Trapping effect on inflation

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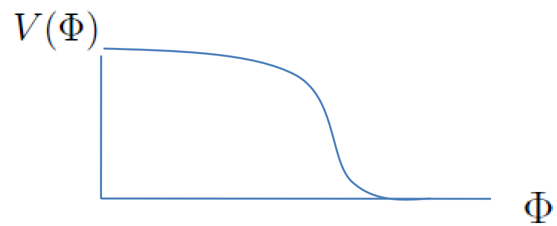
Basic idea

Slow roll

60 e foldings $a(t) = e^{Ht}$

- Flat potential

Slow rolling potential

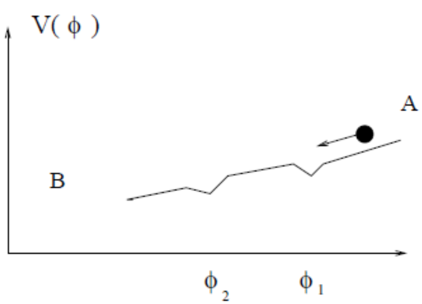


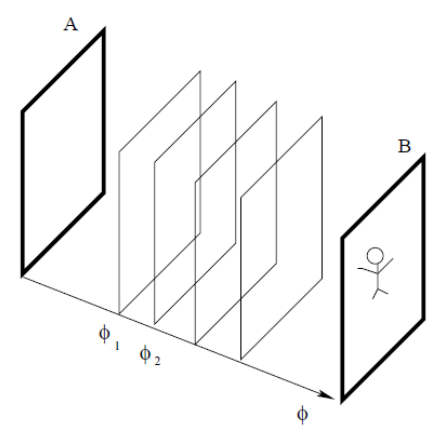
- Steep potential

Trapped inflation , potential is too steep for slow roll

JHEP 0405:030,2004 Kofman et al

Phys.Rev.D80:063533,2009 Green et al





Trapped inflation

- Particle production slow down the velocity while rolling down a steep potential

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + \frac{1}{2}\sum_i(\partial_\mu\chi_i\partial^\mu\chi_i - g^2(\phi-\phi_i)^2\chi_i^2)$$

- Mean field approximation

PRD 80, 043501
JHEP 0405:030,2004
Phys.Rev.D80:063533

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} + g^2(\phi-\phi_0)\langle\chi^2\rangle = 0$$

$$\langle\chi^2\rangle \approx \frac{n_\chi a^{-3}}{g|\phi-\phi_0|}$$

Trapping effect

- Approximate the space-time by de Sitter metric

$$ds^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2)$$

- A trapping point at

$$\phi = \phi_0$$

- Influence functional approach, trace out χ and then obtain the effective action

Ann. Phys. 24, 118(1963)
Feynman and Vernon

- Semiclassical Langevin equation

$$\langle \varphi_{\mathbf{k}}(\eta)\varphi^*_{\mathbf{k}'}(\eta)\rangle = \frac{2\pi^2}{k^3}\Delta_k^\xi(\eta)\delta(\mathbf{k}-\mathbf{k}')$$

Our model

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \, \partial_\nu \Phi + \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \, \partial_\nu \chi - V(\Phi) - \frac{g^2}{2} (\Phi - \Phi_0)^2 \chi^2$$

Shift the field $\phi = \Phi - \Phi_0$

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi + \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \, \partial_\nu \chi - V(\phi) - \frac{g^2}{2} \phi^2 \chi^2$$

trace out χ to obtain the effective action

arXiv:1101.4493 or PRD 84, 063527

$$\begin{aligned} S_{\text{eff}}[\phi, \phi_\Delta, \xi] \; &= \int d^4x \, a^2(\eta) \, \phi_\Delta(x) \Big\{ -\ddot{\phi}(x) - 2aH\dot{\phi}(x) + \nabla^2\phi(x) - a^2 \left[V'(\phi) + g^2 \langle \chi^2 \rangle \phi(x) \right] \\ &\quad - g^4 a^2 \phi(x) \int d^4x' \, a^4(\eta') \, \theta(\eta - \eta') \, iG_-(x, x') \phi^2(x') + g^2 a^2 \phi(x) \xi(x) \Big\} \end{aligned}$$

Main result

$$\ddot{\phi} + 2aH\dot{\phi} - \nabla^2\phi + a^2\left[V'(\phi) + g^2\langle\chi^2\rangle\phi\right] + g^4a^2\phi\int d^4x'a^4(\eta') \times$$

$$\theta(\eta-\eta')\, i\, G_-(x,x')\phi^2(x') = g^2\, a^2\, \phi\, \xi$$

where

$$G_-(x,x') = \langle\chi(x)\chi(x')\rangle^2 - \langle\chi(x')\chi(x)\rangle^2$$

$$\langle\xi(x)\xi(x')\rangle = G_+(x,x') = \langle\chi(x)\chi(x')\rangle^2 + \langle\chi(x')\chi(x)\rangle^2$$

The effect from χ on inflaton are given by dissipation term and noise term.

Approximate solution

- Drop the dissipation term $g^2 < 10^{-7}$

$$\ddot{\phi} + 2aH\dot{\phi} - \nabla^2\phi + a^2\left[V'(\phi) + g^2\langle\chi^2\rangle\phi\right] = g^2\, a^2\, \phi\, \xi$$

- Decompose $\phi(\eta, \mathbf{x}) = \bar{\phi}(\eta) + \varphi(\eta, \mathbf{x})$, then we obtain the equations

$$\ddot{\bar{\phi}} + 2aH\dot{\bar{\phi}} + a^2\left[V'(\bar{\phi}) + g^2\langle\chi^2\rangle\bar{\phi}\right] = 0$$

$$\ddot{\varphi} + 2aH\dot{\varphi} - \nabla^2\varphi + a^2 m_{\varphi\mathrm{eff}}^2\varphi = g^2\, a^2\, \bar{\phi}\, \xi$$

$$\ddot{\chi} + 2aH\dot{\chi} - \nabla^2\chi + a^2 m_{\chi\mathrm{eff}}^2\chi = 0$$

where

$$m_{\varphi\mathrm{eff}}^2 = V''(\bar{\phi}) + g^2\langle\chi^2\rangle$$

$$m_{\chi\mathrm{eff}}^2 = g^2\bar{\phi}^2$$

Weak coupling limit

$g^2 \ll 1$

- EOM for χ

$$\ddot{\chi} + 2aH\dot{\chi} - \nabla^2\chi + a^2 m_{\chi\mathrm{eff}}^2 \chi = 0$$

$$m_{\chi\mathrm{eff}}^2 = g^2\bar{\phi}^2$$

$$\chi_k(\eta) = \frac{1}{2a}(\pi|\eta|)^{\frac{1}{2}}\left[c_1 H_\mu^{(1)}(k\eta) + c_2 H_\mu^{(2)}(k\eta)\right]$$

$$\mu^2 = 9/4 - m_{\chi\mathrm{eff}}^2/H^2$$

- When $\mu = 3/2$, we can select Bunch-Davies vacuum to calculate the noise term and

$$\langle \varphi_{\mathbf{k}}(\eta)\varphi_{\mathbf{k}'}^*(\eta)\rangle = \frac{2\pi^2}{k^3}\Delta_k^{\xi}(\eta)\delta(\mathbf{k}-\mathbf{k}'),$$

The noise-driven power spectrum

$$\Delta_k^{\xi}(\eta) = \frac{g^4 z^2}{8\pi^4} \int_{z_i}^{z} dz_1 \int_{z_i}^{z} dz_2\, \bar{\phi}(\eta_1)\bar{\phi}(\eta_2) F(z_1) F(z_2) \left\{ \frac{\sin z_-}{z_1 z_2 z_-} \left[\sin(2\Lambda z_-/k)/z_- - 1\right] + G(z_1, z_2) \right\}$$

where

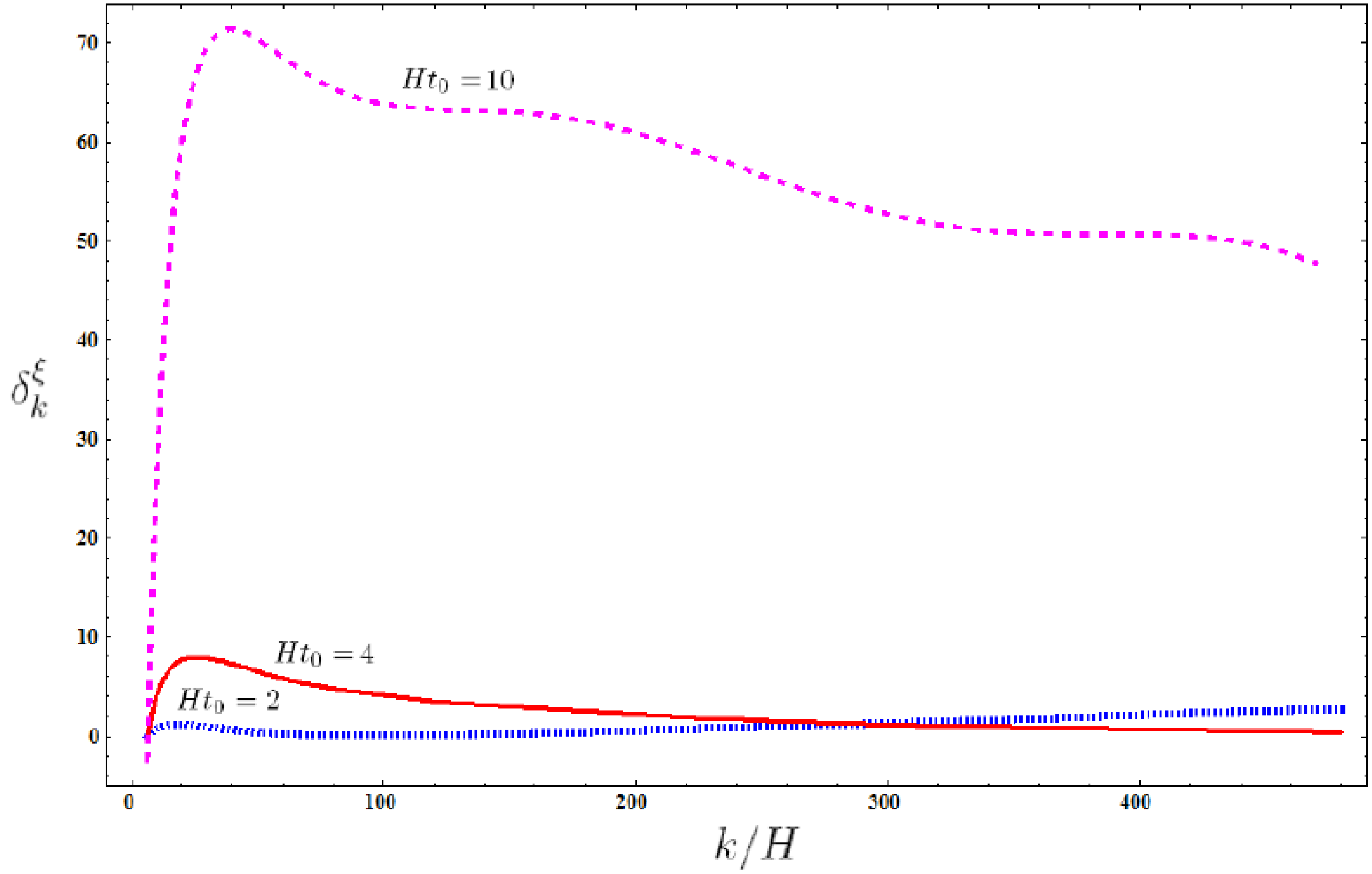
$$\bar{\phi}(\eta) = v(t_0 - t) = \frac{v}{H} \ln \frac{\eta}{\eta_0}$$

$$F(y) = \left(1 + \frac{1}{yz}\right) \sin(y - z) + \left(\frac{1}{y} - \frac{1}{z}\right) \cos(y - z)$$

$$\begin{aligned} & G(z_1, z_2) \\ = & \int_0^{\Lambda} dk_1 \int_{|k-k_1|}^{k+k_1} dq \left\{ \left[\frac{2}{z_1 z_2 q k_1} \left(\frac{k_1}{q} - \frac{z_1}{z_2} - \frac{z_2}{z_1} + 2 + \frac{k^2}{z_1 z_2 k_1 q} \right) \right] \cos\left[\left(\frac{k_1 + q}{k} \right) (z_2 - z_1) \right] \right. \\ & \left. + \frac{2}{kq} \left(\frac{1}{z_1} - \frac{1}{z_2} \right) \left[1 + \frac{k^2}{z_1 z_2 k_1} \left(\frac{1}{q} + \frac{1}{k_1} \right) \right] \sin\left[\left(\frac{k_1 + q}{k} \right) (z_2 - z_1) \right] \right\} \\ & + \frac{2}{z_1 z_2 (z_2 - z_1)} \left\{ \int_0^{\Lambda} \frac{dk_1}{k_1^2} \sin\left[(z_2 - z_1) \left(1 + \frac{2k_1}{k} \right) \right] \right. \\ & \left. - \int_0^{k} \frac{dk_1}{k_1^2} \sin(z_2 - z_1) - \int_k^{\Lambda} \frac{dk_1}{k_1^2} \sin\left[(z_2 - z_1) \left(\frac{2k_1}{k} - 1 \right) \right] \right\} \end{aligned}$$

Power spectrum for a trapping point

Trapping point at $Ht_0 = 2, 4, 10$



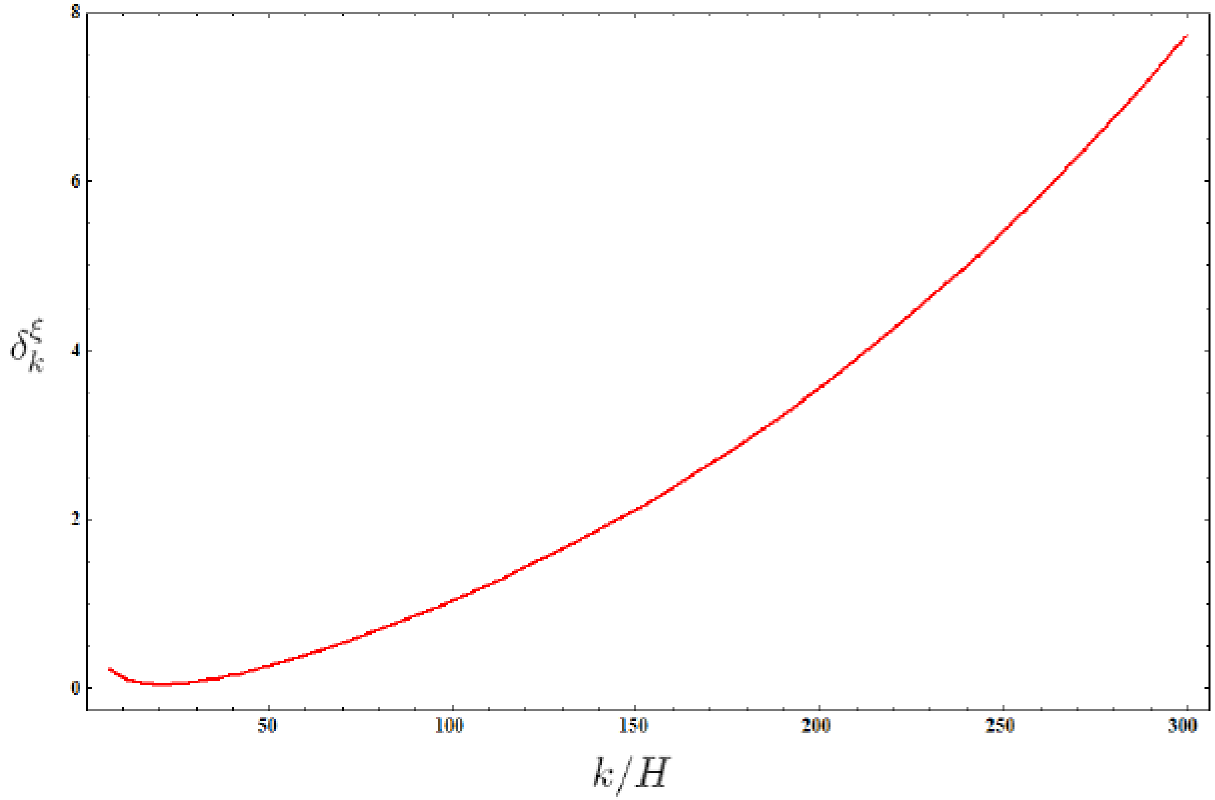
$$\bar{\phi}(\eta) = v(t_0 - t) = \frac{v}{H}\ln\frac{\eta}{\eta_0}$$

$$a(\eta) = a(t) = e^{Ht}$$

Closely spaced trapping points

$$g^2 \simeq 10^{-7}$$

$$\Delta_k^\xi(\eta) = \frac{g^4\Gamma^4 H^2}{8\pi^4 v^2}\int_{z_i}^{z} dz_1 \int_{z_i}^{z} dz_2\, z_1 z_2 \left\{\frac{\sin z_-}{z_-}\left[\sin(2\Lambda z_-/k)/z_- - 1\right] + G(z_1, z_2)\right\}$$



Smaller than de Sitter quantum fluctuation by factor about 400

Conclusion

- Lagrangian approach based on influence functional method

  the noise which is the particle number density fluctuation studied by the authors, and we have dissipation term

- Trapping effect in weak coupling limit

  Single trapping point

  a dip on power spectrum and the noise driven effect gets bigger when inflaton away from the trapping point

  Evenly spaced trapping points

  the power spectrum is blue, however for high k mode, dissipation should damp the power spectrum in late time

- Systematic approach to study the trapping effect

  For strong coupling, one should reexamine the trapping effect and the dissipation to both backreaction and fluctuation