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INFLATION IN TELE-PARALLEL DESCRIPTION OF GENERAL RELATIVITY

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Based on arXiv 1110.3099 Yi-Peng Wu & Chao-Qiang Geng

OUT-LINE

- Introduction
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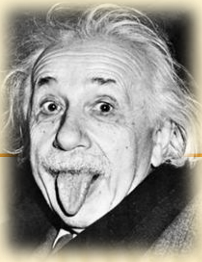
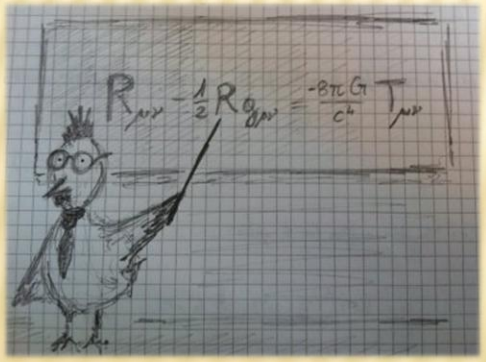
  - teleparallel description of General Relativity
  - higher-order teleparallel theory
- Tele-parallel Theories for Inflation
  - the single field inflationary model
  - inflation driven by higher-order teleparallel theory

TELE-PARALLEL GEOMETRY



- The origin: an unified field theory for gravitation and electromagnetism
- Nowadays: theories for gravity; geometrized by purely torsion

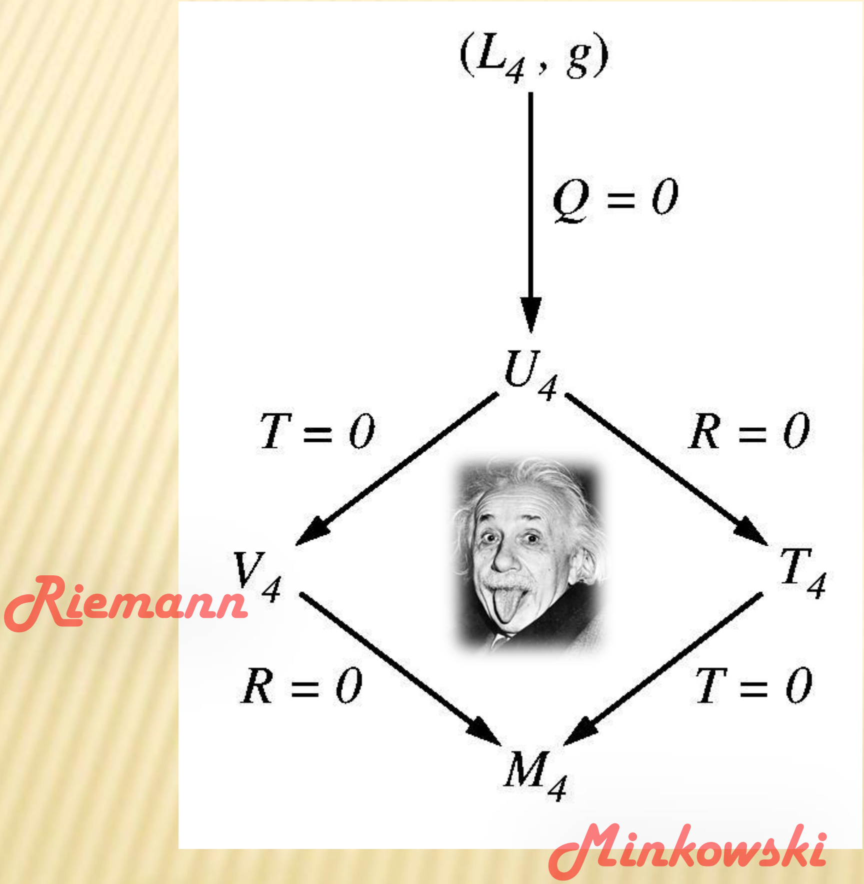
(1916) gravity + (1928) electromagnetism

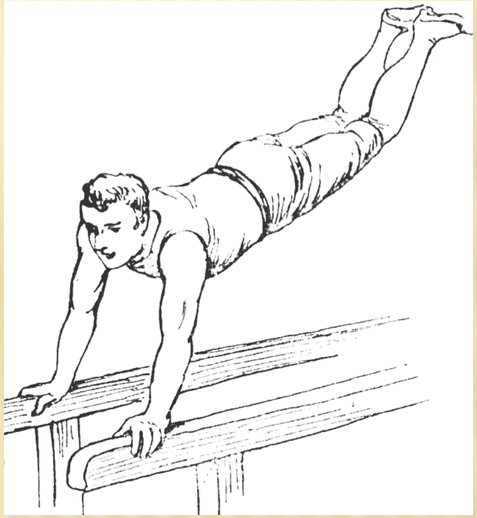


$$g_{\mu\nu}(x) = \eta_{AB} e^A_\mu(x) e^B_\nu(x)$$

$$T^\rho_{\mu\nu} = \Gamma^\rho{}_{\nu\mu} - \Gamma^\rho{}_{\mu\nu}$$

TELE-PARALLEL GEOMETRY





TELE-PARALLEL DESCRIPTION OF GR

- New General Relativity (Hayashi & Shirafuji 1979)

$$\mathcal{L} = a_1 T_\rho{}^{\mu\nu} T^\rho{}_{\mu\nu} + a_2 T^{\mu\nu}{}_\rho T^\rho{}_{\mu\nu} + a_3 T^\rho{}_{\rho\mu} T^\nu{}_\nu{}^\mu + a_0$$

- Teleparallel equivelance of General Relativity (Maluf 1993)

$$\mathcal{L}_{TEGR} = \frac{1}{2}T = \frac{1}{8} T_\rho{}^{\mu\nu} T^\rho{}_{\mu\nu} - \frac{1}{4} T^{\mu\nu}{}_\rho T^\rho{}_{\mu\nu} - \frac{1}{2} T^\rho{}_{\rho\mu} T^\nu{}_\nu{}^\mu$$

$$\Gamma^\rho{}_{\mu\nu} = e^\rho_A \partial_\nu e^A_\mu$$

$$T^\rho{}_{\mu\nu} = \Gamma^\rho{}_{\nu\mu} - \Gamma^\rho{}_{\mu\nu} = e^\rho_A (\partial_\mu e^A_\nu - \partial_\nu e^A_\mu)$$

→ first order Lagrangian; second order field equation

	GR	v.s. TEGR
dynamical variable	metric	$\boxed{\text{tetrad}}$ ⇒ 16 components
connection	Levi-Civita	Weitzenböck
field equation	geometrically equivalence	

(Einstein equation)

TELE-PARALLEL DESCRIPTION OF GR

- The Lagrangian density of TEGR "T" differs from the Ricci scalar "R" only by a total divergence. $T = R + 2\nabla^{\mu} T^{\rho}{}_{\rho\mu}$
- The field equation doesn't determinate the entire dynamic field

$\Rightarrow$ **Local Lorentz symmetry**

Local Lorentz transformations

$$\hat{e}^{\tilde{A}}_{\ \nu} = \Lambda^{\tilde{A}}_{\ B}(x)\hat{e}^{B}_{\ \nu}$$

$$\eta_{\tilde{C}\tilde{D}} = \Lambda_{\tilde{C}}^{\ A}\Lambda_{\tilde{D}}^{\ B}\eta_{AB}$$

metrical quantities are invariant: $g_{\mu\nu}$, $R^{A}{}_{B\mu\nu}$, R

nonmetrical quantities are violated: $\Gamma^{\lambda}{}_{\mu\nu}$, $T^{\lambda}{}_{\mu\nu}$, T

HIGHER-ORDER TELEPARALLEL THEORY

- An *ad hoc* generalization of TEGR inspired from f(R) theories
- Local Lorentz symmetry is broken

⇒ **extra degrees of freedom !!**

(Li, Sotiriou & Barrow 2011)

$$S = \frac{1}{2}\int dx^4 e f(T)$$

$$f_T\left[h^{-1}\partial_\sigma\left(h h_a^\rho S_\rho{}^{\lambda\sigma}\right) - h_a^\sigma S^{\mu\nu\lambda}T_{\mu\nu\sigma}\right] + f_{TT} h_a^\rho S_\rho{}^{\lambda\sigma}\partial_\xi T + \frac{1}{2}h_a^\lambda f(T) = 8\pi G\,\Theta_a^\lambda$$

second order field equation

covariant representation

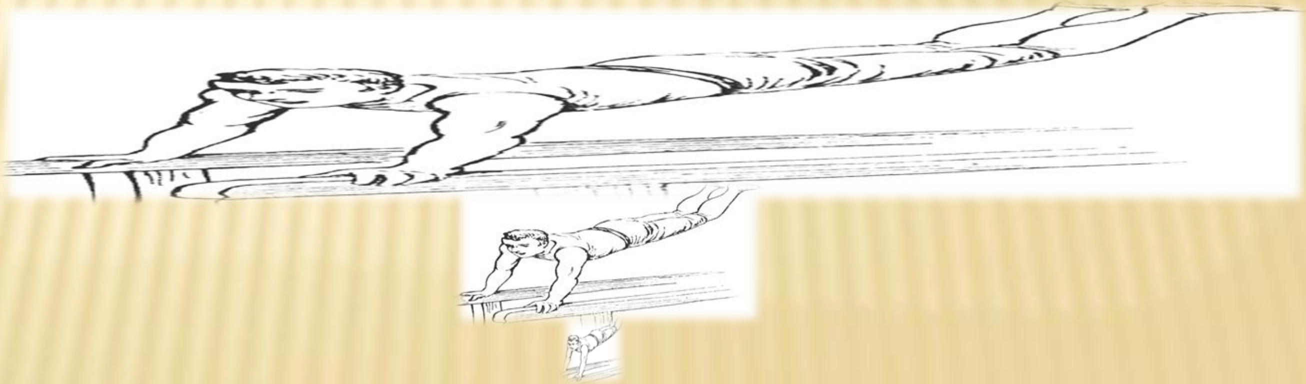
$$H_{(\mu\nu)} = 8\pi G\Theta_{\mu\nu}\,,$$
$$H_{[\mu\nu]} = 0,$$

$$H_{\mu\nu} \equiv f_T G_{\mu\nu} + \frac{1}{2}g_{\mu\nu}\left[f(T) - f_T T\right] + f_{TT} S_{\nu\mu\rho}\nabla^\rho T$$

TELE-PARALLEL THEORIES FOR INFLATION

TELE-PARALLEL THEORIES FOR INFLATION

- Models of inflation with observables in the framework of teleparallel geometry
- Re-exam the "equivalence" of TEGR via the minimal coupling single field inflationary model
- Can higher-order TEGR theories for inflation be available?



TELE-PARALLEL THEORIES FOR INFLATION

- Is the "equivalence" between single field inflationary models trivial ?

$$S = \frac{1}{2}\int d^4x e[T + (\nabla\phi)^2 - 2V(\phi)]$$

$$S = \frac{1}{2}\int d^4x\sqrt{-g}[R + (\nabla\phi)^2 - 2V(\phi)]$$

$$3H^2 = \frac{1}{2}\dot{\phi}^2 + V\ ,\ \dot{H} = -\frac{1}{2}\dot{\phi}^2\ ,\ and\ \ 0 = \ddot{\phi} + 3H\dot{\phi} + V_{,\phi}$$

- Classify the teleparallel geometry:
  - $e_A{}^0$: $\boxed{\text{(i) time-like}}$ (ii) space-like (iii) null

$$e^0_\mu = (N, 0)\quad ,\quad e^a_\mu = (N^a, h^a{}_i)$$
$$e_0^\mu = (1/N, -N^i/N)\quad ,\quad e_a^\mu = (0, h_a{}^i),$$

$$ds^2 = N^2dt^2 - h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

ADM formalism

TELE-PARALLEL THEORIES FOR INFLATION

- Teleparallel geometry with time-like $e_A{}^0$

3- curvature

$$T = \bar{\Sigma}_{ij}\bar{\Sigma}^{ij} - \bar{\Sigma}^2 + R^{(3)} + \mathcal{D}_T$$

extrinsic curvature

$$e^0_\mu = (N, \mathbf{0}) \quad , \quad e^a_\mu = (N^a, h^a_{\ i})$$
$$e_0^{\ \mu} = (1/N, -N^i/N) \quad , \quad e_a^{\ \mu} = (0, h_a^{\ i}),$$

$$T^0_{\ j0} = \partial_j N/N \, ,$$
$$T^i_{\ j0} = D_j N^i - \frac{N^i}{N}\partial_j N - h_a^{\ i}\partial_0 h^a_{\ j}$$
$$T^i_{\ jk} = h_a^{\ i}(\partial_j h^a_{\ k} - \partial_k h^a_{\ j}) \equiv {}^{(3)}T^i_{\ jk}$$

- Fix the time and spatial reparametrizations

$$\delta\phi = 0, \quad h^a_{\ i} = a e^{\zeta}(\delta^a_i + \frac{1}{2}\gamma^a_{\ i})$$

ζ and γ are first order quantities

$$S_\zeta = \frac{1}{2}\int dt d^3x a^3 \frac{\dot{\phi}^2}{H^2}\left(\dot{\zeta}^2 - a^{-2}(\partial\zeta)^2\right)$$

$$S_\gamma = \frac{1}{8}\int dt d^3x \, a^3 \left[(\dot{\gamma}_{ij})^2 - a^{-2}(\partial_i\gamma_{jk})^2\right]$$

The results can be generalized to teleparallel geometry with space-like $e_A{}^0$

expected results!!

TELE-PARALLEL THEORIES FOR INFLATION

- Driven inflation by modified teleparallel gravity opens up the study of f(T) theories (Ferraro & Fiorini 2007)
- Conformal transformation in f(T) theories:

$$S = \frac{1}{2}\int dx^4 e f(T)$$

$$\Downarrow$$

$$S = \frac{1}{2}\int dx^4 \hat{e}\left[\hat{T} - \frac{4}{\sqrt{6}}\hat{\partial}^\mu\varphi\hat{T}^\rho{}_{\rho\mu} + (\hat{\nabla}\varphi)^2 - 2U(\varphi)\right]$$

$$f(T) = FT - 2V,\quad F \equiv \frac{df}{dT},\quad V = \frac{FT - f(T)}{2}$$

$$\hat{e}^A_\mu = \sqrt{F}e^A_\mu \equiv \Omega e^A_\mu$$

$$T = \Omega^2[\hat{T} - 4\hat{\partial}^\mu\omega\hat{T}^\rho{}_{\rho\mu} + 6(\hat{\nabla}\omega)^2]$$

$$\hat{\partial}^\mu\omega \equiv \hat{\partial}^\mu\Omega/\Omega$$

$$d\varphi = \sqrt{6}d\omega = \sqrt{6}dF/2F$$

$$U = V/F^2$$

still non-minimally coupled

TELE-PARALLEL THEORIES FOR INFLATION

- Before calculations....

$f(T)$

$F \equiv \frac{df}{dT}$

$\Omega = \sqrt{F}$

$$S = \frac{1}{2}\int dx^4 \hat{e}\left[\hat{T} - \frac{4}{\sqrt{6}}\hat{\partial}^\mu \varphi \hat{T}^\rho{}_{\rho\mu} + (\hat{\nabla}\varphi)^2 - 2U(\varphi)\right]$$

$T = \Omega^2[\hat{T} - 4\hat{\partial}^\mu \omega \hat{T}^\rho{}_{\rho\mu} + 6(\hat{\nabla}\omega)^2]$

$d\varphi = \sqrt{6}d\omega = \sqrt{6}dF/2F$

$\hat{\partial}^\mu \omega \equiv \hat{\partial}^\mu \Omega / \Omega$

$U = V/F^2$

They are all Lorentz violated!!!

  + (i) we need more assumptions to reduce the degrees of freedom for the flat FRW choice
  + (ii) the unitary field gauge δϕ = 0 ?
  + (iii) the equivalence between conformal frames?

(Faraoni & Nadeau 2007)

TELE-PARALLEL THEORIES FOR INFLATION

- If the gravitational field is protected by gauge invariance (at least in the energy scale before horizon crossing) :
  - Extra dofs induced by Lorentz violation are supressed
  - The previous parametrization is applied $\hat{e}(\varphi\hat{\nabla}^\mu \hat{T}^\rho{}_{\rho\mu}) = -\varphi\partial_t(\sqrt{\hat{h}}\hat{\Sigma}) + \sqrt{\hat{h}}\varphi\bar{D}_i(N^i\hat{\Sigma} + h^{ij}\partial_j N - N\hat{T}^{j}_{j}{}^{i})$
  - A single-field theory with second-order field equations:

(Kobayashi, Yamaguchi & Yokoyama 2011)

quadratic actions

$$S_\zeta = \frac{1}{2}\int dt dx^3 a^3 \Big[-6\mathcal{G}_T\dot{\zeta}^2 - 2\mathcal{F}_T(\partial\zeta)^2 + 2\Xi N_1^2 - 4\Theta N_1\partial^2\psi + 4\mathcal{G}_T\dot{\zeta}\partial^2\psi + 12\Theta N_1\dot{\zeta} - 4\mathcal{G}_T N_1\partial^2\psi\Big]$$

$$\mathcal{G}_T = \mathcal{F}_T = 1\ ,\ \Xi = -U \text{ and } \Theta = H(1 - \dot{\varphi}/\sqrt{6}H)$$

$$S_\gamma = \frac{1}{8}\int dt d^3x\, a^3 \left[\mathcal{G}_T(\dot{\gamma}_{ij})^2 - a^{-2}\mathcal{F}_T(\partial_i\gamma_{jk})^2\right]$$

$$S_\zeta = \frac{1}{2}\int dt d^3x a^3 \left[\mathcal{G}_S\dot{\zeta}^2 - a^{-2}\mathcal{F}_S(\partial\zeta)^2\right],$$

tensor perturbations unchanged

background eqs.

$$0 = \ddot{\varphi} + 3H\dot{\varphi} + U' - \sqrt{6}H\dot{\varphi}/\varphi$$

$$3H^2 = \frac{1}{2}\dot{\varphi}^2 + U + \sqrt{6}\dot{\varphi}H$$

$$\dot{H} = -\frac{1}{2}\dot{\varphi}^2 + \frac{1}{\sqrt{6}}\ddot{\varphi} - \frac{\sqrt{6}}{2}H\dot{\varphi}$$

to avoid ghost and instabilities: $\mathcal{F}_S > 0, \mathcal{G}_S > 0$

SUMMARY

- The single field inflationary model for TEGR, indistinguishable from the standard inflation results, can be realized in the certain classes of teleparallel geometry.
- It is possible to carry out observables from higher-order TEGR inflation if the gravitational field is characterized by local gauge invariance.

Thank you !