Local and global structure of domain wall space-time

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Reference: (1) Yu-Huei Wu and Chih-Hung Wang, now writing up, 2012,
(2) Chih-Hung Wang, Yu-Huei Wu, and Stephen D. H. Hsu, arXiv: gr-qc/1107.1762, 2012,
(3) Chih-Hung Wang, Hing-Tong Cho, and Yu-Huei Wu, PRD, 2011.

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Introduction

1. Surface layers in vacuo. Domain walls are vacuumlike hypersurfaces where the positive tension equals the mass density, i.e., $\tau = \sigma$.
2. Domain walls can be form in the early Universe by the second-order phase transition, which is known as Kibble mechanism [Kibble 1976].
3. A domain wall by itself is a source of repulsive gravity [Ipser and Sikivie, PRD, 1984].
4. What are gravitational effects of DWs on primordial quantum fluctuations of the inflaton field? Can these effects be observed from the CMB temperature anisotropies? [Chih-Hung Wang, Yu-Huei Wu, and Stephen D. H. Hsu, 2012, arXiv: gr-qc/1107.1762]

Motivation

1. The main motivation is to generalize our previous work on planar solution with reflection symmetriy in de-Sitter spacetime [Chih-Hung Wang, Hing-Tong Cho, and Yu-Huei Wu, PRD, 2011] to spherical, planar, and hyperbolic domain wall solutions without reflection symmetry [Yu-Huei Wu and Chih-Hung Wang, to be submitted, 2012].
2. On equivalence of **comoving-coordinate approach** and **moving-wall approach**.
3. Equation of motion can be calculated and compare with the results of moving wall approach.
4. Mass term on spherical domain wall hypersurface can be calculated and we found positivity of mass does not necessarily hold in our construction.
5. Global structure and Penrose diagram.

What does the bubble look like over there? Where does the bubble go?

To find the dynamics of the bubble and the equation of motion or flight with the bubble (comoving -coordinate approaches).

How do these bubbles form? Who can control the bubble dynamics? A small bubble is easy to blow but a large bubble needs more efforts.

Phase transition in the early Universe and perhaps God knows! Energy and global structure of the bubble?

On the equivalence of comoving-coordinate and moving-wall approaches

- Consider a spherical, planar, or hyperbolic domain wall sitting at $r = r_0$, and the metric solutions inside and outside the wall give

$$ds^2_\pm = -4A_\pm(r,\eta)\frac{d\eta^2 - dr^2}{(r-\eta)^2} + B^2_\pm(r,\eta)dV_2 \quad (1)$$

where $A_\pm := -F_\pm G_\pm L_\pm$ and $L_\pm = (k - \frac{2M_\pm}{B_\pm} - \frac{\Lambda_\pm B^2_\pm}{3})$ where $k = -1, 0, +1$. $B_\pm$ needs to satisfy

$$dB_\pm = -L_\pm[(\frac{F_\pm}{(r-\eta)^2} + G_\pm)d\eta^2 + (-\frac{F_\pm}{(r-\eta)^2} + G_\pm)dr^2]. \quad (2)$$

Here, the subscript $\pm$ denotes the solutions for exterior the wall $(+)$ and interior the wall $(-)$, respectively.

- Because of the freedom of choosing coordinates, we have four unknown functions $F_\pm = F_\pm(\frac{1}{r-\eta})$ and $G_\pm = G_\pm(r+\eta)$, which are functions of $\frac{1}{r-\eta}$ and $r+\eta$ respectively.
- Thin domain wall solutions need to further satisfy
  1. Metric continuity

$$B_+|_{r=r_0} = B_-|_{r=r_0} = B, \qquad A_+|_{r=r_0} = A_-|_{r=r_0} \tag{3}$$

  2. Israel's Junction condition

$$\pi_{ab+}|_{r=r_0} - \pi_{ab-}|_{r=r_0} = -\frac{\kappa\sigma}{2} h_{ab} \tag{4}$$

  where $\pi_{ab} = \mathcal{L}_n h_{ab}$.
- The four functions $F_\pm$ and $G_\pm$ at $r = r_0$ should satisfy Eqs. (3) and (4), which have three independent equations. So there remain one unknown function.

- By requiring $A_{\pm}|_{r=r_0} = -1$, which corresponds to coordinate time η being the proper time on the wall, we obtain

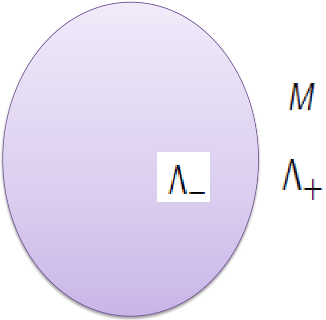
$$\sqrt{L_+ + \dot{B}^2} - \sqrt{L_- + \dot{B}^2} = -\frac{\kappa\sigma}{2} B, \tag{5}$$

which are the same as the well-known equations of motion of domain walls in a static background spacetime (i.e. moving-wall approach).

- We also find that by solving Eq. (5), one can obtain $F_{\pm}$, $G_{\pm}$, and also $B_{\pm}$. It means that by knowing the trajectory of the wall, we can construct a comoving coordinates and obtain the metric solutions in this coordinates.
- We find that the massless bubbles has several types of exact solutions.

A special case: $K = 1, M_- = 0, \Lambda_\pm > 0$

We start from a special case. Surface layers of spherical domain wall bubble in vacuo.



The metric ds_-^2 in the region V_- interior to the shell and the metric ds_+^2 in the region V_+ exterior to the shell are

$$ds_-^2 = -4A_-(r,\eta)\frac{d\eta^2 - dr^2}{(r-\eta)^2} + B_-^2(r,\eta)dV_2 \tag{6}$$

$$ds_+^2 = -4A_+(r,\eta)\frac{d\eta^2 - dr^2}{(r-\eta)^2} + B_+^2(r,\eta)dV_2 \tag{7}$$

where

$$A_- = -F_- G_- L_-, A_+ = -F_+ G_+ L_+ \tag{8}$$

and

$$L_- = (1 - \frac{\Lambda_- B_-^2}{3}), L_+ = (1 - \frac{2M}{B_+} - \frac{\Lambda_+ B_+^2}{3}) \tag{9}$$

and from Einstein field equation we have

$$dB_\pm = -L_\pm[(\frac{F_\pm}{(r-\eta)^2} + G_\pm)d\eta^2 + (-\frac{F_\pm}{(r-\eta)^2} + G_\pm)dr^2]. \tag{10}$$

Mass of the bubble

Mass of the bubble on spherical domain wall hypersurface is

$$M = E_V + E_S \tag{11}$$

where E_V is the volume energy

$$E_V = \frac{1}{6}(\Lambda_- - \Lambda_+)B^3 \tag{12}$$

and E_S is the surface energy of the bubble

$$E_S = \frac{\kappa\sigma}{4}B^2[2(1 - \frac{\Lambda_- B^2}{3} + \dot{B}^2)^{1/2} - \frac{\kappa\sigma}{2}B]. \tag{13}$$

Here we use the metric continuity and Junction condition in our calculation.

Equation of motion of domain wall

From Israel's Junction condition in our comoving wall coordinate, we have

$$\dot{B}^2 = -1 + \frac{M}{B}\left(\frac{4}{3\kappa^2\sigma^2}(\Lambda_+ - \Lambda_-) + 1\right) + H^2 B^2 + \frac{4M^2}{\kappa^2\sigma^2 B^4} \tag{14}$$

where

$$H^2 = \frac{1}{\kappa^2\sigma^2}\left[\frac{(\Lambda_+ - \Lambda_-)^2}{9} + \frac{2}{3}\left(\Lambda_+ - \Lambda_- + \frac{\kappa^2\sigma^2}{256}\right)\right]. \tag{15}$$

It agrees with [Aurilia, Kissack, Mann, and Spallucci, PRD 1987] and we can use it to describe the bubble dynamics.
Similar to the equation for a matter dominated, spatially closed , Friedmann universe

$$\left[\frac{dR}{d\tau}\right]^2 = -1 + \frac{\Lambda}{3}R^2 + \frac{\kappa\sigma R_0^3}{3R}. \tag{16}$$

A spherical solution $k = 1, M_{\pm} = 0, \Lambda_+ > \Lambda_-$

We find a spherical domain wall exact solution $ds^2_{\pm}$

$$ds^2_{\pm} = \frac{3}{\Lambda_{\pm}} \frac{H^2}{\cosh^2(Hr - \rho')}\left(-d\eta^2 + dr^2 + \frac{\cosh^2 H\eta}{H^2} dV_2\right) \tag{17}$$

where $\rho'_{\pm} = Hr_0 - \rho_{\pm}$ and $\sqrt{\frac{3}{\Lambda_{\pm}}}H = \cosh \rho_{\pm}$. This result is agreed with the results from M. Cvetič *et al* (1993). Changing coordinate we can get

$$ds^2 = (\alpha \cos t_c)^{-2}(dt_c^2 - d\psi^2 - \sin^2 \psi dV_2) \tag{18}$$

where $-\pi \leq t_c \pm \psi \leq \pi$, $0 \leq \psi \leq \pi/2$. Note that this will be useful later when we look at global structure.

Planar DWs in de-Sitter space

1. So far, we discuss the spherical, planar, and hyperbolic domain wall solutions by requiring $A_{\pm}|_{r=r_0} = -1$ and these solutions in the case of $M_{\pm} = 0$ agree with the solutions obtained by M. Cvetič *et al.*
2. It is interesting to know how to recover our previous planar DW solution in de-Sitter spacetime [Wang-Cho-Wu, PRD 2011]. It turns out that instead of setting $A_{\pm}|_{r=r_0} = -1$, we should set $A_{\pm}|_{r=r_0} = -\frac{\alpha^2}{\eta^2}$, which means that the coordinate time η on the wall corresponds to conformal time in de-Sitter spacetime. Moreover, this condition on $A_{\pm}$ has also been used to investigate the spherical and hyperbolic domain wall solutions in the comoving coordinates. (Wu & Wang, 2012)

The metric of planar DWs

- Thus the metric of a planar domain wall in de-Sitter space-time (Wang, Cho, & Wu, PRD, 2011) with reflection symmetry has directly calculated from (Wu and Wang 2012), thus we get :

$$ds^2 = \frac{1}{\alpha^2 \left(\eta + \beta|z|\right)^2}\left(-d\eta^2 + dz^2 + dx^2 + dy^2\right), \qquad (19)$$

where the wall is placed at $z = 0$. $\alpha = \sqrt{\Lambda/12\Gamma}(\Gamma + 1)$, $\beta = \frac{\Gamma - 1}{\Gamma + 1}$, satisfying $-1 < \beta \leqslant 0$, and Γ is a dimensionless parameter

$$\Gamma = 1 + \frac{3\epsilon - \sqrt{48\epsilon + 9\epsilon^2}}{8}, \qquad (20)$$

where $\epsilon = \frac{\kappa^2\sigma^2}{\Lambda}$ and σ is the surface tension of the domain wall. Eq. (20), which gives $0 < \Gamma \leqslant 1$, is only valid for the coordinate ranges $-\infty < \eta + \beta|z| < 0$.

Global structure of planar DWs

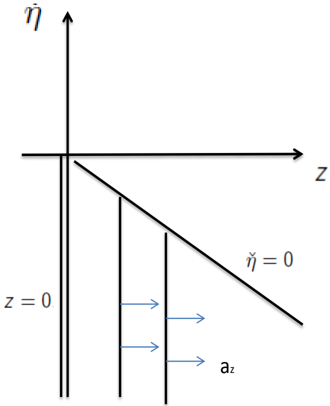
Our planar domain wall solution for $z > 0$ side can be rewritten as

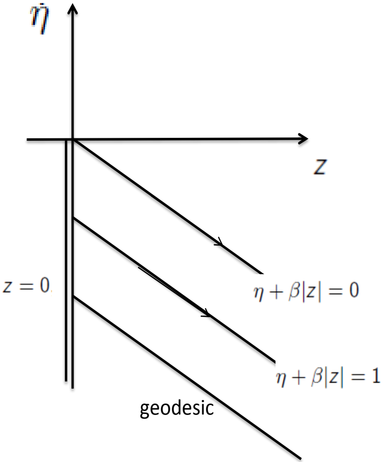
$$ds^2 = \frac{1}{\frac{\Lambda}{3}\check{\eta}^2}[-d\check{\eta}^2 + d\check{z}^2 + d\check{y} + d\check{z}] \tag{21}$$

where $\check{\eta} = \frac{\eta+\beta z}{\sqrt{1-\beta^2}}, -1 < \beta \leqq 0$ and $-\infty < \eta < -\beta z$.
The 4-acceleration, which is defined by $A = \nabla_u u$, gives

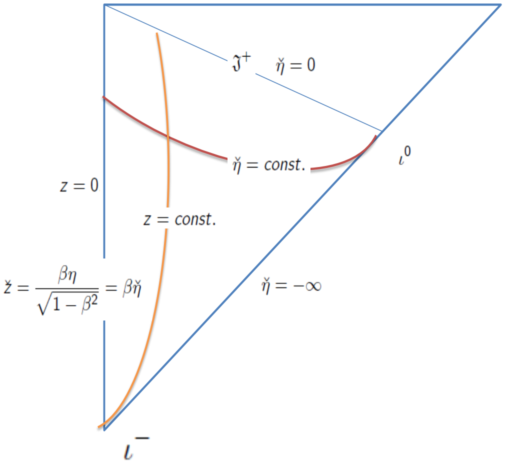
$$A_z = -|\alpha\beta| = -\frac{\kappa\sigma}{4}, \tag{22}$$

where $u = -\alpha(\eta + \beta z)\partial_\eta$

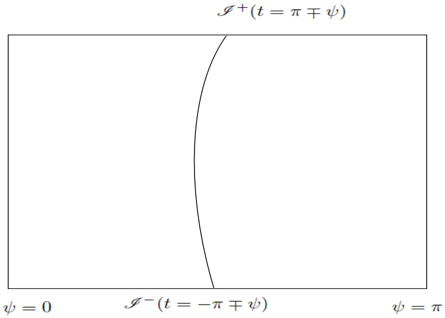

$\dot{\eta}$
z
$\check{\eta} = 0$
$z = 0$
a_z


$\dot{\eta}$
z
$z = 0$
$\eta + \beta|z| = 0$
$\eta + \beta|z| = 1$
geodesic

Penrose diagram of comoving planar DWs



Penrose diagram of spherical domain wall



K=1 Domain Wall solution in De-Sitter spacetime

Conclusion and Outlook

- Mass term can be negative or zero.
- We present a general proof on the equivalence of comoving coordinate approach and moving wall approach.
- The gravitational effects of spherical and hyperbolic domain wall spacetimes on primordial quantum fluctuations will be studied in our future work.
- We also plan to study the primordial gravitational waves in domain wall spacetimes.

Conclusion and Outlook

- Several special case of domain wall solutions are studied. We find that the massless bubbles has several types of exact solutions. [Wu and Wang 2012]
- Off domain wall mass term may be calculated from the quasi-local expressions in the future and would be important for gravitational radiation from the domain wall.
- Particle creation in domain-wall space-time and experiment of Unruh like effect in the Universe.
- If one believe in phase transition could happen in early Universe, then one cannot exclude the possibilities of the existence of domain walls and topological defects and these might further lead to the effect on gravity and CMB.

Thank you!

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