No-boundary measure in cosmic landscape

Study of fuzzy instantons

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2. How to use the no-boundary measure?
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Based on

- Hartle, Hawking and Hertog, arXiv:0711.4630, 0803.1663
- Hwang, Sahlmann and DY, arXiv:1107.4653
- Hwang, Lee, Sahlmann and DY, in preparation

What is the no-boundary measure?

Brief introduction

Problem of singularity

The singularity theorem:

Our universe should begin from the initial singularity. How to resolve?

Maybe, by using the Schrodinger equation for fields:

so-called, **the Wheeler-DeWitt equation.**

$$\left(G_{ijkl}\frac{\delta}{\delta\gamma_{ij}}\frac{\delta}{\delta\gamma_{kl}}+\gamma^{1/2}\ {}^{(3)}R\right)\Psi[{}^{(3)}\mathcal{G}]=0$$

(quantized)
Hamiltonian constraint

Wave function of universe

3-metric (and fields) $\in$ superspace

No-boundary proposal

What is the boundary condition of WDW eqn?

Perhaps, the ground state?

Hartle-Hawking wave function

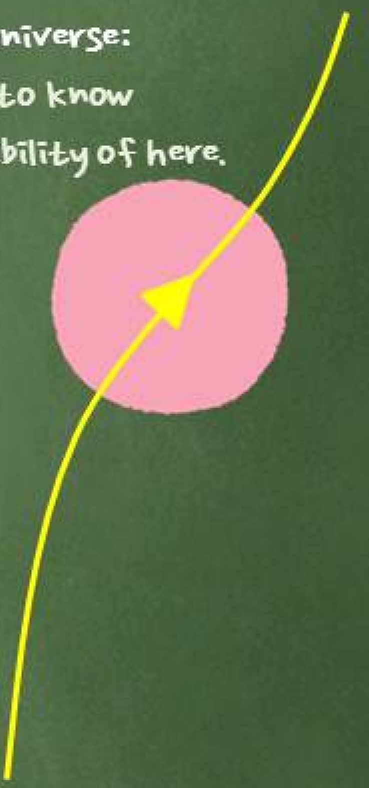
$$\Psi_0[h_{ij}] = N \int \delta g \exp(-I_E[g])$$

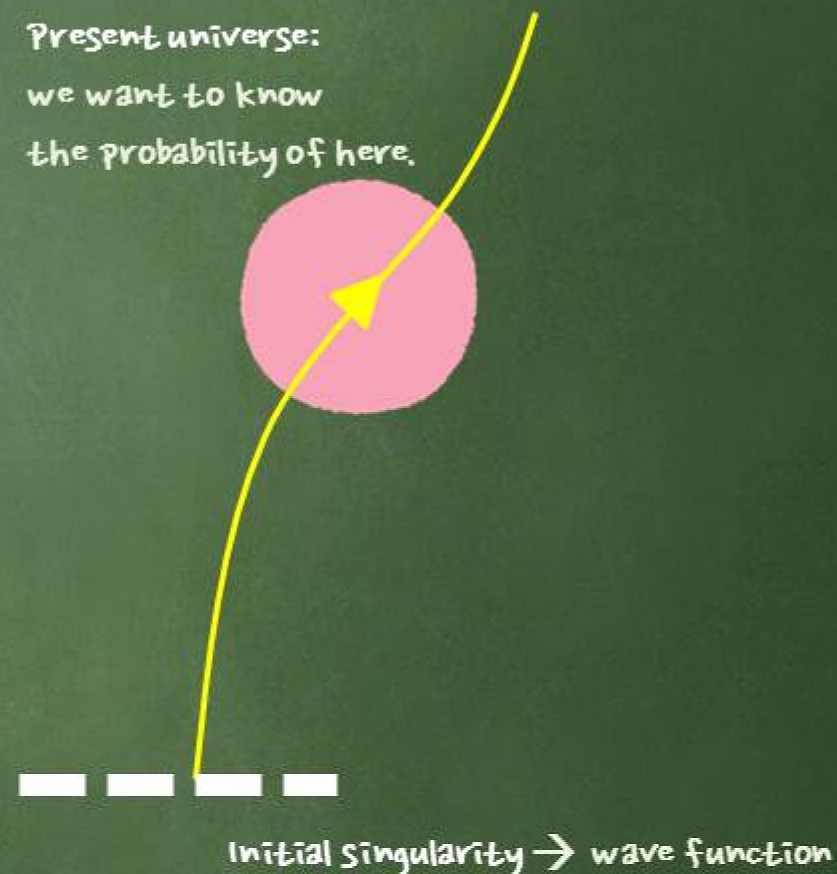
Euclidean action

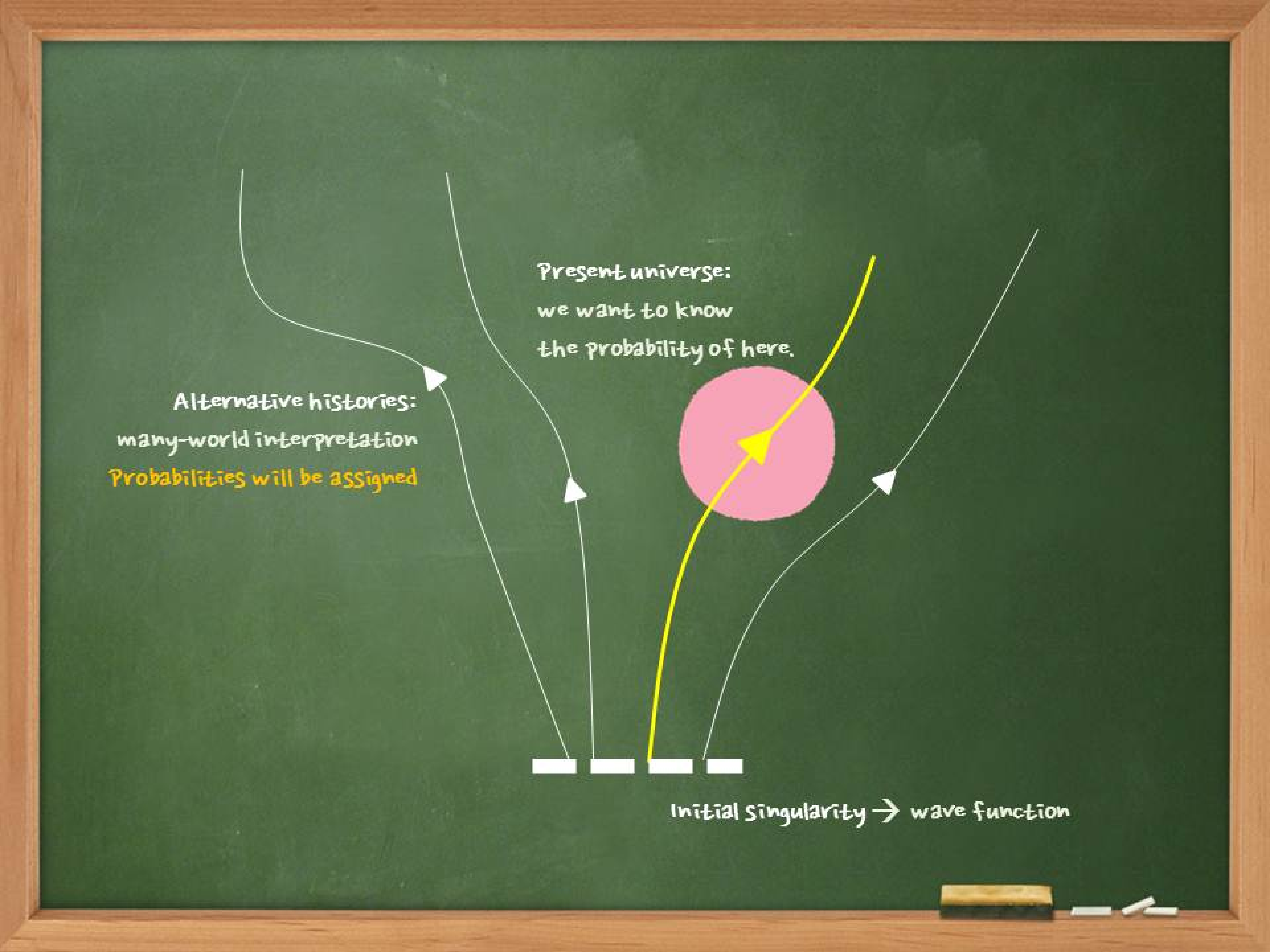
path integral

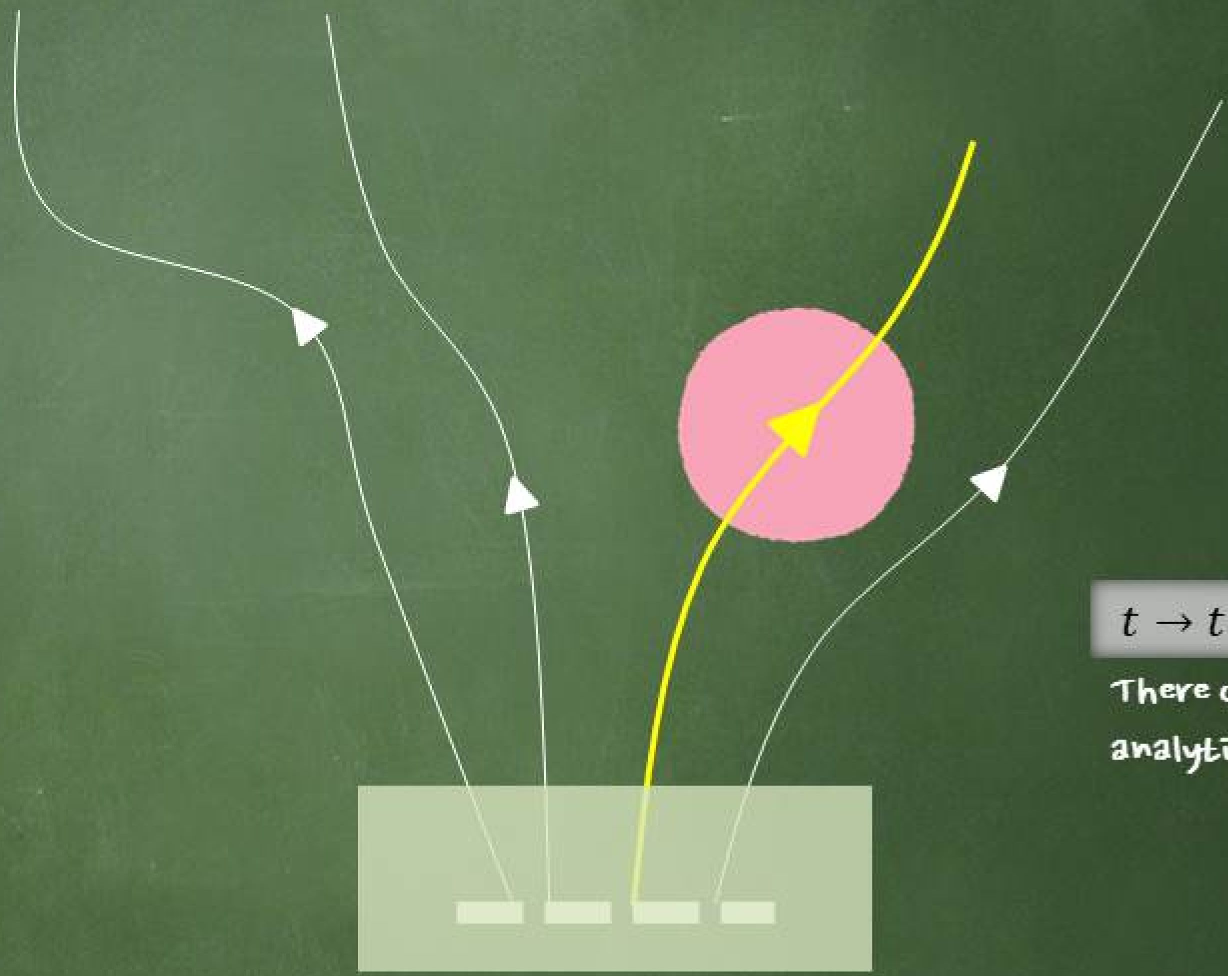
over regular compact manifold

Present universe:
we want to know
the probability of here.



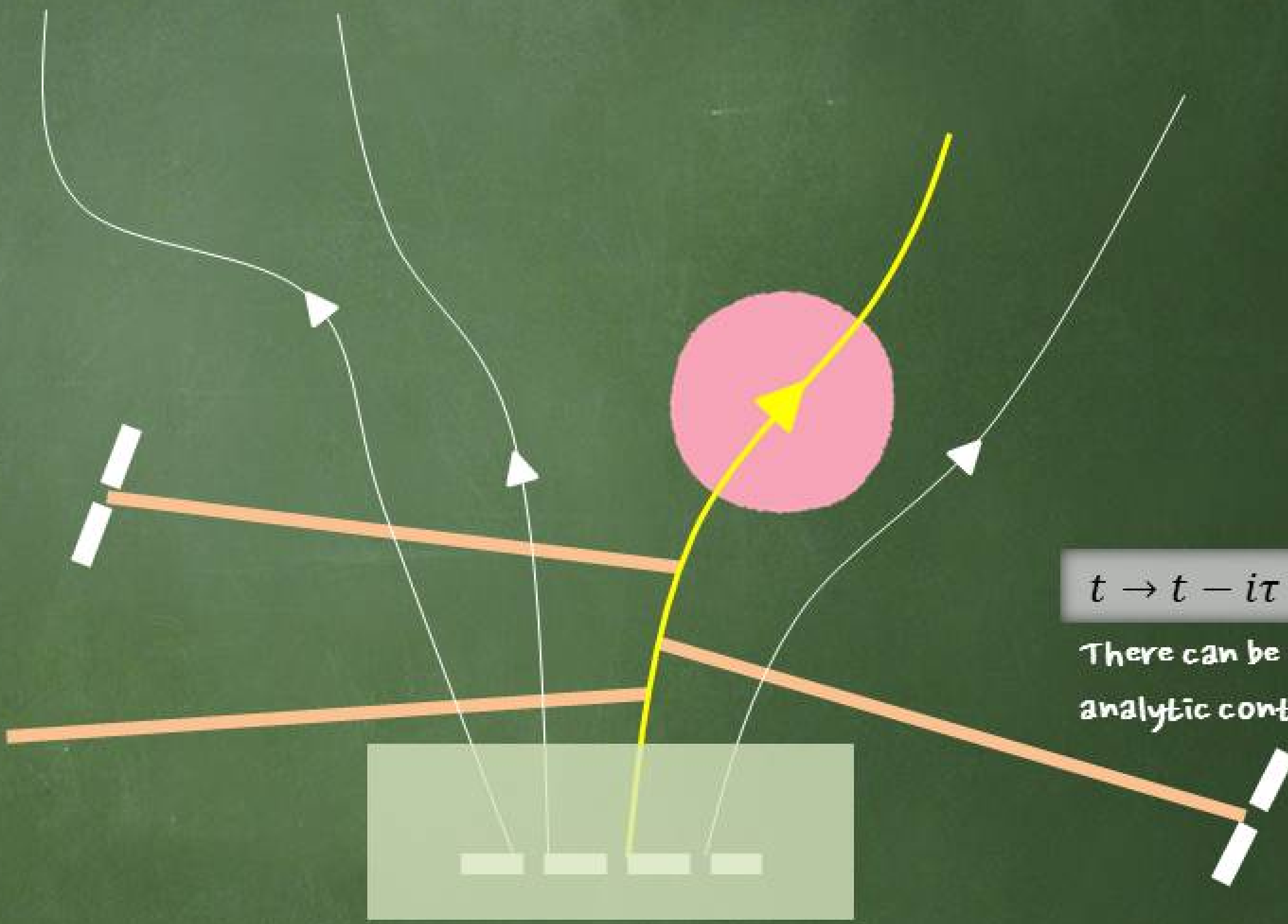

Present universe:
we want to know
the probability of here.
Initial singularity → wave function


Present universe:
we want to know
the probability of here.
Alternative histories:
many-world interpretation
Probabilities will be assigned
Initial singularity → wave function



$$t \to t - i\tau$$

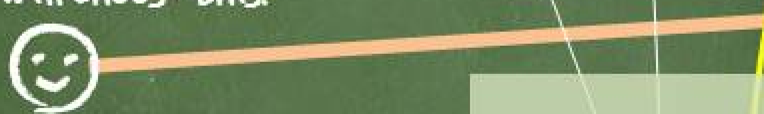
There can be various analytic continuations.

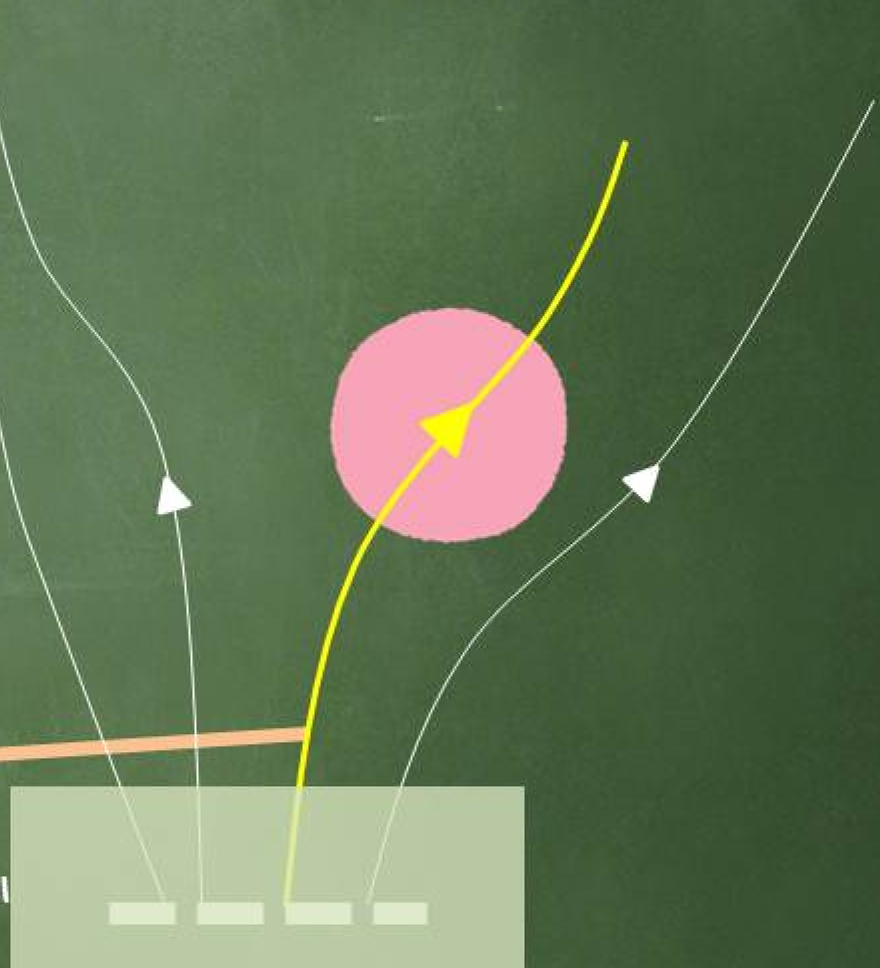


$t \to t - i\tau$

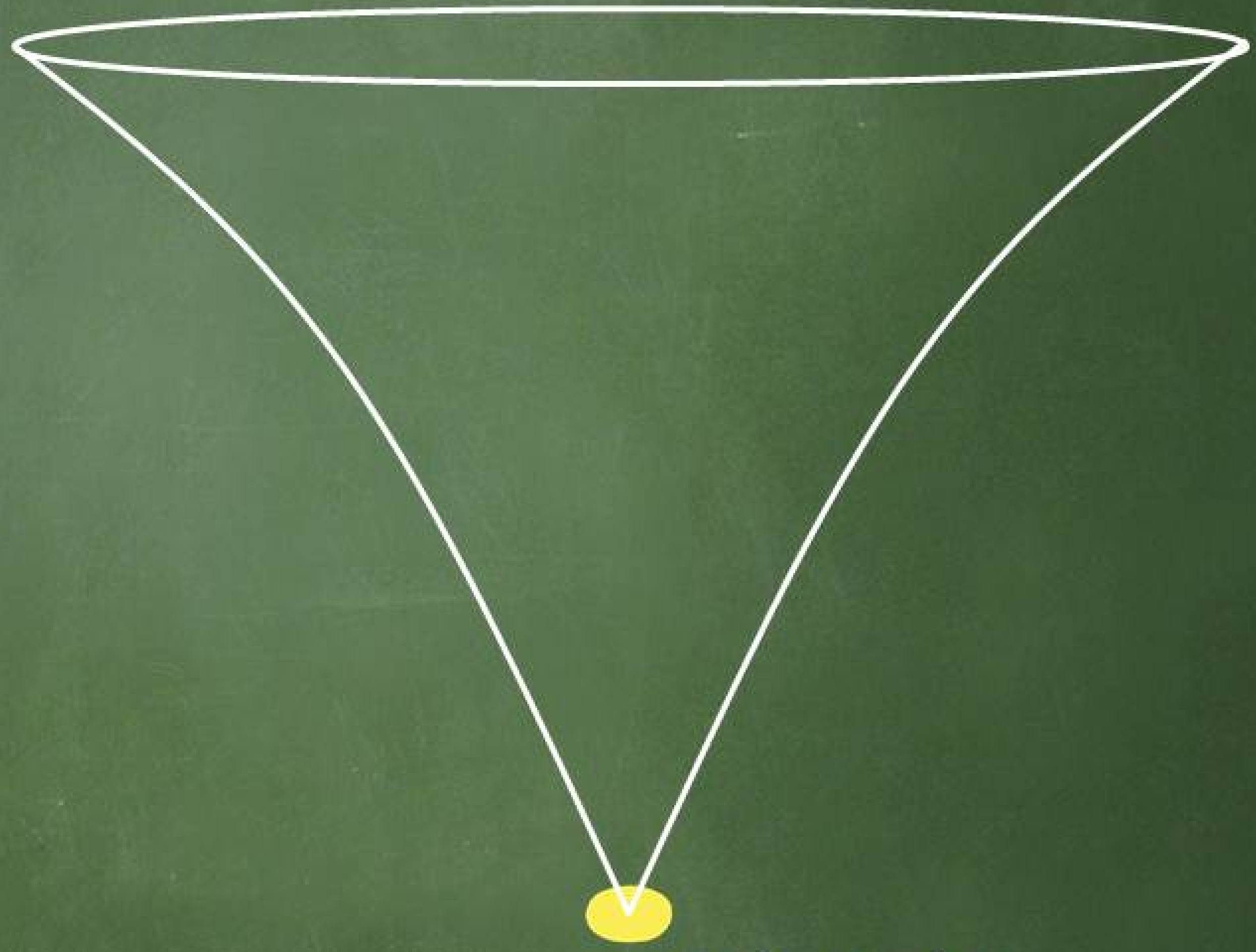
There can be various analytic continuations.

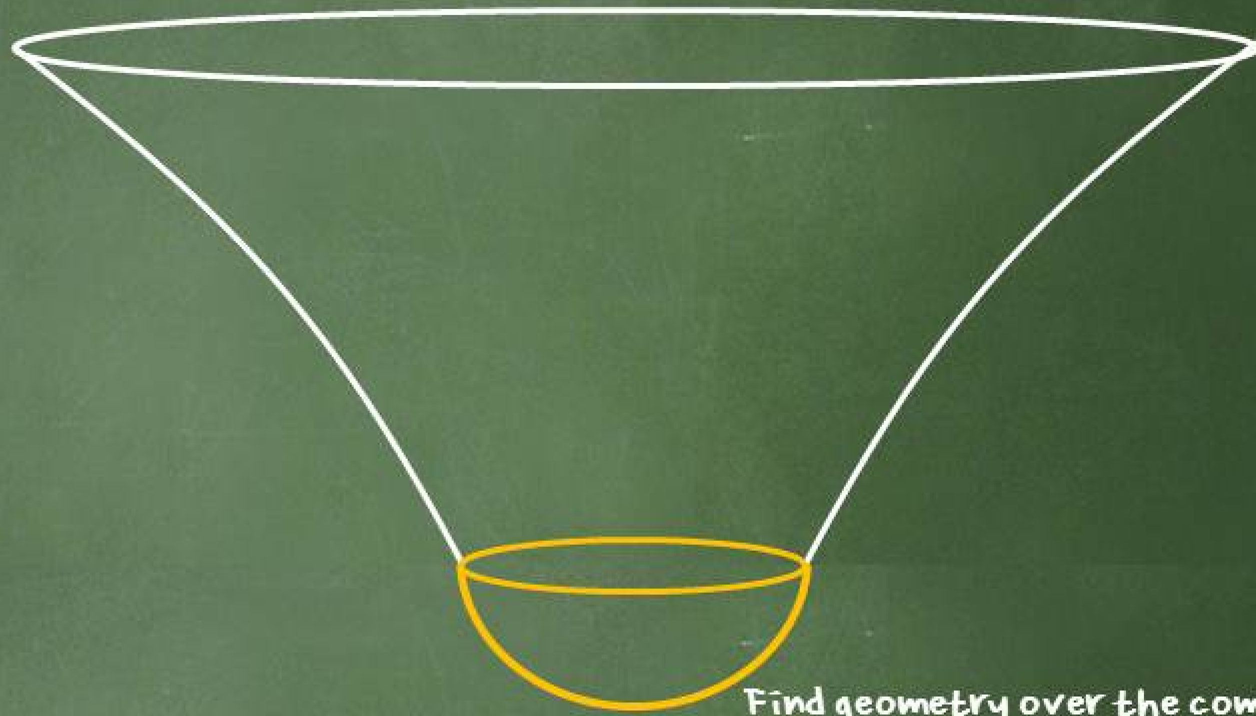
If this path is regular,
then we will choose this.





Due to **analyticity,** the path-integral
still makes sense,
even though we analytically continue
to Euclidean time.


Big-bang singularity



Find geometry over the complex time,
until the geometry to be regular.

How to use the no-boundary measure?

use of fuzzy instantons

Fuzzy instantons

No-boundary wave function may have the problem of divergence.

This can be cured, if we analytically continue all functions (metric and fields) to complex functions (Halliwell and Hartle, 1990).

An on-shell solution of Euclidean complexified fields are called by fuzzy instanton (Hartle, Hawking and Hertog, 2007).

How to calculate path integral?

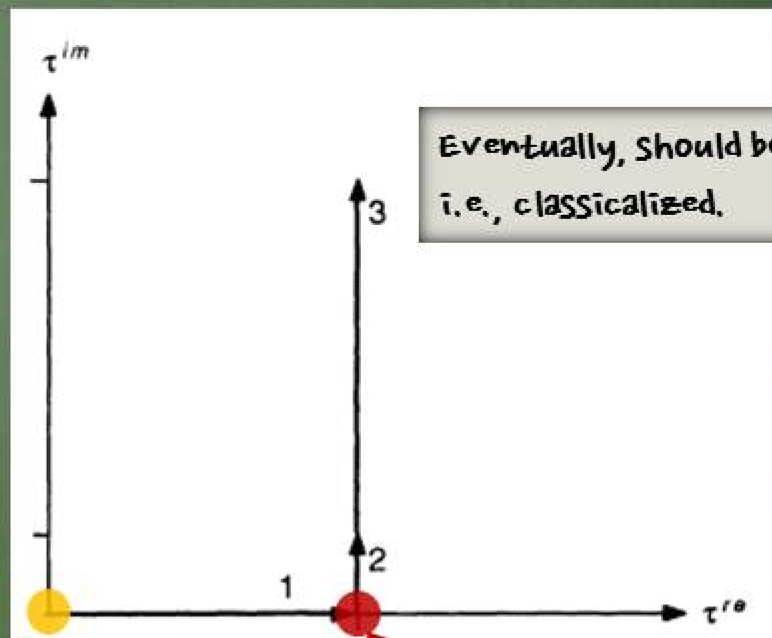
Approximation 1: Mini-superspace

$$ds_{\mathrm{E}}^2 = N^2(\eta)d\eta^2 + \rho^2(\eta)(d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\varphi^2))$$

Approximation 2: Steepest-descent (sum-over fuzzy instantons)

$$\Psi_{\mathrm{HH}}(q) \approx \sum_{\mathrm{ext}} P(q_{\mathrm{ext}}) e^{-S_{\mathrm{E}}[q_{\mathrm{ext}}]/\hbar}$$

classicality



Eventually, should be real, i.e., classicalized.

$$|(\nabla I_R)^2| \ll |(\nabla S)^2|$$

No-boundary condition

Turn to the Lorentzian time

Example (Einstein gravity)

$$S_{\mathrm{E}} = -\int d^4x\sqrt{+g}\left(\frac{1}{16\pi}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right)$$

$$\begin{aligned} \ddot{\phi} &= -3\frac{\dot{\rho}}{\rho}\dot{\phi} \pm V', \\ \ddot{\rho} &= -\frac{8\pi}{3}\rho\left(\dot{\phi}^2 \pm V\right) \end{aligned}$$

$$\begin{aligned} \rho(0)^{\Re e} = \rho(0)^{\Im m} &= 0, \\ \dot{\rho}(0)^{\Re e} &= 1, \\ \dot{\rho}(0)^{\Im m} &= 0, \\ \dot{\phi}(0)^{\Re e} = \dot{\phi}(0)^{\Im m} &= 0. \end{aligned}$$

Initial conditions for no-boundary

$$\phi(0) = \phi_0 e^{i\theta}$$

The remained initial conditions.

$$\underline{\rho}(t=0) = \rho(\eta = X), \quad \underline{\dot{\rho}}(t=0) = i\dot{\rho}(\eta = X),$$
$$\underline{\phi}(t=0) = \phi(\eta = X), \quad \underline{\dot{\phi}}(t=0) = i\dot{\phi}(\eta = X).$$

Junction conditions at the turning point

For given initial field amplitude,
by tuning the phase angle and the turning point,
we find a classical fuzzy instanton!

Summarize

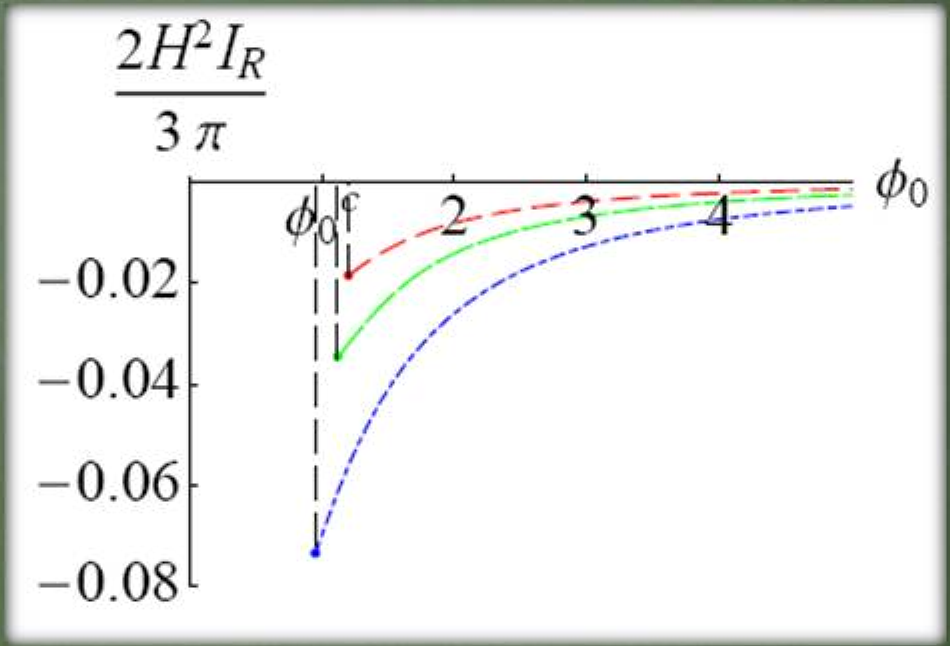
Step 1: We impose the mini-superspace metric.

Step 2: We only consider on-shell solutions (fuzzy instantons), that begins from the no-boundary condition.

Step 3: Between on-shell fuzzy instantons, we only restrict that satisfies classicality.

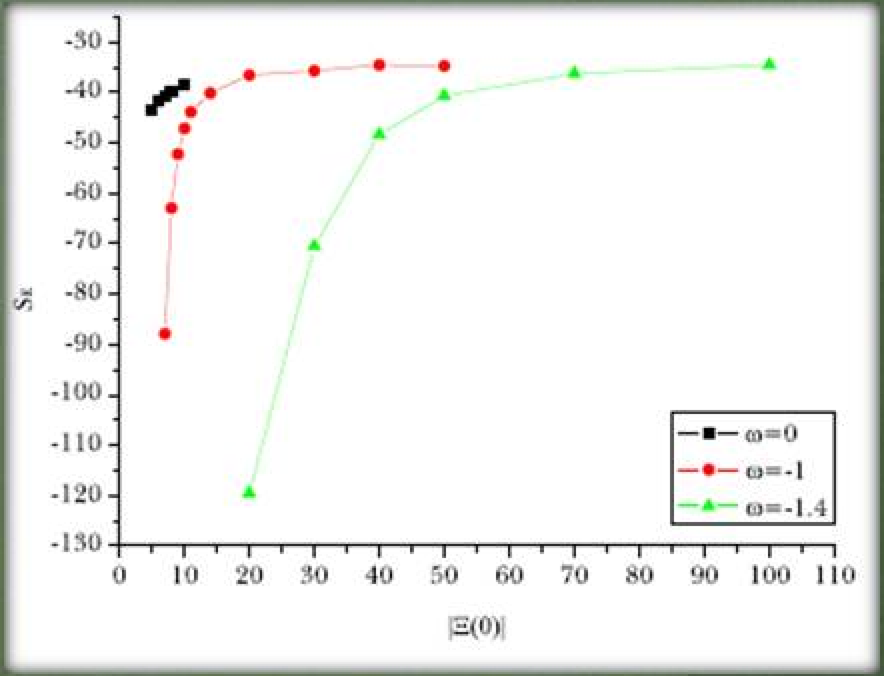
- on-shell: dof = 8+1
- on-shell + no-boundary: dof = 8+1 − 6
- on-shell + no-boundary + classicality: dof = 8+1 − 6 − 2 = 1

Example (Einstein gravity, quadratic potential)



(Hartle, Hawking and Hertog, 2007)

Example (Brans-Dicke gravity, quadratic potential)



(Hwang, Sahlmann and DY, 2011)

Two types of fuzzy instantons

Slow-rolling vs. fast-rolling

Two types of fuzzy instantons

Let us observe near the local maximum.

$$V(\phi) = V_0 - (1/2)m^2\phi^2$$

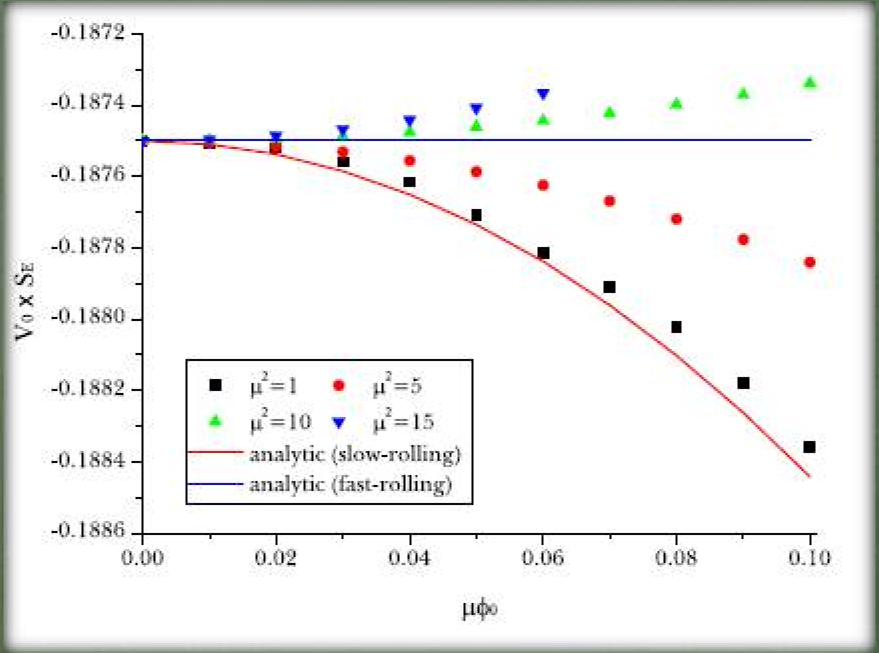
According to Hartle, Hawking and Hertog, the probability will be approximately as follows.

$$\begin{aligned} V_0 S_{\mathrm{E}} &\simeq -\frac{3}{16\tilde{V}(\phi_0)} \\ &\simeq -\frac{3}{16}\left(1 - \frac{1}{2}\mu^2\phi_0^2\right)^{-1} \end{aligned}$$

$$\mu^2 = m^2/V_0$$

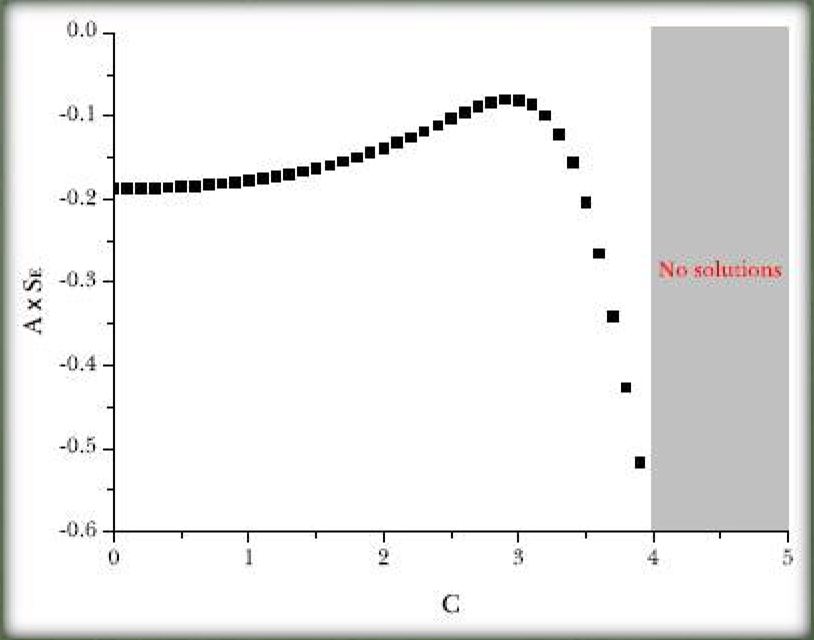
However, in general, it is not true.

Two types of fuzzy instantons



As the slow-roll parameter increases,
the action only depends on
the vacuum energy of the local maximum!

can it be run-away fuzzy instantons?



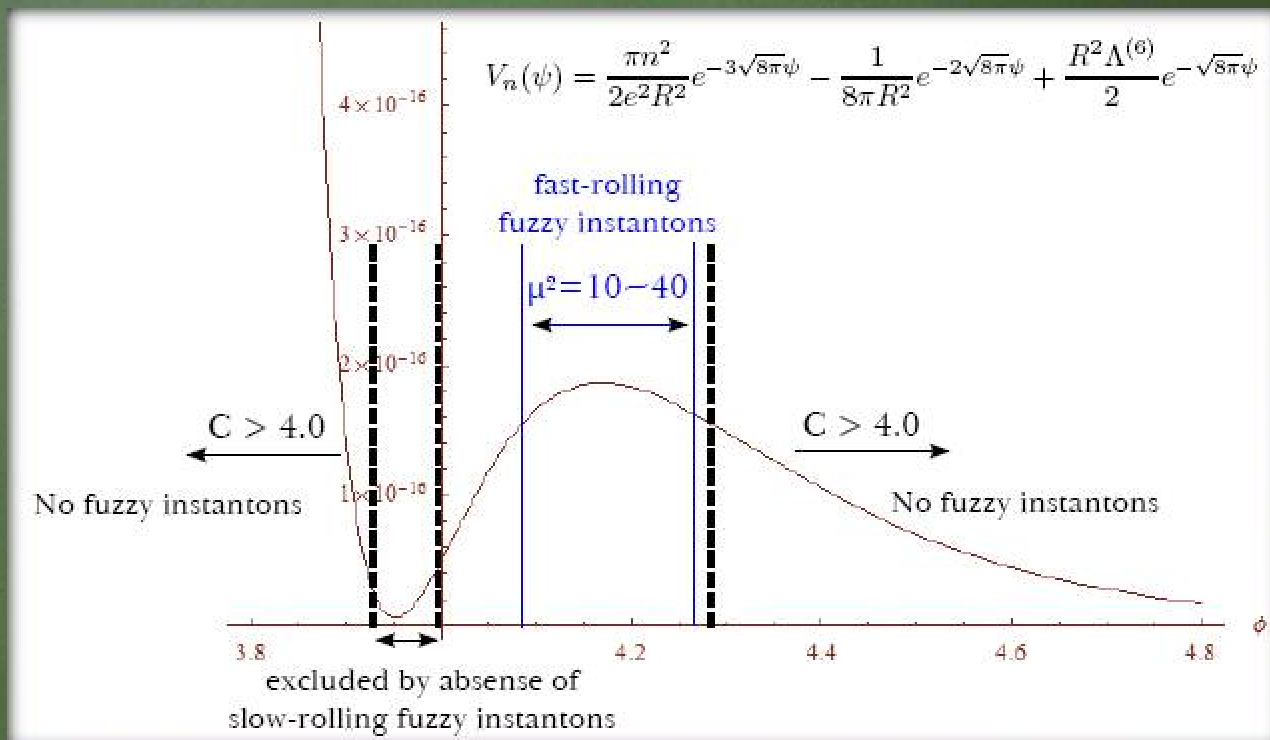
$V(\phi) = Ae^{-C\phi}$

If c > 4, then there is no run-away fuzzy instantons!

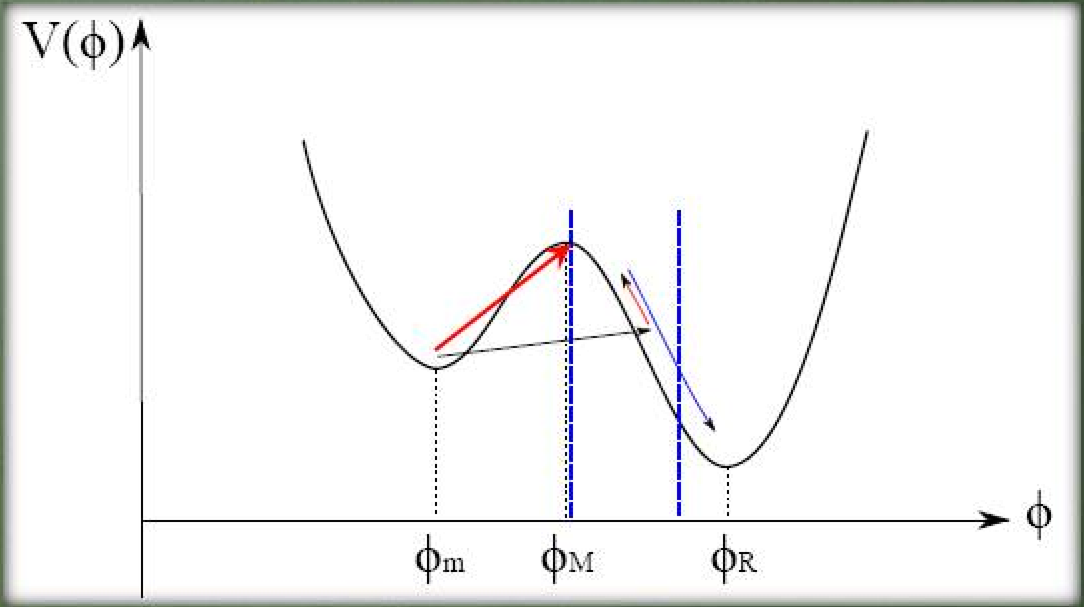
cosmological applications

with string theory

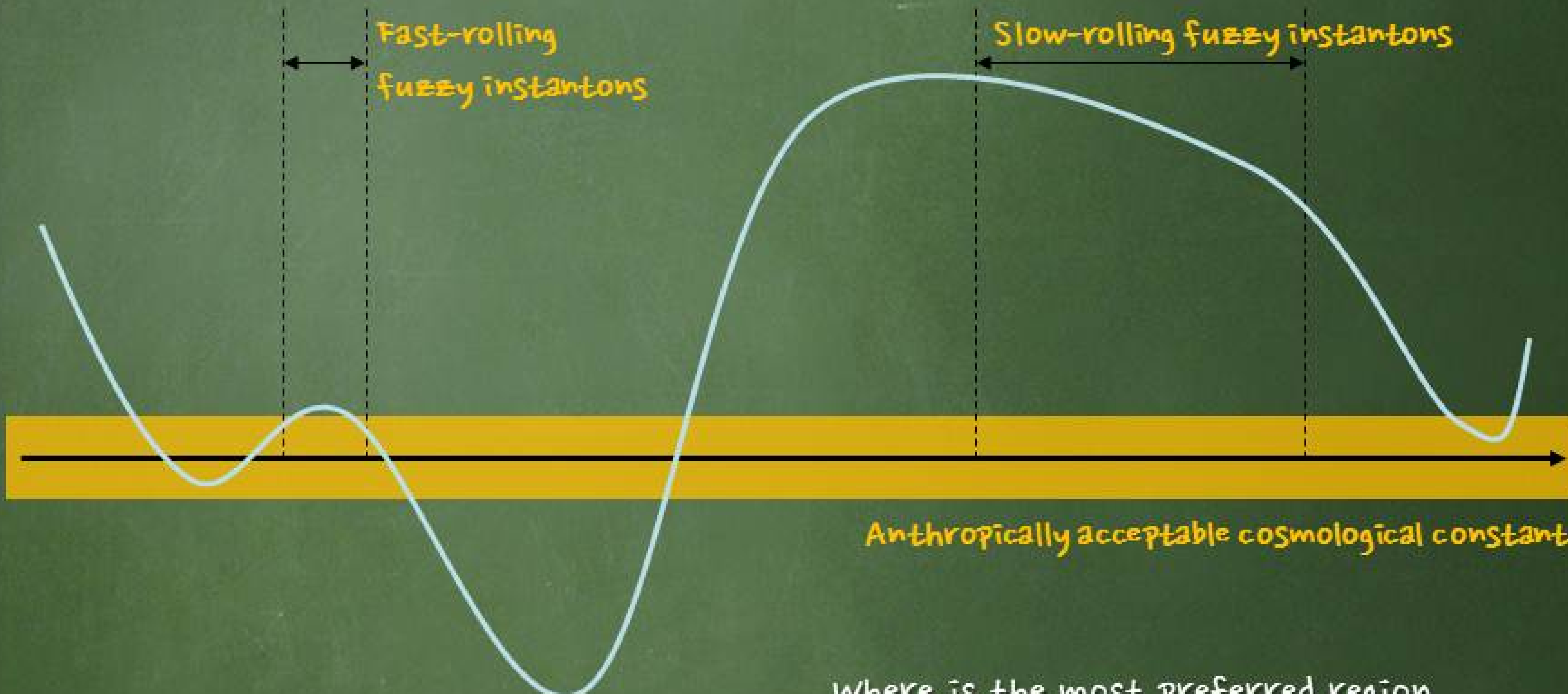
Applications I: Moduli/dilaton stabilization



Applications 2: Generalized HM instanton



Applications 3: Multiverse vs. manyworld?



Where is the most preferred region,
in terms of the wave function?

Conclusion

- The Euclidean path integral/the no-boundary measure can be approximated by sum-over fuzzy instantons.
- We classified two types of fuzzy instantons: slow-rolling and fast-rolling. Also, we know the existence/absence condition for run-away potentials.
- These bring interesting issues in the context of cosmology/string theory.
  - Should be generalized to multifields
  - Should be compared with Hawking-Moss instantons
  - Can it compatible with string theory?