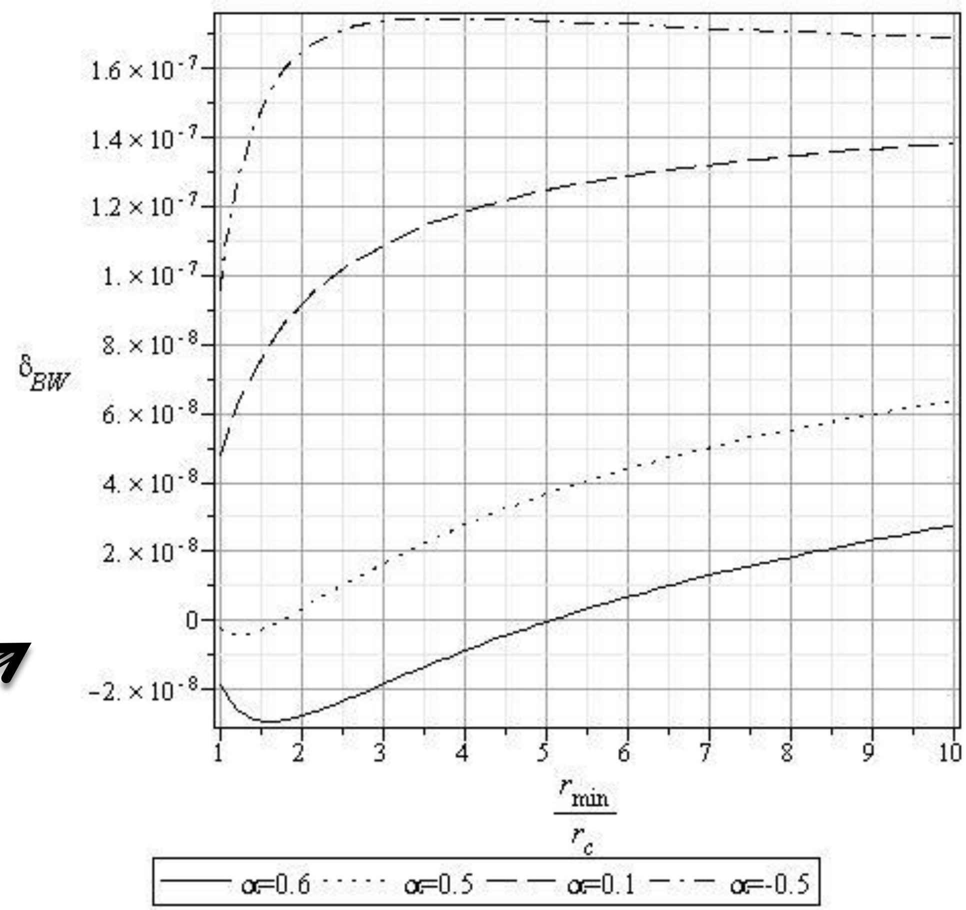
$$-g_{tt} \approx 1 - \frac{2GM}{c^2 r} + \frac{2\gamma}{1-\alpha}\left(\frac{r}{r_c}\right)^{\alpha-1}$$

$$g_{rr} \approx \left\{1 - \frac{2G(M+M_b)}{c^2 r} + \gamma\left[\left(\frac{r}{r_c}\right)^{\alpha-1} - 1\right]\right\}^{-1}$$

- α and γ depend on a and B
- M_b is a degenerate parameter in rotation curve
- r_c is obtained from matching Baryonic mass

Contribution to deflection angle

- The numerical integration of δ_{BW}



$$\gamma = 10^{-7}, M_b = 10^{-7} c^2 r_c / G$$

Solving the geometry of the bulk

- Key Question: Do we need a bulk blackhole to support the picture?

$$^{(5)}ds^2 = -M(r,y)^2 dt^2 + N(r,y)^2 dr^2 + Q(r,y)^2 d\Omega^2 + dy^2$$

- This metric gives dark radiation

$$E^t_t = \frac{N_{,y,y}}{N} + \frac{2Q_{,y,y}}{Q} - \frac{\Lambda_5}{2}$$
$$E^r_r = -\frac{N_{,y,y}}{N} + \frac{\Lambda_5}{6}$$
$$E^\theta_\theta = E^\phi_\phi = -\frac{Q_{,y,y}}{Q} + \frac{\Lambda_5}{6}$$

- The phenomenological brane metric define boundary condition
- 5D Einstein equation form a system of PDE with Neumann boundary condition, and…

Thank you