Van der Waals like Phase Structure of Black Hole

Xiaoning Wu
Institute of Mathematics, AMSS
Cooperated with Y.Tian, C. Niu, Y. Cai and Y. Yang

- Black hole mechanical law

0th law : $\varkappa$ is constant on horizon.

1st law :

$$\delta M = \frac{\kappa}{2\pi}\delta\frac{A_H}{4} + \Omega_H \delta J + \Phi_H \delta Q$$

2nd law : the area of horizon never decrease.

3rd law : Impossible to achieve $\varkappa=0$ by a physical process.

- Compare with ordinary thermodynamic law

0th law : for a system at thermal equilibrium, T is a constant.

1st law : $dU=TdS+PdV$

2nd law : Entropy never decrease.

3rd law : Impossible to achieve $T=0$ by a physical process.

- Black Hole with cosmological constant
  - Reissner-Nordstrom black hole with cosmological constant

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{n-1}^2,$$

$$f(r) = k - \frac{8\Gamma(\frac{n}{2})M}{(n-1)\pi^{\frac{n}{2}-1}r^{n-2}} + \frac{Q^2}{r^{2n-4}} + \frac{r^2}{l^2}$$

$$\Lambda = -\frac{n(n-1)}{2l^2}$$

Where k=1,0,-1, corresponds to spherical, plane and hyperbola symmetry.

- Kerr black hole with cosmological constant

$$ds^2 = -\frac{\Delta_r}{\Sigma}\left(dt - \frac{a\sin^2\theta}{\Xi}d\phi\right)^2 + \frac{\Sigma}{\Delta_r}dr^2 + \frac{\Sigma}{\Delta_\theta}d\theta^2 + \frac{\Delta_\theta \sin^2\theta}{\Sigma}\left(adt - \frac{r^2+a^2}{\Xi}d\phi\right)^2,$$

$$\Delta_r = (r^2+a^2)\left(1+\frac{r^2}{l^2}\right) - 2Mr, \quad \Xi = 1 - \frac{a^2}{l^2},$$

$$\Delta_\theta = 1 - \frac{a^2}{l^2}\cos^2\theta, \qquad \Sigma = r^2 + a^2\cos^2\theta.$$

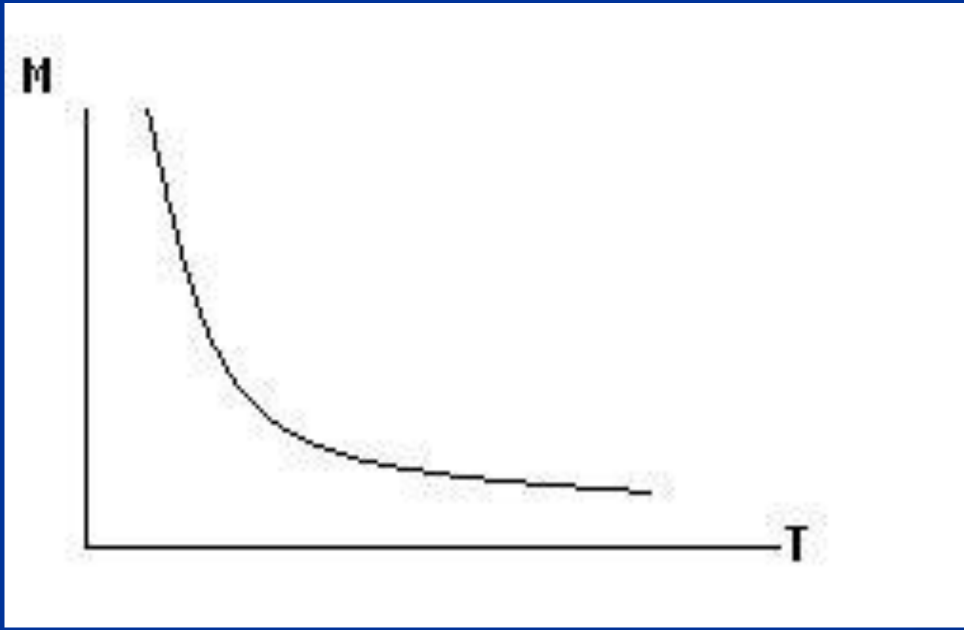
- Phase structure of black hole

  - Schwarzschild black hole

$$M = \frac{1}{8\pi T}$$

$$C := \frac{\partial M}{\partial T} < 0$$

Thermal unstable.

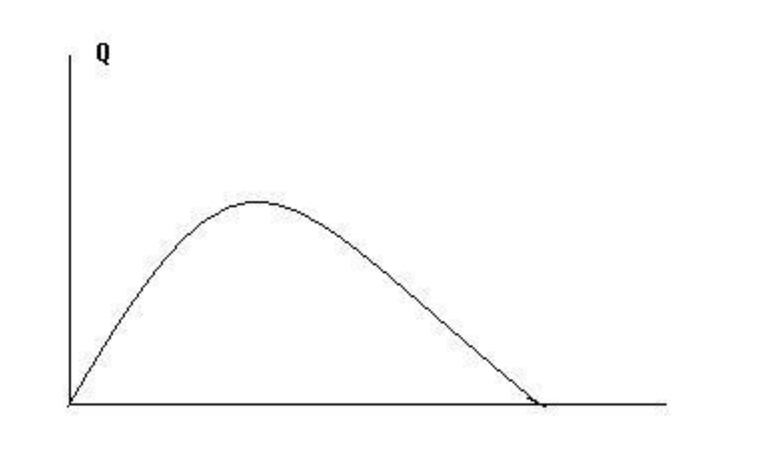


- Reissner-Nordstroem black hole

$$r_+^2 - 2Mr_+ + Q^2 = 0,$$
$$\frac{1}{2\pi}\left(\frac{M}{r_+^2} - \frac{Q^2}{r_+^3}\right) = T$$

$$T = \frac{1}{4\pi}\frac{-\Phi^4 + \Phi^2}{Q\Phi}$$

$$\delta M = T\delta S + \Phi\delta Q.$$

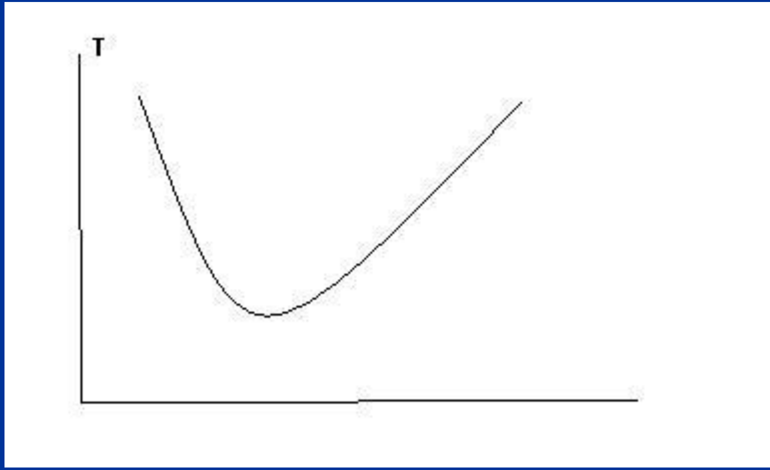


Davis phase transition.

- Schwarzschild-AdS black hole

$$\frac{\Lambda}{3}r_+^3 + r_+ - 2M = 0,$$

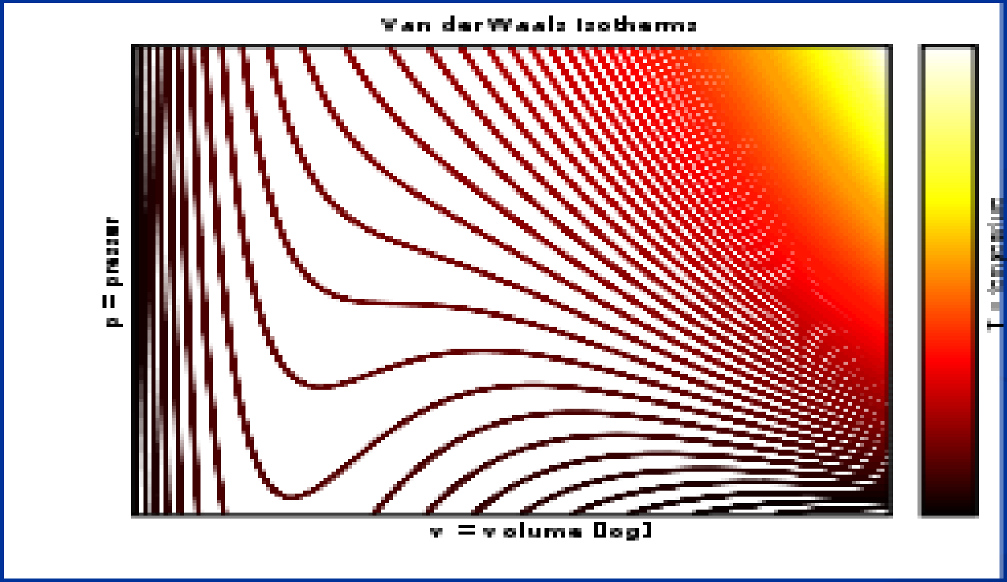
$$\frac{1}{2\pi}\left(\frac{2M}{r_+^2} + \frac{2}{3}\Lambda r_+\right) = T.$$



Hawking-Page phase transition.
(AdS/QCD)

- Thermal structure of black hole
  - Van de Waals gas

$$\left(p + \frac{a'}{v^2}\right)(v - b') = kT$$



- Phase structure of Van de Waals gas

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_V, \quad C_P = T\left(\frac{\partial S}{\partial T}\right)_P, \quad \kappa_T = \left(\frac{\partial V}{\partial P}\right)_T$$

- Critical point

$$State\ equation,$$
$$\left(\frac{\partial P}{\partial V}\right)_c = 0,$$
$$\left(\frac{\partial^2 P}{\partial V^2}\right)_c = 0$$

- Critical exponents (see A modern course in statistical physics / Reichl, L. E.)

$$(1) \quad P - P_c \sim (V - V_c)^{\delta}$$

$$(2) \quad \frac{V_g - V_l}{V_c} \sim (-\epsilon)^{\beta}$$

$$(3) \quad C_P \sim (-\epsilon)^{-\alpha'} \quad (T < T_c)$$

$$\sim \epsilon^{-\alpha} \quad (T > T_c)$$

$$(4) \quad \kappa_T \sim (-\epsilon)^{-\gamma'} \quad (T < T_c)$$

$$\sim \epsilon^{-\gamma} \quad (T > T_c).$$

- Scaling law (see A modern course in statistical physics / Reichl, L. E.)

$$\alpha + 2\beta + \gamma = 2$$

$$\alpha + \beta(\delta + 1) = 2$$

$$\gamma(\delta + 1) = (2 - \alpha)(\delta - 1)$$

$$\gamma = \beta(\delta - 1)$$

$$F(\Lambda^p \epsilon, \Lambda^q \Pi) = \Lambda F(\epsilon, \Pi)$$

Main critical exponents

α	0
β	1/2
γ	1
δ	3

- Phase structure of Reissner-Nordstrom-anti de Sitter black hole
    - State equation of black hole

$$T = \left(\frac{\partial M}{\partial S}\right)_Q = \frac{1}{4\pi}\frac{-\frac{2\Lambda}{n-1}r_+^{2n-2} + (n-2)kr_+^{2n-4} - (n-2)Q^2}{r_+^{2n-3}}$$

$$\Phi = \left(\frac{\partial M}{\partial Q}\right)_S = \frac{(n-1)\pi^{\frac{n}{2}-1}}{4\Gamma(\frac{n}{2})}\frac{Q}{r_+^{n-2}}$$

$$C_Q = T\left(\frac{\partial S}{\partial T}\right)_Q = \frac{2(n-1)\pi^{\frac{n}{2}+1}}{\Gamma(\frac{n}{2})}\frac{r_+^{3n-4}T}{-\frac{2\Lambda}{n-1}r_+^{2n-2} - (n-2)kr_+^{2n-4} + (n-2)(2n-3)Q^2}$$

For non-extreme case, k=0,-1 has no phase transition, i.e. C_Q is always positive.

For spherical case,

With the formal correspondence $(\Phi, Q) \leftrightarrow (V, P)$

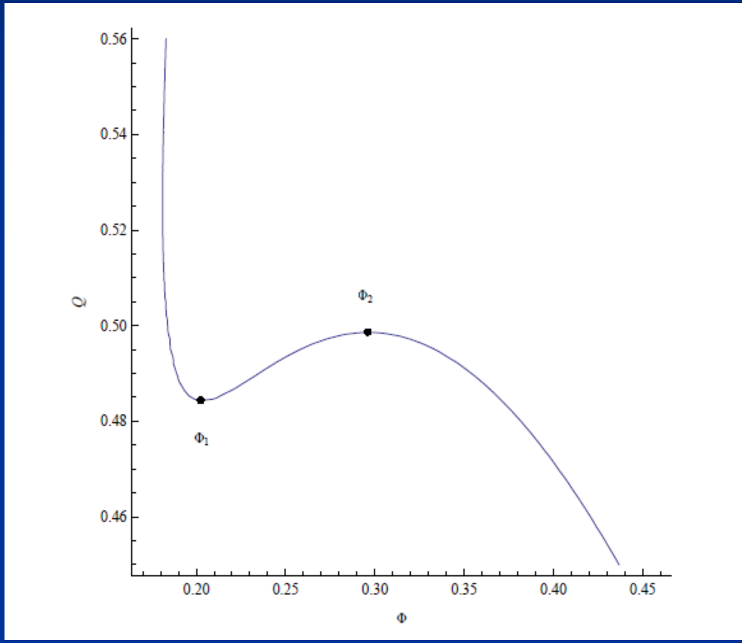
- Critical point

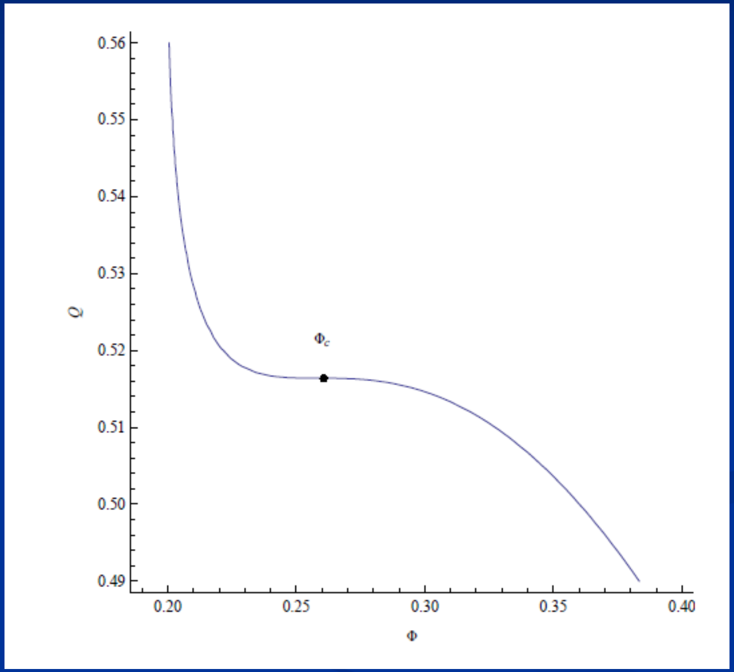
$$\left(\frac{\partial Q}{\partial \Phi}\right)\bigg|_c = 0$$

$$\left(\frac{\partial^2 Q}{\partial \Phi^2}\right)\bigg|_c = 0$$

equation of state

- Isothermal curve





- Critical exponents

Main critical exponents	
α	0
β	1/2
γ	1
δ	3

- Scaling law

$$\begin{aligned} \alpha + 2\beta + \gamma &= 2 \\ \alpha + \beta(\delta - 1) &= 2 \\ \gamma(\delta - 1) &= (2 - \alpha)(\delta - 1) \\ \gamma &= \beta(\delta - 1). \end{aligned}$$

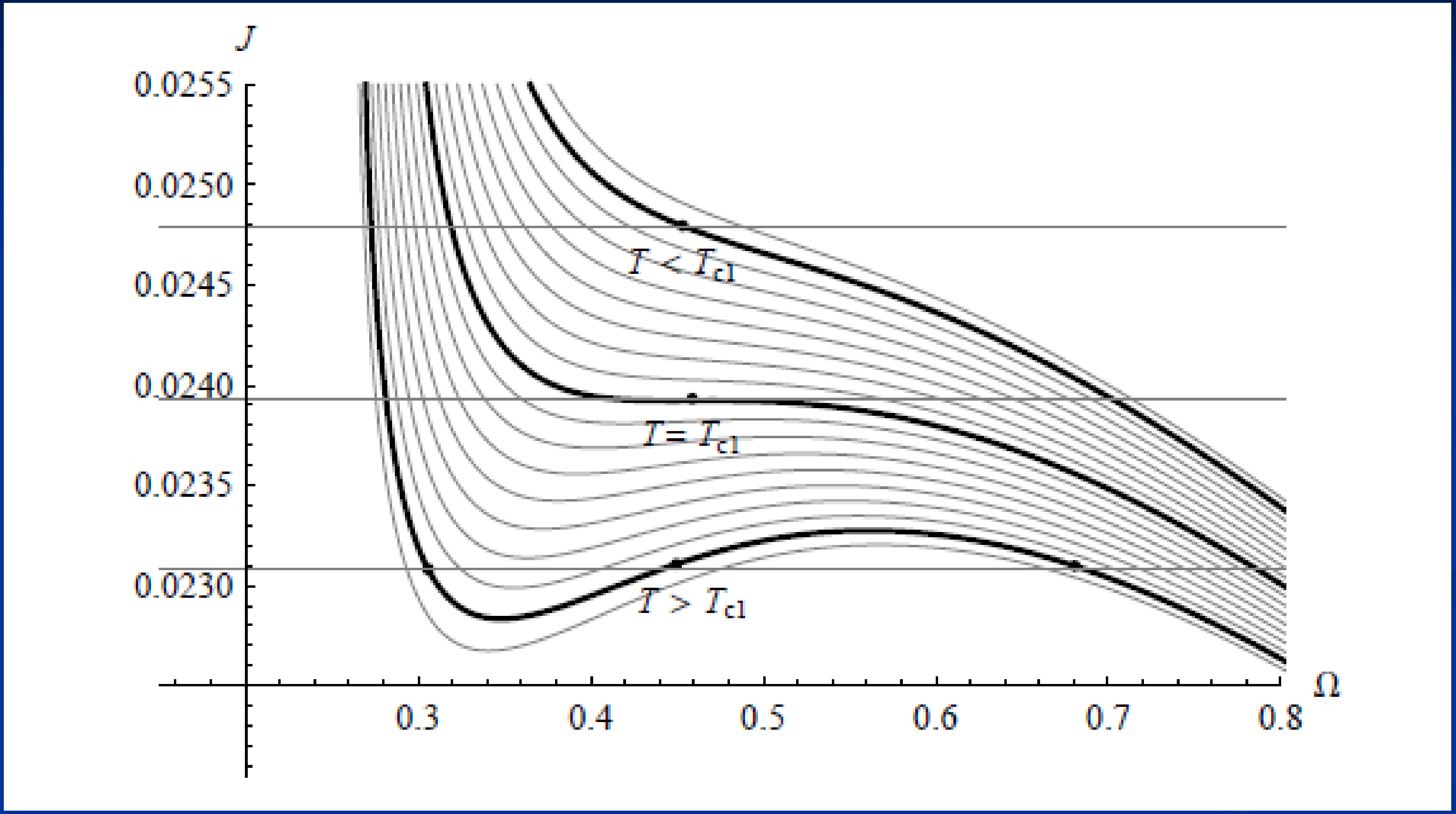
$$\begin{aligned} \alpha &= 1 - \tfrac{1}{p} \\ \beta &= \tfrac{1-q}{p} \\ \gamma &= \tfrac{2q-1}{p} \\ \delta &= \tfrac{q}{1-q}. \end{aligned}$$

- Phase structure of Kerr-anti de Sitter black hole
  - State equation of black hole

$$J = \frac{r_+^2\sqrt{(1 - r_+^2 + 4\pi r_+ T)(1 + 3r_+^2 - 4\pi r_+ T)}}{(1 - 3r_+^2 + 4\pi r_+ T)^2},$$

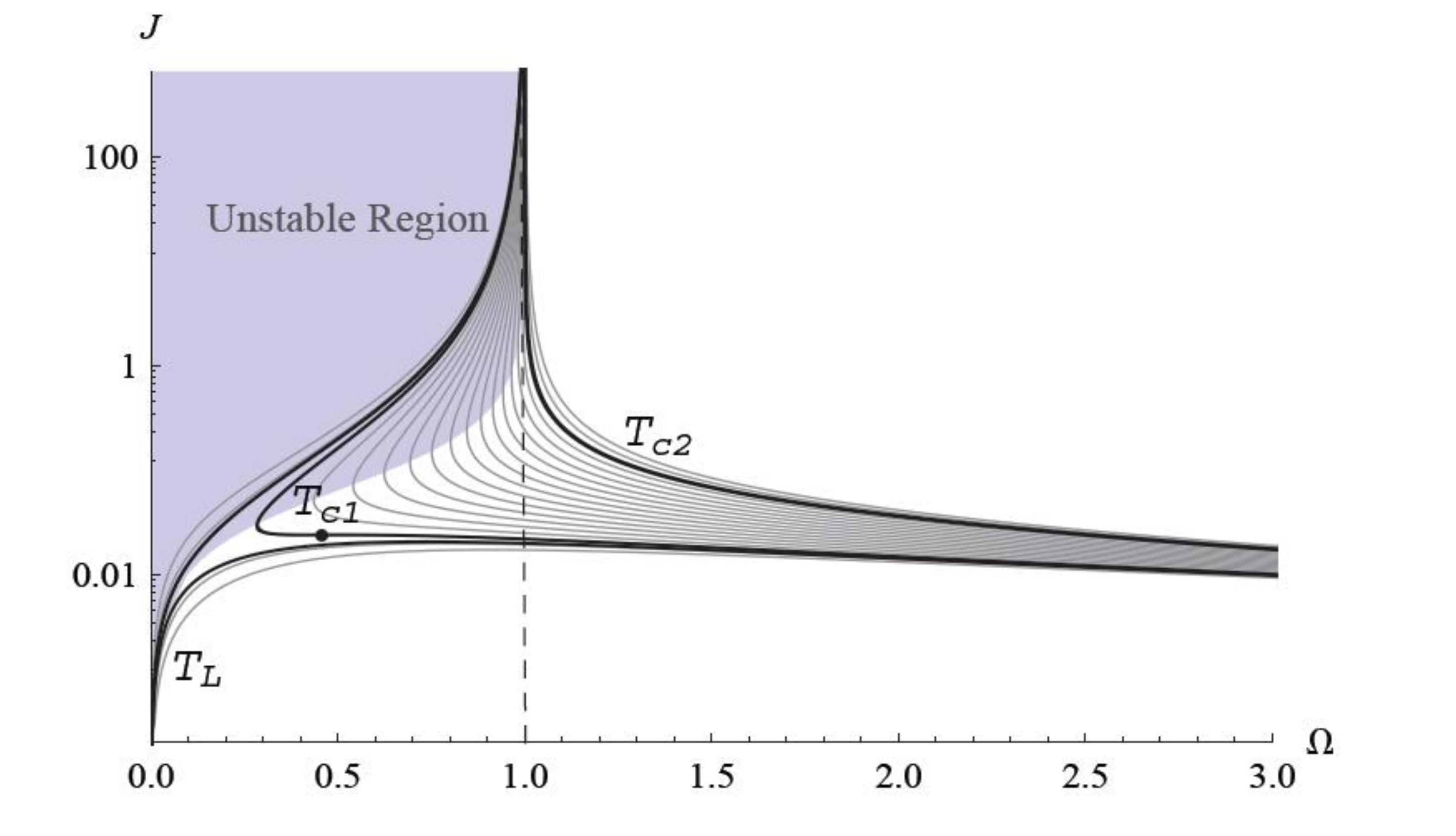
$$\Omega = \frac{\sqrt{(1 - r_+^2 + 4\pi r_+ T)(1 + 3r_+^2 - 4\pi r_+ T)}}{2r_+}.$$

- Isothermal curve



Main critical exponents	
α	0
β	1/2
γ	1
δ	3

- Another phase structure



Remark :

1. How to understand such phase structure?
2. Relations with AdS/CFT
3. Relations with AdS/condense matter
4. ……

Thank you