Quantum stress tensor fluctuations of a conformal field and inflationary cosmology

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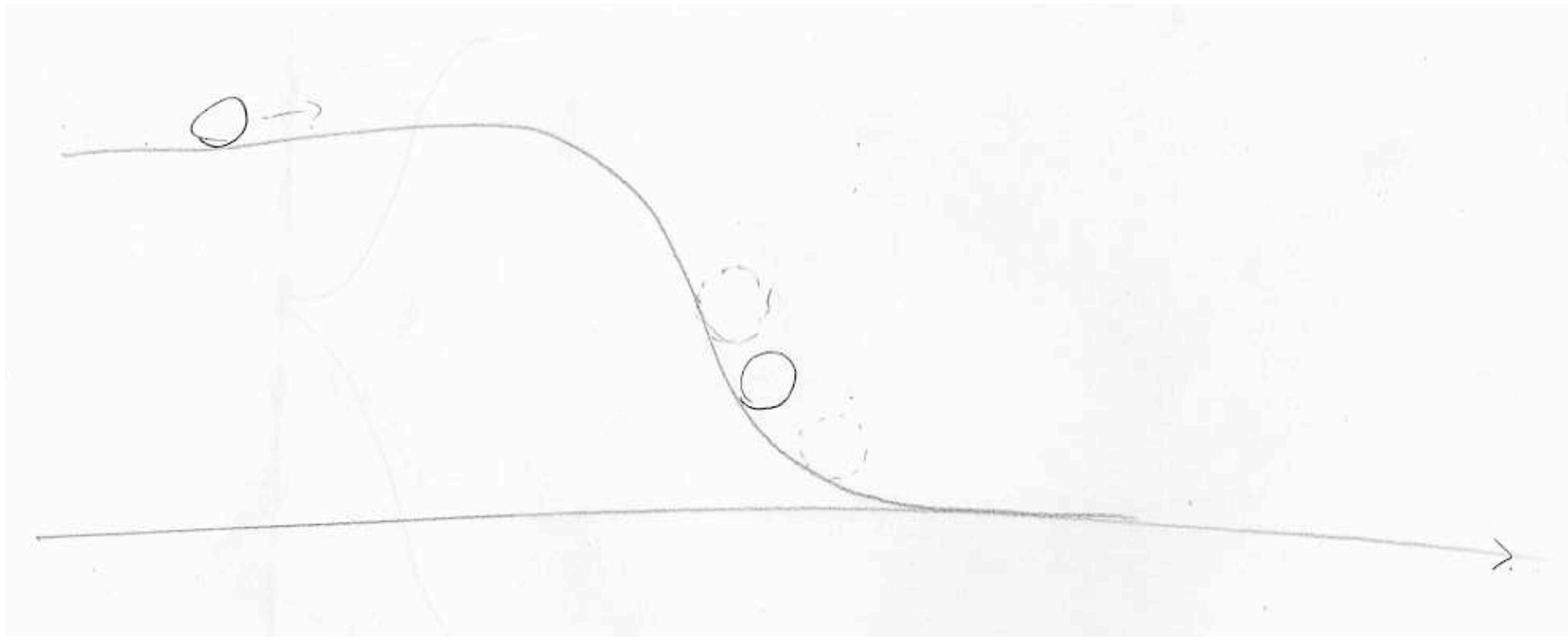
Plan of talk

- Review of density perturbation in inflation
- Different models and Langenvin equation
- A dynamical model: basic ideas and results
- Conclusion

- The usual picture is that inflation washed out the initial classical density perturbation.

- The primordial density perturbation is due to quantum fluctuation of inflaton field.

- The basic idea is that spatial variations in inflaton field cause different regions to reheat at different times.



Other quantum matter

+

Inflaton → Inflaton fluctuation → Density fluctuation

+

Background + Active + Passive

spacetime metric fluctuation metric fluctuation

Stress tensor fluctuation
due to
other quantum matter

A Dynamical Model

Consider a single scalar inflaton field φ and
a fluctuating conformal field stress tensor.

Basic idea:

Solve the coupled Einstein-φ equations to find $\Delta\varphi$, the fluctuation in φ induced by the stress tensor fluctuations.

$$\Box\varphi + V'(\varphi) = 0$$

$$G_{\mu\nu} = 8\pi T^{\rm total}_{\mu\nu} = 8\pi(T^{\rm infl}_{\mu\nu} + T^{\rm conf}_{\mu\nu})$$

Then use the conservation law to find density perturbation.

First order perturbations:

Inflaton: $\varphi(t, \mathbf{x}) = \varphi_0(t) + \delta\varphi(t, \mathbf{x})$

Metric:

$$ds^2 = -dt^2 + 2a(t)h_{0i}(t, \mathbf{x})dtdx^i + a^2(t)\Big[\delta_{ij} + h_{ij}(t, \mathbf{x})\Big]dx^i dx^j$$

Unit normal to surfaces of constant φ

$$u^\mu = -\frac{g^{\mu\nu}\partial_\nu\varphi}{\sqrt{-g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi}} = \delta^\mu_0$$

Obtain a pair of 2nd order coupled equations for $\delta\varphi$ and $h_{\mu\nu}$

Eliminate $h_{\mu\nu}$ and obtain an equation for

$$B(t,k) = \delta\varphi(t,k)/\dot{\varphi}_0$$

(spatial Fourier transform)

$$\left(\partial_t^2 + 5H\,\partial_t + 6H^2 + \frac{k^2}{a^2}\right)\dot{B}(t,k) = 8\pi\, T_{tt}^{\rm conf}$$

(slow roll approximation)

Set $B = 0$ at $\eta = \eta_0$

Result for the momentum space correlation function:

$$\langle B^2 \rangle_k$$

$$= (8\,\pi)^2\,\frac{\ell_p^4}{k^2}\int_{\eta_0}^{0}\frac{d\eta_1}{a(\eta_1)}\int_{\eta_0}^{0}\frac{d\eta_2}{a(\eta_2)}\int_{\eta_0}^{\eta_1} d\eta\,\sin[k(\eta-\eta_1)]\int_{\eta_0}^{\eta_2} d\eta'\,\sin[k(\eta'-\eta_2)]\,\mathcal{E}(\Delta\eta,k)$$

$\mathcal{E}(\Delta\eta, r)$ = flat space energy density correlation function

Conformally invariant fields:

$$C^{RW}_{\mu\nu\alpha\beta}(x,x') = a^{-4}(\eta)\,a^{-4}(\eta')\,C^{flat}_{\mu\nu\alpha\beta}(x,x')$$

Stress tensor correlation function

$$C_{\mu\nu\alpha\beta}(x,x') = \langle T_{\mu\nu}(x)\,T_{\alpha\beta}(x')\rangle - \langle T_{\mu\nu}(x)\rangle\langle T_{\alpha\beta}(x')\rangle$$

(conformal anomaly cancels)

$$\langle B^2 \rangle_k$$

$$= (8\,\pi)^2\,\frac{\ell_p^4}{k^2}\int_{\eta_0}^{0}\frac{d\eta_1}{a(\eta_1)}\int_{\eta_0}^{0}\frac{d\eta_2}{a(\eta_2)}\int_{\eta_0}^{\eta_1} d\eta\,\sin[k(\eta-\eta_1)]\int_{\eta_0}^{\eta_2} d\eta'\,\sin[k(\eta'-\eta_2)]\,\mathcal{E}(\Delta\eta,k)$$

$$\sim -\frac{\ell_p^4\,H^2\,|\eta_0|^3}{480\pi^2}+\frac{\ell_p^4\,H^2\,\eta_0^2}{600\pi\,k}$$

Then calculate $\mathcal{P}_k$ from $\langle B^2 \rangle_k$ using the conservation law.

$$C_\rho(x_1, x_2) = \left\langle \frac{\delta\rho(x_1)}{\rho} \; \frac{\delta\rho(x_2)}{\rho} \right\rangle$$

$$= (8\,\pi)^2\,\ell_p^4\,(1+w)^2 \int_0^{\eta_s} \frac{d\eta_1}{a(\eta_1)} \int_0^{\eta_s} \frac{d\eta_2}{a(\eta_2)} \int_{\eta_0}^{\eta_1} \frac{d\eta}{a(\eta)} \int_{\eta_0}^{\eta_2} \frac{d\eta'}{a(\eta')}\,\mathcal{E}(\Delta\eta, r)$$

$\ell_p =$ Planck length

$\eta_s =$ conformal time of last scattering

(Non-Gaussian fluctuations)

Power spectrum of the density fluctuations:

$$\mathcal{P}(k) = \frac{k^3}{2\pi^2} \int d^3r\, \mathrm{e}^{-ikr}\, C_\rho(r, \eta_s)$$

Associated power spectrum of density perturbations:

$$\mathcal{P}_k \approx \frac{2\,\pi\,\ell_p^4 H^4}{15}\left(-|k\eta_0|^3 + \frac{4\pi}{5}|k\eta_0|^2 + \cdots\right)$$

leads to an unobservable delta function term in the spatial correlation function, so ignore this term

Power spectrum is not scale invariant and depends upon the duration of inflation.

In any case, here we use the positive term:

$$\mathcal{P}_k \approx \frac{8\pi\,\ell_p^4\,H^4}{75}\,k^2\,\eta_0^2$$

It must be a small overall contribution to the total power spectrum, which leads to a constraint on the scale factor change during inflation:

$$\frac{1}{a_0} < 10^{42}\left(\frac{10^{12}\ \mathrm{GeV}}{T_R}\right)^3$$

reheat temperature

Allows enough inflation to solve the horizon and flatness problems

Does the η_0^2 dependence contradict Weinberg's result that radiative effects in inflation should grow no faster than ln(a)?

One interpretation of our result is that it is not so much growing, as always large - the contribution of very energetic modes is preserved.

Use of transplanckian modes.

Dominant contribution comes from modes with

$$\lambda \approx 10^{-8}\,\ell_p \quad (T_R = 10^{12}\,\mathrm{GeV})$$

Is this a problem?

This effect might offer a probe of transplanckian physics in the form of a small non-Gaussian, non-scale invaraint component in the CMB.

Summary

Stress tensor fluctuation in inflation produce a non-Gaussian, non-scale invariant component in the density fluctuations.

This suggests an upper bound on the length of inflation, but adequate to solve the horizon problem.

Possibility of an observational test, and a possible probe of transplanckian physics and quantum gravity.