Gravitational effects of domain walls on primordial quantum fluctuations

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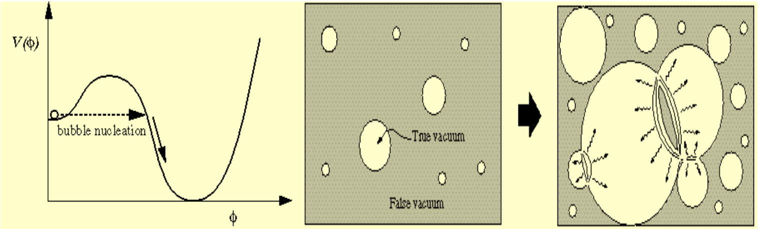
Formations of domain walls (DWs)

- Kibble mechanism: In the second-order phase transition, a scalar field, for expamle, may fall into different degenerate vacua once the temperature cools down to critical temperture T_c. Below the Ginzburg temperature T_G, temperature fluctuations will be insufficient to lift it from one minimum into the other, so DW, a boundary interpolating between two different degenerate vacua, effectively freeze-out. (Kibble 1976)

False vacuum decay (First-order phase transition):



- Coleman-de Luccia bubbles: A single Coleman-de Luccia bubble in four dimensional spacetime has SO(3,1) symmetry, i.e. 3 generators of spatial rotations and 3 generators of Lorentz boosts. (Coleman & de Luccia 1980)

$$ds^2 = d\zeta^2 + \rho^2(\zeta)\, dH_3^2, \tag{1}$$

where dH_3^2 is the line element of 3-dimensional unit time-like hyperboloid.

- Two-bubble colliding: Two bubbles in four dimensional spacetime has SO(2,1) symmetry, i.e. 1 generator of spatial rotation and 2 generators of Lorentz boosts. (Hawking, Moss & Stewart 1982)

$$ds^2 = A^2(\tau,\zeta)(-d\tau^2 + d\zeta^2) + B^2(\tau,\zeta)\, dH_2^2, \tag{2}$$

where dH_2^2 is the line element of 2-dimensional unit space-like hyperboloid.

General properties of thin DWs

- For thin DWs, it is useful to apply thin-wall approximation.
- DWs are vacuumlike hypersurfaces with surface tension $\sigma = constant$
- Domain walls produce repulsively gravitational forces (Ipser & Sikivie, PRD, 1984).
- The surface stress-energy tensor: $S^{ab} = \sigma h^{ab}$

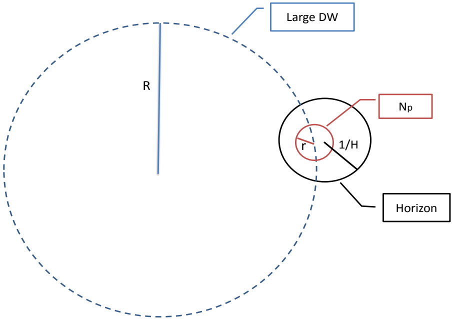
Motivation

It is generally believed that the phase transitions happened in the early Universe, so domain walls should naturally form after phase transitions. During inflation, domain walls will be inflated away from our observable Universe and leave no direct interaction with CMB. However, it is still not clear whether their gravitational effects will affect primordial quantum fluctuations during inflation. So it motivate us to study the following questions.

1. Can gravitational fields of domain walls affect primordial quantum fluctuations during inflation? If yes, how does it modify primordial power spectrum?
2. Can these domain wall effects be observed from CMB anisotropies?

Planar DW space-time

1. The simulations of domain wall evolution in radiation and matter dominated Universes indicate that each horizon typically contains one large domain wall, which extends across the horizon. (Press, Ryden, & Spergel, ApJ 1990)
2. For any point p on a such large closed DW with its radius R, one can define a local neighborhood $\mathcal{N}_p$ with radius r satisfying $r << R$ and center at p. In $\mathcal{N}_p$, gravitational effects of the closed DW can be well approximated as an infinite planar DW, so we may consider plane symmetry and reflection symmetry in $\mathcal{N}_p$.



- The size of comoving horizon decreases exponentially during inflation, but the size of $\mathcal{N}_p$ in comoving scale is nearly the same by studying equations of motion of large spherical DW in de-Sitter space. (A. Aurilia, M. Palmer & E. Spallucci PRD, 1989). It means that our observable Universe can be well inside the $\mathcal{N}_p$ after inflation.

The metric of planar DWs

- The metric of a planar domain wall in de-Sitter space-time with reflection symmetry has been obtained (Wang, Cho, & Wu, PRD, 2011):

$$ds^2 = \frac{1}{\alpha^2 \left(\eta + \beta|z|\right)^2}\left(-d\eta^2 + dz^2 + dx^2 + dy^2\right), \tag{3}$$

where the wall is placed at $z = 0$. $\alpha = \sqrt{\Lambda/12\Gamma}(\Gamma + 1)$, $\beta = \frac{\Gamma-1}{\Gamma+1}$, satisfying $-1 < \beta \leqslant 0$, and Γ is a dimensionless parameter

$$\Gamma = 1 + \frac{3\epsilon - \sqrt{48\epsilon + 9\epsilon^2}}{8}, \tag{4}$$

where $\epsilon = \frac{\kappa^2\sigma^2}{\Lambda}$ and σ is the surface tension of the domain wall. Eq. (4), which gives $0 < \Gamma \leqslant 1$, is only valid for the coordinate ranges $-\infty < \eta + \beta|z| < 0$.

- It is useful to introduce a proper-time coordinate: $\tau = -\frac{1}{\alpha}\ln[-\alpha(\eta \pm \beta z)]$ and $z' = \sqrt{1-\beta^2}z$, so the metric (3) becomes

$$ds^2 = -d\tau^2 \pm \frac{2\beta\, e^{\alpha\tau}}{\sqrt{1-\beta^2}} d\tau dz' + e^{2\alpha\tau}(dz'^2 + dx^2 + dy^2), \tag{5}$$

  where $\pm$ corresponds to $z' > 0$ and $z' < 0$ sides, respectively. It is clear that the metric (5) also has the reflection symmetry about $z' = 0$.

- The appearance of the cross term $g_{\tau z}$ indicates that the gravitational effects of planar domain walls will break the rotational invariance, i.e. $O(3)$ symmetry, of space-time geometry. In the post-Newtonian theory, the metric components g_{0i} are associated with the boost of gravitating sources.

Gravitational fields of planar DWs

- To understand the gravitational effects of metric (3), we consider observers stationary relative to the wall on the $z > 0$ side, with 4-velocities described by a future-pointing unit time-like vector field $U = -\alpha(\eta + \beta z)\partial_\eta$. Their 4-acceleration

$$\mathcal{A} \equiv \nabla_U U, \tag{6}$$

has a constant magnitude $|\mathcal{A}| \equiv \sqrt{g(\mathcal{A}, \mathcal{A})} = |\alpha\beta| = \kappa\sigma/4$ and z-direction component $A_z \equiv g(\nabla_U U, -\alpha(\eta + \beta z)\partial_z) = -\kappa\sigma/4$, where the minus sign denotes the acceleration toward the wall.

- It yields that the gravitational field of a planar domain wall produces a constant repulsive force on each observer, independent of their distance from the wall.

- The trajectories of geodesic observers represented in the coordinates (η, z, x, y) are $z = -\beta\eta + constant$, which are straight lines away from the wall.
- We conclude that the stationary observers ($z = constant$) and straight-line observers ($z = -\beta\eta + constant$) correspond to uniformly accelerated observers and geodesic observers, respectively.

Quantum fluctuations in planar DW spacetime

- Since background metric of planar DWs break isotropy (but still preserve homogeneity), one should expect that quantum fluctuations of a inflaton field in planar DW space-time will have rotational violation without violating translation.
- In planar DW space-times, the stationary observers have uniformly acceleration associated with surface tension of planar DW, so they may detect extra particles due to their accelerations (the Unruh effect).

Vacuum states in curved space-times

- Quantized fields in flat space-time, i.e. Minkowski space-time, have a well-defined vacuum state.
- Vacuum states become ambiguous in curved space-times since the decompositions of fields into positive and negative frequency mode-functions are coordinate dependent, i.e. positive and negative frequencies have no invariant meaning in curved space-times.
- In some highly symmetric space-times, e.g conformally flat space-time, it is possible to define physically reasonable vacuum state.

Exact solutions of a massless scalar field

- To understand the gravitational effects of a planar DW on inflaton fluctuations, we study a massless scalar field ϕ, which has the field equation

$$\mathrm{d} \star \mathrm{d}\phi = 0, \tag{7}$$

  where d is the exterior derivative and $\star$ is the Hodge map associated with the metric g.
- We assume that the scalar field do not have direct interaction with DWs, so no boundary condition on DWs are imposed.

- A general exact solution of mode functions $\phi_{\mathbf{k}}(x^i)$ gives (for $z > 0$):

$$\phi_{\mathbf{k}}(x^i) = \sqrt{\frac{\Lambda}{6}}\frac{1}{k\sqrt{k}}\left(\frac{1+\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{z}}}{\sqrt{1-\beta^2}}\right)^{-\frac{3}{2}}\left(i+\frac{k(1+\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{z}})}{\alpha(1-\beta^2)}e^{-\alpha\tau}\right) \times e^{i(k_z+\beta k)z+ik_x x+ik_y y+i\frac{k}{\alpha}e^{-\alpha\tau}}. \qquad (8)$$

- To obtain the solution (8), we have assumed that for the high k modes, $\phi_{\mathbf{k}}(x^i) \propto \frac{1}{\sqrt{k}}\tilde{\eta}e^{ik\tilde{\eta}}$, where $\tilde{\eta} = \alpha e^{-\alpha\tau}$, and it corresponds to positive frequency mode-function in Minkowski space-time. Actually, when $\beta = 0$, the vacuum state becomes the well-known Bunch-Davies vacuum (Bunch & Davies, Pro. Roy. Soc. **A**, 1978).

Quantization & primordial power spectrum

- To quantize the ϕ field, one may expand ϕ in creation and annihilation operators, $a_{\mathbf{k}}\dagger$ and $a_{\mathbf{k}}$, as

$$\phi = \int \frac{d^3k}{(2\pi)^{3/2}} \;\; a_{\mathbf{k}}\phi_{\mathbf{k}}(x^i) + a_{\mathbf{k}}^{\dagger}\phi_{\mathbf{k}}^{*}(x^i), \tag{9}$$

  with the vacuum state $|0\rangle$, satisfying $a_{\mathbf{k}}|0\rangle = 0$.
- The vacuum expectation value of ϕ^2 is

$$\langle\phi^2(x^i)\rangle = \frac{1}{(2\pi)^3}\int |\phi_{\mathbf{k}}(x^i)|^2 d^3k. \tag{10}$$

- It is convenient to introduce physical momenta $p = ke^{-\alpha\tau}$, which are exponentially decreasing during inflation, to obtain

$$\langle 0|\phi^2(x^i)|0\rangle = \int \frac{d^3p}{(2\pi p)^3}\left[\frac{\Lambda}{6}\left(\frac{1\pm\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{z}}}{\sqrt{1-\beta^2}}\right)^{-3} + \frac{\sqrt{1-\beta^2}\,p^2}{2(1\pm\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{z}})}\right] \quad (11)$$

where $\pm$ denotes $\langle\phi^2(x^i)\rangle$ for $z > 0$ and $z < 0$ sides, respectively.

The vacuum fluctuations of the scalar field $\langle 0|\phi^2(x^i)|0\rangle$ have two interesting features:

1. For those p modes with physical wavelengths λ_p well inside the horizon, i.e. $p \gg \sqrt{\Lambda/3}$, the $\langle 0|\phi^2(x^i)|0\rangle$, which is proportional to $\int \frac{d^3p}{2p}$, corresponds the vacuum fluctuations in Minkowski space-time, i.e. $\mid 0\rangle =\mid 0\rangle_M$ for high p modes.
2. When λ_p crosses the horizon, i.e. $p \lesssim \sqrt{\Lambda/3}$, the $\langle 0|\phi^2(x^i)|0\rangle$ becomes time-independent and $|0\rangle$ for large wavelength λ_p modes may correspond to particle states $|n_{\mathbf{k}}\rangle_M$ in Minkowski space-time. Taking $\beta = 0$ yields the well-known scale-invariant Harrison-Zeldovich spectrum.

- For the fluctuation modes which are well outside the horizon at $\tau = \tau_*$ during inflation, the spectrum of the scalar field fluctuations $P_\phi(\mathbf{k}, \tau_*) = |\phi_{\mathbf{k}}(\tau_*)|^2$ gives

$$P_\phi = \frac{\Lambda(1-\beta^2)^{\frac{3}{2}}}{12\,k^3}\left[(1+\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{z}})^{-3} + (1-\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{z}})^{-3}\right], \qquad (12)$$

  where $P_\phi(\mathbf{k})$ satisfies $P_\phi(\mathbf{k}) = P_\phi(-\mathbf{k})$ and is valid for both $z > 0$ and $z < 0$ regions.

- The $\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{z}}$ terms indicate the existence of a preferred direction in the primordial density spectrum.

Primordial curvature perturbations

- To understand how does $P_\phi(\mathbf{k})$ affect CMB anisotropies, we shall transfer $P_\phi(\mathbf{k})$ to primordial curvature perturbations $\mathcal{R}_{\mathbf{k}}$, which are the initial values for density perturbations $\delta_{\mathbf{k}}$ and have constant value outside the horizon.
- Since the evolution of perturbed classical quantities are described in a background homogenous and isotropic Universe, we should study those quantities in the geodesic coordinates $(\check{\eta}, \check{z}, \check{x}, \check{y})$, where the planar DW metric becomes homogeneous and isotropic for $z > 0$ side.

- Taking our observable Universe to be located at $z > 0$, P_ϕ in the coordinates $(\check{\eta}, \check{z}, \check{x}, \check{y})$ becomes

$$P_\phi(\check{\mathbf{k}}) = \frac{\Lambda}{12}\left[\check{k}^{-3} + \left(\check{k} - \frac{2\,\beta}{1-\beta^2}\,\check{\mathbf{k}}\cdot\hat{\mathbf{z}}\right)^{-3}\right], \tag{13}$$

which is only valid for $z > 0$.

- In the slow-roll inflation, we obtain

$$\mathcal{P}_{\mathcal{R}}(\check{\mathbf{k}}) = \frac{V(\varphi)}{48\pi^2 M_{\rm Pl}^4\,\varepsilon}\left[1 + \left(1 - \frac{2\,\beta}{1-\beta^2}\,\frac{\check{\mathbf{k}}\cdot\hat{\mathbf{z}}}{\check{k}}\right)^{-3}\right], \tag{14}$$

where $V(\varphi)$ is a slow-roll potential and $\varepsilon = \frac{1}{2}M_{\rm Pl}^2(V'/V)$ is one of the slow-roll parameters

- The requirement $|\beta| \ll 1$ yields a constraint on the surface tension of the domain wall: $\sigma \ll M_{\rm Pl} V^{1/2}$. In this limit the leading-order effect of rotational violation of $\mathcal{P}_{\mathcal{R}}(\check{\mathbf{k}})$ is

$$\mathcal{P}_{\mathcal{R}}(\check{\mathbf{k}}) = \frac{V(\varphi)}{24\pi^2 M_{\rm Pl}^4\,\varepsilon}\left[1 + 3\beta\,\frac{\check{\mathbf{k}}\cdot\hat{\mathbf{z}}}{\check{k}} + \cdots\right], \tag{15}$$

which may correspond to a primordial dipole.

- In order to associate $P_{\mathcal{R}}(\mathbf{k})$ to CMB anisotropies, one should translate the original point to the location of our galaxy. However, because of the translational invariance of $P_{\mathcal{R}}(\mathbf{k})$, such translation does not change the results.
- It should be point out that our results only apply to the local neighborhood $\mathcal{N}_p$, where one can approximate a realistic domain wall by a planar infinite wall. If our observable Universe is not well inside the $\mathcal{N}_p$ defined at the end of inflation, one should expect to obtain not only rotational but also translational violation of primordial power spectrum, due to the curvature (deviation from planarity) of the wall.

Extension beyond planar DW

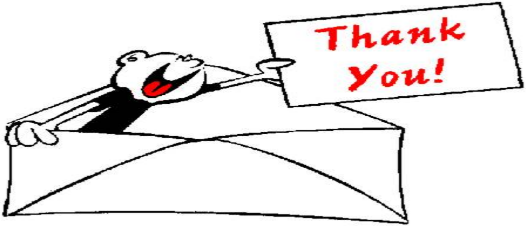
- Supposing gravitational effects of spherical DW is characterized by one metric component $F(\tau, 1/r)$, one can expand F around $r = r_0$ to obtain

$$F(\tau, 1/r) = F(\tau, 1/r_0) + \left.\frac{\partial F}{\partial r}\right|_{r_0} (r - r_0) + \left.\frac{\partial^2 F}{2\,\partial r^2}\right|_{r_0} (r - r_0)^2 + \cdots (16)$$

  where $\frac{\partial^2 F}{\partial r^2}$ denotes curvature effects of DW.
- Recently, we have discovered spherical, planar, and hyperbolic DW solutions without reflection symmetry, i.e. the two regions separated by DW having different cosmological constants and Schwartzschild mass. (to be submitted. Dr Wu Yu-Huei's talk)

Future Works

- A detail quantitative study on rotational violation of primordial power spectrum is necessary.
- Understanding the next-order gravitational effects (deviation from planarity) of domain wall on primordial power spectrum?
- What are the effects of rotational violation of P_ϕ on CMB temperature anisotropies?
- What are gravitational effects of DW on primordial gravitational waves?



$$G_{ab} - \Lambda g_{ab} = \kappa T_{ab}$$

By WYH