General Relativity without paradigm of space-time covariance: sensible quantum gravity and resolution of the "problem of time"

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The purposes of this talk:

Main Themes of this work:

Theory of gravity with only spatial covariance, construction of local Hamiltonian for dynamical evolution and resolution of "problem of time"

- Any sensible quantum theory of time has to link quantum time developments to passage of time measured by physical clocks in classical space-times! Where/what is physical time in Quantum Gravity?

Outlines of this talk:

1. Hints/Ingredients for a sensible theory of Quantum Gravity
2. Theory of gravity without full space-time covariance
    - General framework, and quantum theory
    - Emergence of classical space-time
    - Paradigm shift and resolution of "problem of time"
    - Improvements to the quantum theory
3. Further discussions

Time: Fundamental, emergent , or illusionary, ...?

The Measure of Time

From Wikisource

The Measure of Time (1898)
by Henri Poincaré, translated by George Bruce Halsted

In French: *Poincaré, Henri (1898), "La mesure du temps", Revue de métaphysique et de morale* ***6****: 1-13*

If now it be supposed that another way of measuring time is adopted, the experiments on which Newton's law is founded would none the less have the same meaning. Only the enunciation of the law would be different, because it would be translated into another language; it would evidently be much less simple. So that the definition implicitly adopted by the astronomers may be summed up thus: Time should be so defined that the equations of mechanics may be as simple as possible. In other words, there is not one way of measuring time more true than another; that which is generally adopted is only more *convenient*. Of two watches, we have no right to say that the one goes true, the other wrong; we can only say that it is advantageous to conform to the indications of the first.

Importance of time:
Time translation <-> Hamiltonian as generator

Hints/Ingredients for a sensible theory of quantum gravity

1. Dynamics of spacetime doesn't make sense, geometrodynamics evolves in Superspace
2. QG wave functions are generically distributional, $\therefore$ concept of a particular spacetime cannot be fundamental, then why 4D covariance?
3. GR cannot enforce full 4D spacetime covariance *off-shell*, fundamental symmetry (classical and quantum) is 3D diffeomorphism invariance(arena $=$ Superspace)
4. The local Hamiltonian should not be the generator of symmetry, but determines only dynamics
5. Local Hamiltonian constraint $H = 0$ replaced by Master Constraint; $\mathrm{M} := \int_\Sigma \frac{[H(x)]^2}{\sqrt{q(x)}} = 0$, not just math. trick but Paradigm shift
6. DeWitt supermetric has one -ive eigenvalue $\Rightarrow$ intrinsic time mode
7. A theory of QG should be described by a S-eqt *first order in intrinsic time* with $+$ive semi-definite probability density

General Framework

1. Decomposition of the spatial metric $q_{ij} = q^{\frac{1}{3}}\bar{q}_{ij}$ on Σ
2. Symplectic potential, $\int \tilde{\pi}^{ij}\delta q_{ij} = \int \bar{\pi}^{ij}\delta\bar{q}_{ij} + \pi\delta \ln q^{\frac{1}{3}} \Rightarrow \ln q^{\frac{1}{3}}$ and $\bar{q}_{ij}$ are respectively conjugate to $\pi = q_{ij}\tilde{\pi}^{ij}$ and traceless $\bar{\pi}^{ij} = q^{\frac{1}{3}}[\tilde{\pi}^{ij} - q^{ij}\frac{\pi}{3}]$ parts of the original momentum variable
3. Non-trivial Poisson brackets are

$$\left\{\bar{q}_{kl}(x), \bar{\pi}^{ij}(x')\right\} = P^{ij}_{kl}\,\delta(x,x'),\ \left\{\ln q^{\frac{1}{3}}(x), \pi(x')\right\} = \delta(x,x')$$

$P^{ij}_{kl} := \frac{1}{2}(\delta^i_k\delta^j_l + \delta^i_l\delta^j_k) - \frac{1}{3}\bar{q}^{ij}\bar{q}_{kl}$; trace-free projector depends on $\bar{q}_{ij}$

4. Separation carries over to the quantum theory, the $\ln q^{\frac{1}{3}}$ d.o.f separate from others to be identified as temporal information carrier. However, physical time intervals associated with $\delta \ln q^{\frac{1}{3}}$ can be consistently realized only when dogma of full spacetime covariance is relinquished

5. DeWitt supermetric, $G_{ijkl} = \frac{1}{2}(q_{ik}q_{jl} + q_{il}q_{jk}) - \frac{\lambda}{3\lambda-1}q_{ij}q_{kl}$, has signature $[\mathrm{sgn}(\frac{1}{3}-\lambda), +, +, +, +, +]$, and comes equipped with intrinsic temporal intervals $\delta \ln q^{\frac{1}{3}}$ provided $\lambda > \frac{1}{3}$
6. The Hamiltonian constraint is of the general form,

$$\begin{aligned}0 = \frac{\sqrt{q}}{2\kappa}H &= G_{ijkl}\tilde{\pi}^{ij}\tilde{\pi}^{kl} + V(q_{ij}) \\ &= -\frac{1}{3(3\lambda-1)}\pi^2 + \bar{G}_{ijkl}\bar{\pi}^{ij}\bar{\pi}^{kl} + V[\bar{q}_{ij}, q] \\ &= -\beta^2\pi^2 + \bar{H}^2[\bar{\pi}^{ij}, \bar{q}_{ij}, q] = -(\beta\pi - \bar{H})(\beta\pi + \bar{H})\end{aligned}$$

$$\bar{H}[\bar{\pi}^{ij}, \bar{q}_{ij}, q] = \sqrt{\tfrac{1}{2}(\bar{q}_{ik}\bar{q}_{jl} + \bar{q}_{il}\bar{q}_{jk})\bar{\pi}^{ij}\bar{\pi}^{kl} + V[\bar{q}_{ij}, q]}, \quad \beta^2 := \frac{1}{3(3\lambda-1)}$$

Einstein's GR ($\lambda = 1$ and $V[\bar{q}_{ij}, q] = -\frac{q}{(2\kappa)^2}(R - 2\Lambda_{eff})$) is a particular realization of a wider class of theories, all of which factorizes marvelously as in the last step

Note several important features:

- Only spatial diffeomorphism is intact
- Master constraint, $\mathrm{M} = \int H^2/\sqrt{q} = 0$ equivalently enforces the local constraint and its physical content. H determines dynamical evolution but not generates symmetry. M decouples from H_i is attained, paving the road for quantization$\Rightarrow$ For, theories with only spatial diff. inv. will have physical dynamics dictated by H, but encoded in M
- M itself does not generate dynamical evolution, but only spatial diff.; $\{q_{ij}, m(t)\mathrm{M} + H_k[N^k]\}|_{\mathrm{M}=0 \Leftrightarrow H=0} \approx \{q_{ij}, H_k[N^k]\} = \mathcal{L}_{\vec{N}} q_{ij}$. Therefore, true physical evolution can only be w.r.t to an intrinsic time extracted from the WDW eqt
- only $(\beta\pi + \bar{H}) = 0$ is all that is needed to recover the classical content of $H = 0$. This is a breakthrough
  (i) π is conjugate to $\ln q^{\frac{1}{3}}$, therefore semiclassical HJ eqt is first order in intrinsic time with consequence of completeness
  (ii) QG will now be dictated by a corresponding WDW eqt which is a S-eqt first order in intrinsic time

Quantum Gravity

1. Consistent quantum theory of gravity starts with spatial diff. inv. $\mathrm{M}|\Psi\rangle = 0; \mathrm{M} := \int(\beta\pi + \bar{H})^2/\sqrt{q}$.
   Positive-semi-definite inner product for $|\Psi\rangle$ will equivalently imply $(\beta\hat{\pi} + \hat{\bar{H}}[\hat{\bar{\pi}}^{ij}, \hat{\bar{q}}_{ij}, \hat{q}])|\Psi\rangle = 0$ and $\hat{H}_i|\Psi\rangle = 0$
2. In metric representation; $\hat{\pi} = \frac{3\hbar}{i}\frac{\delta}{\delta \ln q}$, $\hat{\bar{\pi}}^{ij} = \frac{\hbar}{i}P^{ij}_{lk}\frac{\delta}{\delta \bar{q}_{lk}}$ operates on $\Psi[\bar{q}_{ij}, q]$; S-eqt and HJ-eqt for semi-classical states $Ce^{\frac{i\mathcal{S}}{\hbar}}$ are:

$$i\hbar\beta\frac{\delta\Psi}{\delta \ln q^{\frac{1}{3}}} = \bar{H}[\hat{\bar{\pi}}^{ij}, q_{ij}]\Psi$$

$$\beta\frac{\delta\mathcal{S}}{\delta \ln q^{\frac{1}{3}}} + \bar{H}[\bar{\pi}^{ij} = P^{ij}_{kl}\frac{\delta\mathcal{S}}{\delta\bar{q}_{kl}}; \bar{q}_{ij} \ln q] = 0$$

$\nabla_j\frac{\delta\Psi}{\delta q_{ij}} = 0$ enforces spatial diffeomorphism symmetry

3. True Hamiltonian $\bar{H}$ generating intrinsic time evolution w.r.t. $\frac{1}{\beta}\delta \ln q^{\frac{1}{3}}$

Emergence of classical spacetime

1. The first order HJ equation bridges quantum and classical regimes, has complete solution $\mathcal{S} = \mathcal{S}({}^{(3)}\mathcal{G}; \alpha)$
2. Constructive interference; $\mathcal{S}({}^{(3)}\mathcal{G}; \alpha + \delta\alpha) = \mathcal{S}({}^{(3)}\mathcal{G}; \alpha)$; $\mathcal{S}({}^{(3)}\mathcal{G} + \delta^{(3)}\mathcal{G}; \alpha + \delta\alpha) = \mathcal{S}({}^{(3)}\mathcal{G} + \delta^{(3)}\mathcal{G}; \alpha)$ $\Rightarrow \frac{\delta}{\delta\alpha}\left[\int \frac{\delta\mathcal{S}({}^{(3)}\mathcal{G};\alpha)}{\delta q_{ij}}\delta q_{ij}\right] = 0$ subject to $\mathrm{M} = H_i = 0$.

$$0 = \frac{\delta}{\delta\alpha}\left[\int(\pi^{ij}\delta q_{ij} + \delta N_i H^i) + \delta m \mathbf{M}\right]$$
$$= \frac{\delta}{\delta\alpha}\left[\int(\pi\delta \ln q^{\frac{1}{3}} + \bar{\pi}^{ij}\delta\bar{q}_{ij} + \frac{q^{ij}}{3}\delta N_i \nabla_j \pi + q^{-\frac{1}{3}}\delta N_i \nabla_j \bar{\pi}^{ij})\right]$$

$$\frac{\delta\bar{q}_{ij}(x) - \mathcal{L}_{\vec{N}dt}\bar{q}_{ij}(x)}{\delta \ln q^{\frac{1}{3}}(y) - \mathcal{L}_{\vec{N}dt}\ln q^{\frac{1}{3}}(y)} = P^{kl}_{ij}\frac{\delta[\bar{H}(y)/\beta]}{\delta\bar{\pi}^{kl}(x)} = P^{kl}_{ij}\frac{\bar{G}_{klmn}\bar{\pi}^{mn}}{\beta\bar{H}}\delta(x-y)$$

M generates no evolution w.r.t unphysical coordinate time, but w.r.t intrinsic time $\ln q^{\frac{1}{3}}$ through constructive interferences at deeper level

4. EOM relates mom. to coord. time derivative of the metric which can be interpreted as extrinsic curvature to allow emergence of spacetime

$$\frac{2\kappa}{\sqrt{q}}G_{ijkl}\tilde{\pi}^{kl} = \frac{1}{2N}\left(\frac{dq_{ij}}{dt} - \mathcal{L}_{\vec{N}}q_{ij}\right), \quad Ndt := \frac{\delta \ln q^{\frac{1}{3}} - \mathcal{L}_{\vec{N}dt}\ln q^{\frac{1}{3}}}{(4\beta\kappa\bar{H}/\sqrt{q})}$$

In Einstein's GR with *arbitrary* lapse function N, the EOM is,

$$\frac{dq_{ij}}{dt} = \left\{q_{ij}, \int d^3x[NH + N_iH^i]\right\} = \frac{2N}{\sqrt{q}}(2\kappa)G_{ijkl}\tilde{\pi}^{kl} + \mathcal{L}_{\vec{N}}q_{ij}$$

This relates the extrinsic curvature to the momentum by
$K_{ij} := \frac{1}{2N}\left(\frac{dq_{ij}}{dt} - \mathcal{L}_{\vec{N}}q_{ij}\right) = \frac{2\kappa}{\sqrt{q}}G_{ijkl}\tilde{\pi}^{kl} \Rightarrow \frac{1}{3}Tr(K) = \frac{2\kappa}{\sqrt{q}}\beta\bar{H}$
proves that the lapse function and intrinsic time are precisely related (a posteriori by the EOM) by the same formula in the above for reconstruction of spacetime

5. For theories with full 4-d diff. invariance(i.e. GR), this relation is *an identity* which does not compromise the arbitrariness of N

Paradigm shift and resolution of "problem of time"

1. Starting with only spatial diff. invariance and constructive interference, EOMs with physical evolution in intrinsic time generated by $\bar{H}$, can be obtained
2. Possible to interpret the emergent classical space-time from constructive interference to possess extrinsic curvature which corresponds precisely to the lapse function displayed in the above
3. Only the freedom of spatial diff. invariance is realized, the lapse is now completely described by the intrinsic time $\ln q^{\frac{1}{3}}$ and $\vec{N}$
4. All EOM w.r.t coordinate time t generated by $\int NH + N^i H_i$ in Einstein's GR can be recovered from evolution w.r.t. $\ln q^{\frac{1}{3}}$ and generated by $\bar{H}$ iff N assumes the form in the above
5. Full 4-dimensional space-time covariance is a red herring which obfuscates the physical reality of time, all that is necessary to consistently capture the classical physical content of even Einstein's GR is a theory invariant only w.r.t. spatial diff. accompanied by a master constraint which enforces the dynamical content

6. ADM metric,

$$ds^2 = -\left(\frac{d\ln q^{\frac{1}{3}}(x,t) - dt\mathcal{L}_{\vec{N}}\ln q^{\frac{1}{3}}(x,t)}{[4\beta\kappa\bar{H}(x,t)/\sqrt{q}(x,t)]}\right)^2 + q^{\frac{1}{3}}\bar{q}_{ij}(x,t)(dx^i + N^i dt)(dx^j + N^j dt)$$

   emerges from constructive interference of a spatial diff. invariant quantum theory with Schrodinger and HJ equations first order in intrinsic time development

7. Correlation (for vanishing shifts) between classical proper time $d\tau$ and quantum intrinsic time $\ln q^{\frac{1}{3}}$ through $d\tau^2 = [\frac{d\ln q^{\frac{1}{3}}}{(4\beta\kappa\bar{H}/\sqrt{q})}]^2$.
   Physical reality of intrinsic time intervals cannot be denied

8. In particular, by Eqs of the extrinsic curvature, proper time intervals measured by physical clocks in space-times which are solutions of Einstein's equations always agree with the result in the above

Improvements to Quantum Theory

1. Real physical Hamiltonian $\bar{H}$ compatible with spatial diff. symmetry suggests supplementing the kinetic term with a quadratic form, i.e.

$$\bar{H} = \sqrt{\bar{G}_{ijkl}\bar{\pi}^{ij}\bar{\pi}^{kl} + [\frac{1}{2}(q_{ik}q_{jl} + q_{jk}q_{il}) + \gamma q_{ij}q_{kl}]\frac{\delta W}{\delta q_{ij}}\frac{\delta W}{\delta q_{kl}}}$$
$$= \sqrt{[\bar{q}_{ik}\bar{q}_{jl} + \gamma\bar{q}_{ij}\bar{q}_{kl}]Q^{ij}_{+}Q^{kl}_{-}}$$

2. $\bar{H}$ is then real if $\gamma > -\frac{1}{3}$
3. $W = \int\sqrt{q}(aR - \Lambda) + CS$ and $Q^{ij}_{\pm} := \bar{\pi}^{ij} \pm iq^{\frac{1}{3}}\frac{\delta W}{\delta q_{ij}}$ and Einstein's theory with cosmological constant is recovered at low curvatures

$$\kappa = \frac{8\pi G}{c^3} = \sqrt{\frac{1}{10\pi^2 a\Lambda(1+3\gamma)}} \text{ and } \Lambda_{eff} = \frac{3}{2}\kappa^2\Lambda^2(1+3\gamma) = \frac{3\Lambda}{20a\pi^2}$$

4. New parameter γ in the potential, positivity of $\bar{H}$ (with $\gamma > -\frac{1}{3}$) is correlated with *real* κ and *positive* Λ_{eff}
5. Zero modes in $\bar{H}$ occurs i.e. $\gamma \to -\frac{1}{3}$, for fixed $\kappa \Rightarrow \Lambda_{eff} \to 0$

Further Discussions

1. Although there is only spatial diff. invariance, Lorentz symmetry of the tangent space is intact, the ADM metric $ds^2 = \eta_{AB} e^A_\mu e^B_\nu dx^\mu dx^\nu = -N^2 dt^2 + q^{\frac{1}{3}} \bar{q}_{ij}(x,t)(dx^i + N^i dt)(dx^j + N^j dt)$ is invariant under local Lorentz transformations $e'^A_\mu = \Lambda^A{}_B(x) e^B_\mu$ which do not affect metric components $g_{\mu\nu} = \eta_{AB} e^A_\mu e^B_\nu$
2. 2 physical canonical degrees of freedom in $(\bar{q}^{phys.}_{ij}, \bar{\pi}^{ij}_T)$, and an extra pair $((\ln q^{\frac{1}{3}})_{phys.}, \pi_T)$ to play the role of time and Hamiltonian (which, remarkably, is consistently tied to π_T by the dynamical equations)
3. Inverting, $\bar{\pi}^{ij}$ in terms of $\frac{\delta \bar{q}_{ij}}{\delta \ln q}$ from the EOM, yield the action,

$$S = -\int \sqrt{V}\sqrt{(\delta \ln q^{\frac{1}{3\beta}} - \mathcal{L}_{\delta\vec{N}} \ln q^{\frac{1}{3\beta}})^2 - \bar{G}^{ijkl}(\delta\bar{q}_{ij} - \mathcal{L}_{\delta\vec{N}}\bar{q}_{ij})\delta(\bar{q}_{kl} - \mathcal{L}_{\delta\vec{N}}\bar{q}_{kl})}$$

Just the superspace proper time with $\sqrt{V}$ playing the role of “mass”.

Summary

1. Paradigm shift from full space-time covariance to spatial diff. invar.
2. Master constraint + Clean decomposition of the canonical structure $\Rightarrow$ physical dynamics + resolution of the problem of time free from arbitrary lapse and gauged histories
3. Intrinsic time provide a simultaneity instant for quantum mechanics
4. Difficulties with Klein-Gordon type WDW equations is overcome with a S-eqt with positive semi-definite probability density at any instant
5. Gauge invariant observables can be constructed from integrations constants of the first order HJ equation which is also complete
6. Classical space-time with direct correlation between its proper times and intrinsic time intervals emerges from constructive interference
7. Framework not only yields a physical Hamiltonian for GR, but also prompts natural extensions and improvements towards a well-behaved quantum theory of gravity

$$\psi_E(x,t) = \begin{pmatrix}\text{SLOWLY VARYING}\\ \text{AMPLITUDE FUNCTION}\end{pmatrix} \exp\left(\frac{i}{\hbar}\right) S_E(x,t) \tag{1}$$

It is of no help in localizing the probability distribution that the Hamilton–Jacobi function S has in many applications a value large in comparison with the quantum of angular momentum $\hbar = 1.02 \times 10^{-27}$ g-cm^2/sec. It is of no help that this "dynamical phase"—to give S another name—obeys the simple Hamilton–Jacobi law of propagation,

$$\begin{aligned}\frac{\partial S}{\partial t} &= H\left(\frac{\partial S}{\partial x}, x\right)\\ &= \left(\frac{1}{2m}\right)\left(\frac{\partial S}{\partial x}\right)^2 + V(x)\end{aligned} \tag{2}$$

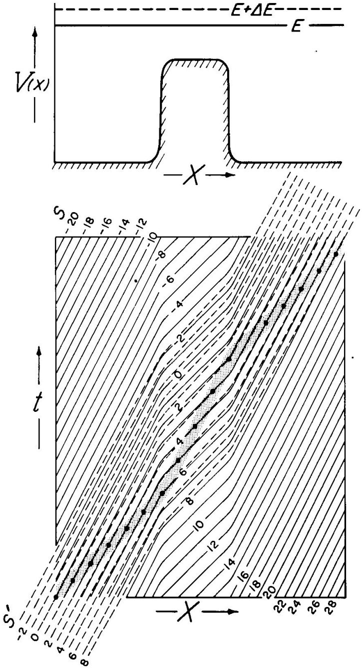
And finally, it is of no help that the solution of this equation for a particle of energy E is extraordinarily simple,

$$S(x,t) = -Et + \int^x \{2m[E - V(x)]\}^{1/2}\, dx + \delta_E \tag{3}$$

The probability is still spread all over everywhere! There is not the slightest trace of anything like a localized world line, $x = x(t)$!

How old the idea of building wave packets out of monofrequency waves and how easy! The probability amplitude is now a superposition of terms, qualitatively of the form

$$\psi(x,t) = \psi_E(x,t) + \psi_{E+\Delta E}(x,t) + \cdots \tag{4}$$



E+ΔE
E
V(x)
X
S
-20 -18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16 18 20
t
X
22 24 26 28
S'
-2 0 2 4 6 8

At last a world line! And how easy to find the Newtonian motion from this condition of constructive interference:

$$0 = S_{E+\Delta E} - S_E$$
$$\Downarrow$$
$$0 = -t\,\Delta E + \int^x \Delta p_E(x)\,dx + (\delta_{E+\Delta E} - \delta_E)$$
$$\Downarrow$$
$$t = \int^x \frac{dx}{v_E(x)} + t_0 \qquad (\text{Newton}) \tag{6}$$

Here $v_E(x)$ denotes the velocity at the location x,

$$\frac{\Delta[2m(E-V)]^{1/2}}{\Delta E} = \frac{\Delta p_E}{\Delta E} \to \frac{\partial(\text{momentum})}{\partial E} = \frac{1}{v_E(x)} = \begin{pmatrix}\text{time to cover} \\ \text{a unit distance}\end{pmatrix} \tag{7}$$

and the quantity t_0 is an abbreviation for

$$\frac{\delta_{E+\Delta E} - \delta_E}{\Delta E} \to \frac{d\delta_E}{dE} \equiv t_0 \tag{8}$$

Marvelously, not one trace of the quantum of action appears in the final solution for the motion. Yet the quantum principle supplies the whole rationale and motivation for talking about "constructive interference." The quantum comes in only when one recognizes the finite spread of the wave packet (Fig. 1). Then the idea of a world line has to be renounced. A whole range of histories contribute to the propagation of the particle from start to finish. This is the way the real world of quantum physics operates!

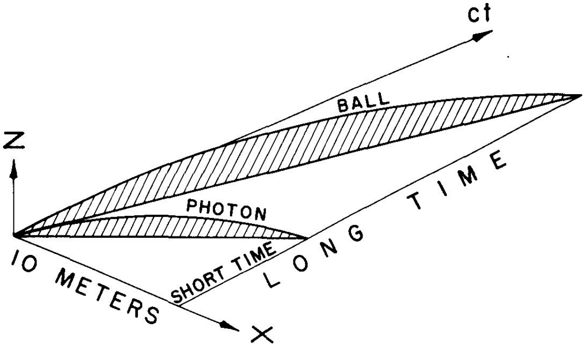
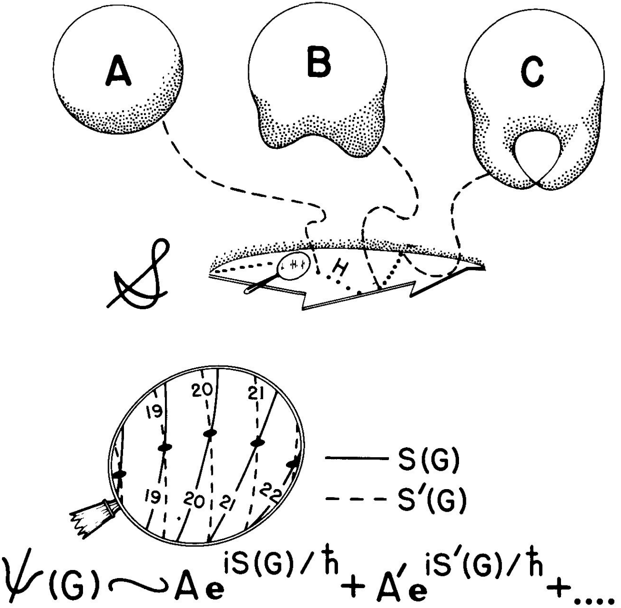
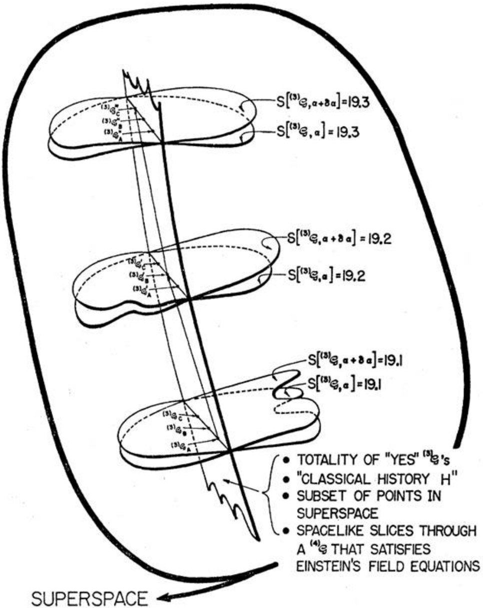


FIGURE 2. The track of the ball and the track of the photon through space (x, z plane) have very different curvatures, but in space time (x, z, ct space) the curvatures are comparable.


A
B
C
H
19
20
21
22
S(G)
S′(G)
ψ(G) ∼ A e^{iS(G)/ℏ} + A′ e^{iS′(G)/ℏ} + ...


$S[{}^{(3)}\mathcal{G}, a+\delta a] = 19.3$
$S[{}^{(3)}\mathcal{G}, a] = 19.3$
$S[{}^{(3)}\mathcal{G}, a+\delta a] = 19.2$
$S[{}^{(3)}\mathcal{G}, a] = 19.2$
$S[{}^{(3)}\mathcal{G}, a+\delta a] = 19.1$
$S[{}^{(3)}\mathcal{G}, a] = 19.1$
• TOTALITY OF "YES" ${}^{(3)}\mathcal{G}$'s
• "CLASSICAL HISTORY H"
• SUBSET OF POINTS IN SUPERSPACE
• SPACELIKE SLICES THROUGH A ${}^{(4)}\mathcal{G}$ THAT SATISFIES EINSTEIN'S FIELD EQUATIONS
SUPERSPACE

Superspace is the arena for geometrodynamics, just as Lorentz–Minkowski space-time is the arena for particle dynamics (Table 1). The momentary configuration of the particle is an event, a single point in space-time. The momentary configuration of space is a 3-geometry, a single point in superspace.

TABLE 1 *Geometrodynamics Compared with Particle Dynamics*

Quality	Particle	Geometrodynamics
Dynamical entity	Particle	Space
Descriptors of momentary configuration	x, t ("event")	${}^{(3)}\mathscr{G}$ ("3-geometry")
History	$x = x(t)$	${}^{(4)}\mathscr{G}$ ("4-geometry")
History is a stockpile of configurations?	Yes. Every point on world line gives a momentary configuration of particle	Yes. Every spacelike slice through ${}^{(4)}\mathscr{G}$ gives a momentary configuration of space
Dynamic arena	Spacetime (totality of all points x, t)	Superspace (totality of all ${}^{(3)}\mathscr{G}$'s)