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鑑三

Quantum properties of the inflaton

☆ Similar to the behavior of massless scalar field $\varphi(\boldsymbol{x}, t)$ in de Sitter space whose square expectation value behaves as $\langle\varphi(\boldsymbol{x},t)^2\rangle = \left(\frac{H}{2\pi}\right)^2 Ht$.

(Bunchi & Davis 78, Vilenkin & Ford 82…)

☆ $$\varphi(\boldsymbol{x},t) = \int \frac{\mathrm{d}^3k}{(2\pi)^{3/2}} \left(\hat{a}_{\boldsymbol{k}}\varphi_k(t)e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{a}^\dagger_{\boldsymbol{k}}\varphi^*_k(t)e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\right) \equiv \int \frac{\mathrm{d}^3k}{(2\pi)^{3/2}} \hat{\varphi}_{\boldsymbol{k}}(t)e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

☆ If we impose the normalization condition $\varphi_k(t)\dot{\varphi}^*_k(t) - \dot{\varphi}_k(t)\varphi^*_k(t) = \frac{i}{a^3(t)}$,
the canonical commutation relation $[\varphi(\boldsymbol{x},t), \pi(\boldsymbol{x}',t)] = i\delta(\boldsymbol{x}-\boldsymbol{x}')$
yields $[\hat{a}_{\boldsymbol{k}}, \hat{a}^\dagger_{\boldsymbol{k}'}] = \delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')$
where the conjugate momentum is given by $\pi(\boldsymbol{x},t) = a^3(t)\dot{\varphi}(\boldsymbol{x},t)$.

☆ The mode function satisfies $$\left[\frac{\mathrm{d}^2}{\mathrm{d}t^2} + 3H\frac{\mathrm{d}}{\mathrm{d}t} + \frac{k^2}{e^{2Ht}}\right]\varphi_k(t) = 0$$

in de Sitter space and its normalized solution is given by

$$\varphi_k(t) = \sqrt{\frac{\pi}{4}}H(-\eta)^{3/2}H^{(1)}_{3/2}(-k\eta) = \frac{iH}{\sqrt{2k^3}}(1+ik\eta)e^{-ik\eta}$$

$\eta \equiv \int^t \frac{dt}{a(t)} = \int^t \frac{dt}{e^{Ht}} = -\frac{1}{He^{Ht}}$ is the conformal time and $-k\eta = \frac{k}{Ha(t)}$.

☆ $$\varphi_k = \frac{iH}{\sqrt{2k^3}}(1+ik\eta)e^{-ik\eta} = \frac{iH}{\sqrt{2k^3}}\left(1 - \frac{ik}{Ha}\right)e^{i\frac{k}{Ha}}$$

$$\to \frac{iH}{\sqrt{2k^3}}\left[1 + O\left(\left(\frac{k}{Ha}\right)^2\right)\right], \quad \text{for } \; k \ll a(t)H$$

$\varphi_k^*(t) = -\varphi_k(t)$ in the superhorizon regime

So we find $\hat{\varphi}_{\boldsymbol{k}}(t) = \varphi_k(t)(\hat{a}_{\boldsymbol{k}} - \hat{a}^\dagger_{-\boldsymbol{k}})$

and its conjugate momentum reads

$$\hat{\pi}_{\boldsymbol{k}}(t) = a(t)^3\dot{\varphi}_k(t)(\hat{a}_{\boldsymbol{k}} - \hat{a}^\dagger_{-\boldsymbol{k}})$$

The same operator dependence!

☆ When the decaying mode is negligible, $\hat{\varphi}_{\boldsymbol{k}}$ and $\hat{\pi}_{\boldsymbol{k}}$ have the same operator dependence and commute with each other.

Long-wave quantum fluctuations behave as if classical statistical fluctuations.

Origin of large scale structures and CMB anisotropy

★ In the short wave regime well inside the Hubble horizon, $k \gg aH$

$$\varphi_k = \frac{iH}{\sqrt{2k^3}}(1+ik\eta)e^{-ik\eta} \longrightarrow \frac{-H\eta}{\sqrt{2k}}e^{-ik\eta} = \frac{1}{a\sqrt{2k}}e^{-ik\eta} = \frac{1}{a^{3/2}\sqrt{2k/a}}e^{-i\frac{k}{a}t} = \boxed{\frac{1}{a^{3/2}\sqrt{2k_{phys}}}e^{-ik_{phys}t}}$$

In a short time interval when cosmic expansion is negligible, we may set $dt = a(\eta)d\eta \longrightarrow t = a\eta$.

This is the usual positive frequency mode for the Minkowski vacuum with an unusual normalization

$$\varphi_k(t)\dot{\varphi}_k^*(t) - \dot{\varphi}_k(t)\varphi_k^*(t) = \frac{i}{a^3(t)}$$

$$\varphi_k(t) = \sqrt{\frac{\pi}{4}}H(-\eta)^{3/2}H_{3/2}^{(1)}(-k\eta)$$

defines the vacuum state with the appropriate Minkowski limit.

☆ The power spectrum reads

$$|\varphi_k(t)|^2 = \frac{H^2}{2k^3}(1+(k\eta)^2) \to \frac{H^2}{2k^3} \quad \text{for} \quad \frac{k}{Ha(t)} \longrightarrow 0$$

constant and proportional to k^{-3}

☆ Multiplying the phase space density, we find

$$|\varphi_k(t)|^2 \frac{4\pi k^3}{(2\pi)^3} d\ln k = \left(\frac{H}{2\pi}\right)^2$$

: scale-invariant fluctuation

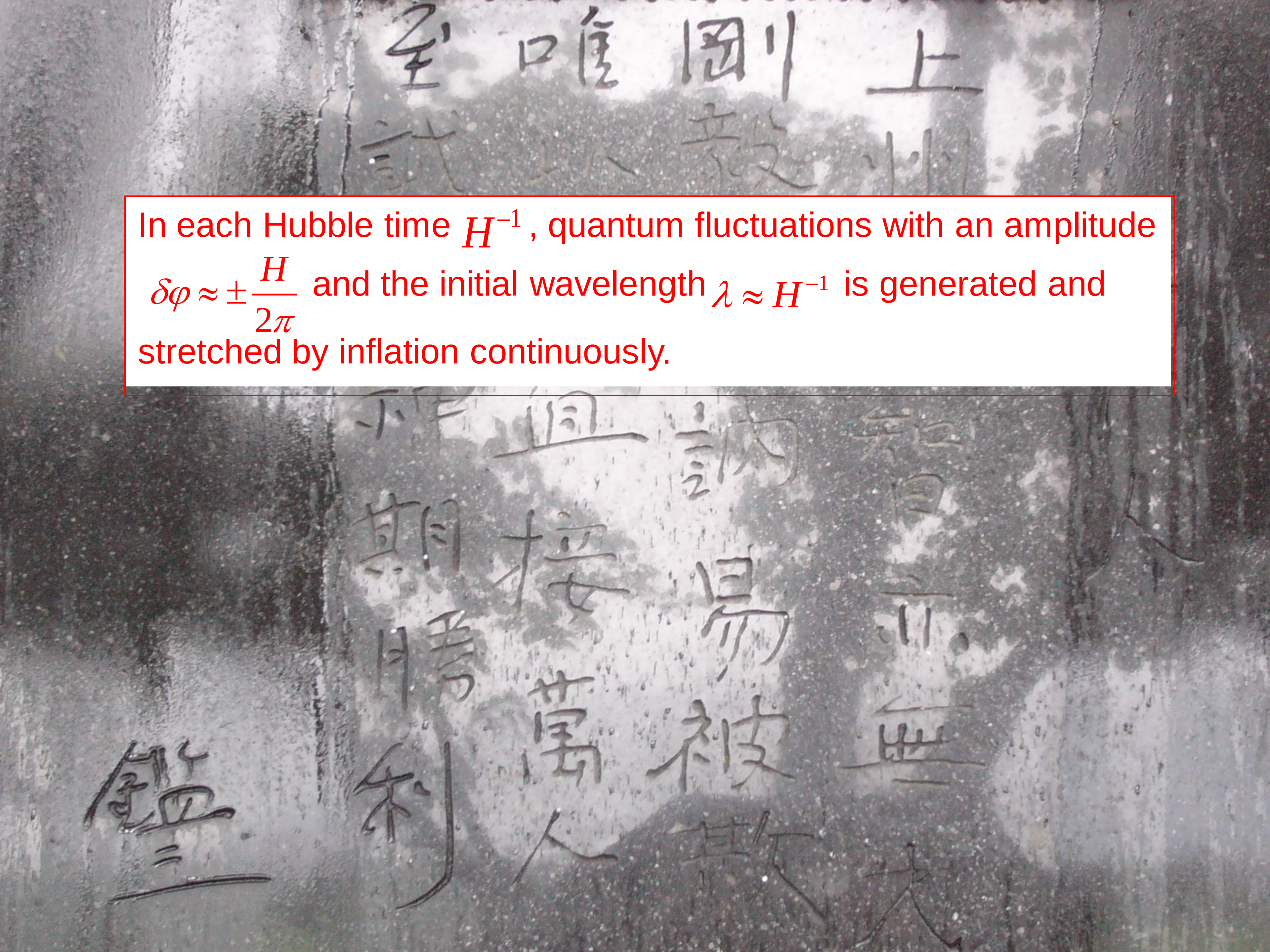
$\langle \varphi(\boldsymbol{x},t)^2 \rangle = \left(\frac{H}{2\pi}\right)^2 Ht$ can be obtained by introducing IR and UV cutoffs as

$$\langle \varphi(\boldsymbol{x},t)^2 \rangle \simeq \int_H^{He^{Ht}} |\varphi_k(t)|^2 \frac{\mathrm{d}^3 k}{(2\pi)^3} = \left(\frac{H}{2\pi}\right)^2 Ht$$

: summing up superhorizon components generated during inflation

≈ Brownian motion with step $\pm\frac{H}{2\pi}$ and interval H^{-1}

In each Hubble time H^{-1}, quantum fluctuations with an amplitude $\delta\varphi \approx \pm\frac{H}{2\pi}$ and the initial wavelength $\lambda \approx H^{-1}$ is generated and stretched by inflation continuously.



In each Hubble time H^{-1}, quantum fluctuations with an amplitude $\delta\varphi \approx \pm \frac{H}{2\pi}$ and the initial wavelength $\lambda \approx H^{-1}$ is generated and stretched by inflation continuously.

- ☆ For later convenience, we derive the same result starting from the action with the conformal time in the metric $ds^2 = a^2(\eta)(-d\eta^2 + d\boldsymbol{x}^2)$.

$$S = \int \sqrt{-g} d^4x \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right] = \frac{1}{2} \int d\eta d^3x \left\{ a^2 \left[\phi'^2 - (\nabla\phi)^2 \right] - a^4 m^2 \phi^2 \right\}$$

- ☆ Using a rescaled field, $\chi \equiv a\phi$ the action is rewritten as

$$S = \frac{1}{2} \int d\eta d^3x \left[\chi'^2 - (\nabla\chi)^2 - \left(a^2 m^2 - \frac{a''}{a} \right) \chi^2 \right] \qquad \chi' = \frac{\partial\chi}{\partial\eta}$$

after integration by parts. So it is of the same form as a free-scalar action with a time dependent mass.

- ☆ In the de Sitter background, $a(\eta) = -1/(H\eta)$, the mode function χ_k satisfies

$$\chi_k'' + k^2 \chi_k - \frac{2}{(-\eta)^2} \chi_k = 0$$

- ☆ The solution satisfying the normalization condition $\chi' \chi^* - \chi \chi^{*'} = i$ as in the Minkowski space is given by

$$\chi_k(\eta) = \left(-\frac{\pi\eta}{4} \right)^{1/2} H^{(1)}_{3/2}(-k\eta) = \frac{\varphi_k(t)}{a(t)}$$

in agreement with the previous calculation.

Cosmological perturbation theory

★ Incorporate linear perturbation to the FLRW background $\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2 \mathrm{d}\boldsymbol{x}^2.$

$$\mathrm{d}s^2 = -(1+2A)\mathrm{d}t^2 - 2aB_j \mathrm{d}t\mathrm{d}x^j + a^2(\delta_{ij} + 2H_L\delta_{ij} + 2\underline{H_{Tij}})\mathrm{d}x^i\mathrm{d}x^j$$

traceless $\quad i, j = 1,2,3$

★ Decompose perturbation variables to spatial scalar, vector, and tensor.

$$B_j = \partial_j B + \widetilde{B}_j, \quad \partial_j \widetilde{B}_j = 0$$ (rotation free mode + divergence free mode)

$$H_{Tij} = \left(\partial_i\partial_j - \frac{\delta_{ij}}{3}\nabla^2\right) H_T + \partial_i \widetilde{H}_{Tj} + \partial_j \widetilde{H}_{Ti} + H_{TTij}$$

$$\partial_j \widetilde{H}_{Tj} = 0, \quad \partial_j {H_{TT}}^k{}_j = 0, \quad {H_{TT}}^j{}_j = 0$$

transverse-traceless mode

$A,\ B,\ H_L$ & H_T Scalar modes・・・Density/Curvature Fluctuations

$\widetilde{B}_j$ & $\widetilde{H}_{Tj}$ Vector modes・・・Decaying modes only

H_{TTij} Tensor modes・・・Gravitational Waves

★ In the linear perturbation theory, scalar, vector, and tensor modes are decoupled from each other. Each Fourier mode also behaves independently.

behave as growing modes in a contracting universe

★ First consider scalar modes in Fourier space

$$ds^2 = -(1+2AY)dt^2 - 2aBY_j dt dx^j + a^2(\delta_{ij} + 2H_L Y \delta_{ij} + 2H_T Y_{ij})dx^i dx^j$$

Y, Y_i, Y_{ij} are scalar harmonics defined by

$$Y = Y_{\boldsymbol{k}} \equiv e^{i\boldsymbol{k}\cdot\boldsymbol{x}}, \quad Y_i \equiv -i\frac{k_i}{k}e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \quad Y_{ij} \equiv \left(-\frac{k_i k_j}{k^2} + \frac{1}{3}\delta_{ij}\right) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

★ Here "AY" means $AY = \sum_{\boldsymbol{k}} A_{\boldsymbol{k}} Y_{\boldsymbol{k}} = \int \frac{d^3k}{(2\pi)^3} A_{\boldsymbol{k}} Y_{\boldsymbol{k}}$ etc.

★ Each perturbation variable is a quantity in Fourier space, e.g. $A = A_{\boldsymbol{k}}(t)$.

★ Physical meaning of each perturbation variable.

A: Fluctuation of the lapse function (Newtonian Potential)

B: Fluctuation of the shift vector

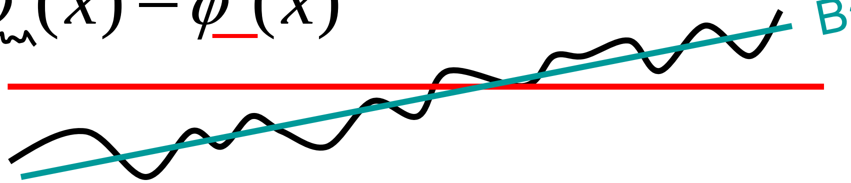
H_L: Fluctuation of the spatial volume

H_T: Spatial anisotropy

Issues of the Gauge

- ☆ Here we started from the background FLRW spacetime and then incorporated perturbations. But actually the real entity is an inhomogeneous spacetime which may be decomposed to a background and perturbations around it. The definition of the background is not unique. We have gauge modes corresponding to the freedoms associated with the definition of the background.

$$\delta\phi(x) = \underset{\sim}{\phi}(x) - \underline{\phi}(x)$$



- ☆ To see how the gauge modes appear, we introduce two coordinate systems corresponding to Background 1 (x^μ) and 2 ($\overline{x}^\mu$) and compare expressions of perturbation variables at the same coordinate value.

- ☆ Suppose that two coordinates are related by the following scalar-type transformation.

$$\overline{x}^0 = x^0 + \delta x^0 = x^0 + TY \qquad \overline{x}^i = x^i + \delta x^i = x^i + \underline{LY^i}$$

gradient of a scalar

- ☆ Then the metrices of the two coordinates are related as

$$\overline{g}_{\mu\nu}(x) = \frac{\partial x^\alpha}{\partial \overline{x}^\mu}\frac{\partial x^\beta}{\partial \overline{x}^\nu} g_{\alpha\beta}(x - \delta x)$$

$$= g_{\mu\nu}(x) - g_{\alpha\nu}(x)(\delta x^\alpha)_{,\mu} - g_{\mu\beta}(x)(\delta x^\beta)_{,\nu} - g_{\mu\nu,\lambda}(x)\delta x^\lambda$$

★ In terms of perturbation variables we find

$$\overline{A} = A - \dot{T}, \qquad \overline{H}_L = H_L - \frac{k}{3}L - HT,$$

$$\overline{B} = B + a\dot{L} + \frac{k}{a}T, \qquad \overline{H}_T = H_T + kL,$$

★ We can constitute two functions independent of generators L and T , namely, gauge invariant quantities. (Bardeen 80)

$$\Phi_A \equiv A + \frac{a}{k}\dot{B} + \frac{\dot{a}}{k}B - \frac{a^2}{k^2}\left(\ddot{H}_T + 2\frac{\dot{a}}{a}\dot{H}_T\right) = \Psi = \Phi$$

$$\Phi_H \equiv H_L + \frac{1}{3}H_T + \frac{\dot{a}}{k}B - \frac{a\dot{a}}{k^2}\dot{H}_T = \Phi = -\Psi$$

$$\mathcal{R}_s \equiv H_L + \frac{1}{3}H_T$$

curvature perturbation

Japanese notation

Kodama&Sasaki PTP Suppl 78(1984)1

★ Gauge-invariant variables can be defined similarly for matter contents, too.

(Example) A scalar field transforms as $\phi(x) = \overline{\phi}(\overline{x})$ by definition.

$$\phi(t, \boldsymbol{x}) = \phi(t) + \Delta\phi Y$$

$$\begin{aligned}\overline{\phi}(t, \boldsymbol{x}) &= \phi(t - TY, x^j - LY^j) = \phi(t - TY) + \Delta\phi Y \\ &= \phi(t) - \dot{\phi}(t) TY + \Delta\phi Y\end{aligned}$$

$$\therefore \ \overline{\Delta\phi} = \Delta\phi - \dot{\phi} T$$

$$\overline{B} = B + a\dot{L} + \frac{k}{a}T,$$

$$\overline{H}_T = H_T + kL,$$

$$\delta\phi = \Delta\phi + \frac{a}{k}\left(B - \frac{a}{k}\dot{H}_T\right)\dot{\phi}$$

gauge-invariant scalar field perturbation

Fixing the gauge

- ☆ In fact, we do not need to start with the most general metric and consider gauge transformation to find invariant quantities, but it is sufficient if the gauge degrees of freedom, L and T are fixed.

$$\overline{A} = A - \dot{T}, \qquad \overline{H}_L = H_L - \frac{k}{3}L - HT,$$

$$\overline{B} = B + a\dot{L} + \frac{k}{a}T, \qquad \overline{H}_T = H_T + kL, \qquad \overline{\Delta\phi} = \Delta\phi - \dot{\phi}T$$

- ☆ **Longitudinal Gauge**

  Let $H_T \equiv 0$ then L is fixed. Then let $B \equiv 0$, then T is also fixed.

$$A \equiv \Phi_A,\ H_L \equiv \Phi_H \Longleftarrow \begin{aligned} \Phi_A &\equiv A + \frac{a}{k}\dot{B} + \frac{\dot{a}}{k}B - \frac{a^2}{k^2}\left(\ddot{H}_T + 2\frac{\dot{a}}{a}\dot{H}_T\right) \\ \Phi_H &\equiv H_L + \frac{1}{3}H_T + \frac{\dot{a}}{k}B - \frac{a\dot{a}}{k^2}\dot{H}_T\ . \end{aligned}$$

$$\mathrm{d}s^2 = -(1 + 2\Phi_A Y)\mathrm{d}t^2 + a^2(1 + 2\Phi_H Y)\mathrm{d}\boldsymbol{x}^2$$

$$\delta\phi = \Delta\phi \Longleftarrow \delta\phi = \Delta\phi + \frac{a}{k}\left(B - \frac{a}{k}\dot{H}_T\right)\dot{\phi}$$

Fixing the gauge

⋆ In fact, we do not need to start with the most general metric and consider gauge transformation to find invariant quantities, but it is sufficient if the gauge degrees of freedom, L and T are fixed.

$$\overline{A} = A - \dot{T}, \qquad \overline{H}_L = H_L - \frac{k}{3}L - HT,$$

$$\overline{B} = B + a\dot{L} + \frac{k}{a}T, \qquad \overline{H}_T = H_T + kL, \qquad \overline{\Delta\phi} = \Delta\phi - \dot{\phi}T$$

⋆ **Unitary Gauge**

Let $\overline{\Delta\phi} = \Delta\phi - \dot{\phi}T = 0$, then T is fixed.

Also let $H_T \equiv 0$, then L is fixed, too.

$\Delta\phi = 0$: scalar field is homogeneous.

Evolution equations for perturbations

- For the moment, we work in the longitudinal gauge

$$\mathrm{d}s^2 = -(1+2\Phi_A Y)\mathrm{d}t^2 + a^2(1+2\Phi_H Y)\mathrm{d}\boldsymbol{x}^2$$

and introduce scalar-type perturbations to the perfect fluid matter.

$$T^{\mu\nu} = Pg^{\mu\nu} + (\rho+P)u^\mu u^\nu \quad (u^\mu u_\mu = -1)$$

$$\rho \to \rho + \delta\rho Y, \qquad u^\mu = (1,0,0,0) \to (1-AY, vY^j/a),$$

$$P \to P + \delta P Y, \qquad u_\mu = (-1,0,0,0) \to (-1-AY, avY_j)$$

Since gauge is already fixed, these variables are already gauge-invariant.

- Write down the perturbed Einstein equations $\delta G^\mu{}_\nu = 8\pi G \delta T^\mu{}_\nu$

Hamiltonian constraint

$$\delta T^0{}_0 = -\rho\delta Y$$

$$\delta \equiv \delta\rho/\rho$$

$$\pi_L \equiv \delta P/P$$

$$\delta T^j{}_0 = -(\rho+P)\frac{v}{a}Y^j$$

$$\pi_T = 0$$

anisotropic stress

$$\delta G^0{}_0 = \left(6H^2\Phi_A - 6H\dot{\Phi}_H - 2\frac{k^2}{a^2}\Phi_H\right)Y$$

$$\delta T^0{}_j = a(\rho+P)vY_j$$

$$\delta G^j{}_0 = \left(-2\frac{kH}{a^2}\Phi_A + 2\frac{k}{a^2}\dot{\Phi}_H\right)Y^j$$

$$\delta T^i{}_j = P(\pi_L\delta^i_j Y + \pi_T Y^i{}_j)$$

$$\delta G^0{}_j = (2kH\Phi_A - 2k\dot{\Phi}_H)Y_j$$

Dynamical eq.

$$\delta G^i{}_j = \left[\left(2H^2 + 4\frac{\ddot{a}}{a}\right)\Phi_A + 2H\dot{\Phi}_A - \frac{2}{3}\frac{k^2}{a^2}\Phi_A - 6H\dot{\Phi}_H - 2\ddot{\Phi}_H - \frac{2}{3}\frac{k^2}{a^2}\Phi_H\right]\delta^i_j Y - \frac{k^2}{a^2}(\Phi_A+\Phi_H)Y^i{}_j$$

★ From Hamiltonian and momentum constraints we find

$$2\frac{k^2}{a^2}\Phi_H = 8\pi G\rho\Delta = 3H^2\Delta \qquad \Delta \equiv \delta + 3(1+w)\frac{aH}{k}v \quad (w \equiv P/\rho)$$

is the comoving density perturbation.

★ $Y^i{}_j$ term yields $\Phi_H + \Phi_A = 0$ ①

★ As a result we find the Poisson equation $-\frac{k^2}{a^2}\Phi_A = 4\pi G\rho\Delta$ ②

★ Dynamical equation may be found from $\delta^i_j Y$ term or from $\delta T^\mu{}_{\nu;\mu} = 0$.

$$\dot{\Delta} - 3Hw\Delta = -(1+w)\frac{k}{a}v$$ ③ continuity eqn.

$$\dot{v} + Hv = \frac{1}{\rho+P}\frac{k}{a}(\delta P - c_s{}^2\delta\rho + c_s{}^2\rho\Delta) + \frac{k}{a}\Phi_A$$ ④ Euler eqn. $c_s^2 \equiv \frac{dP}{d\rho} = \frac{\dot{P}}{\dot{\rho}}$

★ From ①②③④, we find

$$\dot{\Phi}_H + H\Phi_H = -4\pi G(\rho+P)\frac{a}{k}v \equiv -\frac{3}{2}(1+w)H\Upsilon \qquad \Upsilon \equiv \frac{aH}{k}v$$

Momentum constraint

$$\dot{\Upsilon} + \frac{3}{2}H(1+w)\Upsilon = -H\Phi_H + \frac{H}{1+w}(c_s{}^2\Delta + w\Gamma)$$

Euler eqn.

$$P\Gamma \equiv \delta P - c_s^2\delta\rho$$

☆ Subtracting each other we find

$$\frac{\mathrm{d}}{\mathrm{d}t}(\Phi_H - \Upsilon) = -\frac{H}{1+w}({c_s}^2\Delta + w\Gamma)$$

$$2\frac{k^2}{a^2}\Phi_H = 8\pi G\rho\Delta = 3H^2\Delta$$

If there are onlv adiabatic fluctuations, e.g., single fluctuating component, we find $P\Gamma \equiv \delta P - c_s^2\delta\rho = 0$, since $c_s^2 = \dot{P}/\dot{\rho} = \delta P/\delta\rho$ holds. Then

$$\frac{\mathrm{d}}{\mathrm{d}t}(\Phi_H - \Upsilon) = -\frac{H}{1+w}{c_s}^2\Delta = -\frac{2{c_s}^2H}{3(1+w)}\left(\frac{k}{aH}\right)^2\Phi_H$$

$0 \text{ for } \dfrac{k}{aH} \to 0$

superhorizon limit

☆ Comoving curvature perturbation

$$\Phi_H - \Upsilon = \mathcal{R}_s - \frac{aH}{k}v \equiv \mathcal{R}_c$$

is conserved outside the Hubble radius if only adiabatic fluctuations are present.

☆ In the case of single scalar-field matter with $\mathcal{L} = K(X,\phi)$, we find

$${c_s}^2\Delta + w\Gamma = \tilde{c}_s^2\Delta \quad \text{with} \quad \tilde{c}_s^2 \equiv \frac{P_X}{\rho_X} = \frac{K_X}{K_X + 2XK_{XX}}$$

so the conservation of comoving curvature perturbation also holds.

This gives the sound velocity of a scalar field.
It is unity for canonical fields.

★ Using the momentum constraint we can express $\mathcal{R}_c$ with Φ_H only.

$$\mathcal{R}_c = \Phi_H - \Upsilon = \Phi_H + \frac{2}{3(1+w)}(\Phi_H + H^{-1}\dot{\Phi}_H) \equiv \zeta$$ Bardeen's ζ

★ $\zeta = \mathrm{const} \equiv C_1$ can be solved as a first-order differential equation

★ The solution is given by $\Phi_H = C_1 \left(1 - \frac{H}{a}\int^t a(t')\mathrm{d}t'\right)$

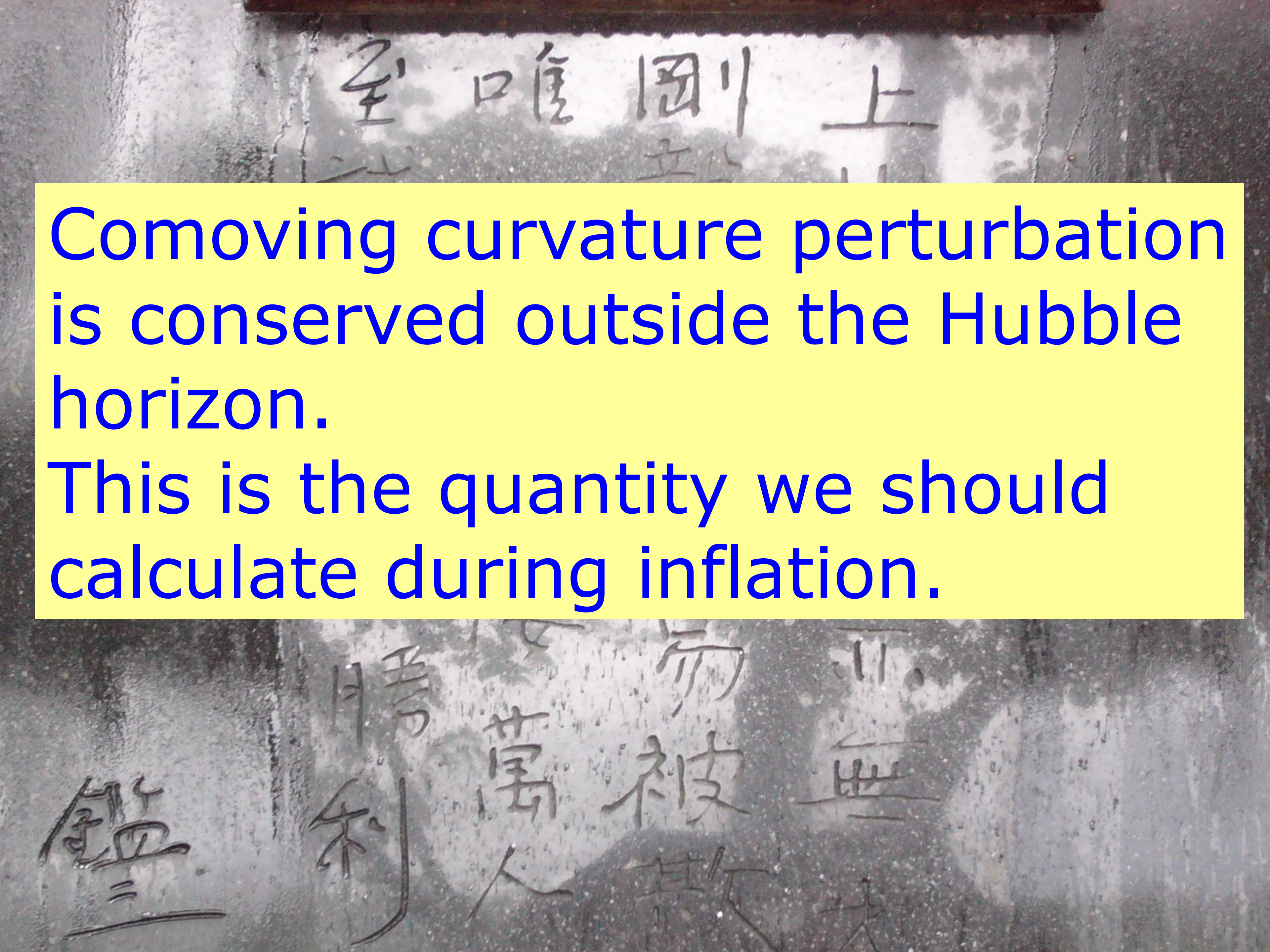
namely,

Growing adiabatic mode $\Phi_H^G = C_1 \left(1 - \frac{H}{a}\int^t a(t')\mathrm{d}t'\right)$

Decaying adiabatic mode $\Phi_H^D = \frac{H}{a}$

★ When $w = P/\rho = \mathrm{const}$ we find $\Phi_H^G = C_1 \left(1 + \frac{2}{3(1+w)}\right)^{-1} = \begin{cases} \frac{2}{3}C_1 & \text{for } w = \frac{1}{3} \\ \frac{3}{5}C_1 & \text{for } w = 0 \end{cases}$

★ In a contracting phase the "Decaying mode" $\Phi_H^D = \frac{H}{a}$ grows severely.



Comoving curvature perturbation is conserved outside the Hubble horizon.

This is the quantity we should calculate during inflation.

Curvature perturbation from inflation

★ Incorporate curvature perturbation to FLRW Universe and calculate its action in the Einstein+scalar model.

$$S = \int d^4x\sqrt{-g}\left[\frac{M_G^2}{2}R + K(X,\phi)\right]$$

include both potential-driven and k-inflation models

★ Background equations

$$3M_G^2H^2 = \rho = 2XK_X - K, \quad 2M_G^2\dot{H} + 3M_G^2H^2 = -P = -K$$

$$\ddot{\phi} + 3Hc_s^2\dot{\phi} + \frac{K_{X\phi}}{K_X}c_s^2\dot{\phi}^2 - \frac{K_\phi}{K_X}c_s^2 = 0, \quad \text{with} \quad c_s^2 \equiv \frac{P_X}{\rho_X} = \frac{K_X}{K_X + 2XK_{XX}}$$

sound speed of perturbation

★ We adopt 3+1 ADM decomposition which is useful to separate constraint equations.

$$ds^2 = -N^2dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \quad h_{ij} \equiv a^2(t)e^{2\mathcal{R}}\delta_{ij}$$

$\mathcal{R} \equiv \mathcal{R}_c$ represents comoving curvature perturbation conserved outside the horizon.

No gauge mode in L, since we have $\overline{H_T} = H_T + kL = 0$.

Setting $\overline{\Delta\phi} = \Delta\phi - \dot{\phi}T = 0$, gauge in T is also fixed.

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \quad h_{ij} \equiv a^2(t) e^{2\mathcal{R}} \delta_{ij}$$

☆ The action then reads

$$S = \frac{1}{2} \int d^4x \sqrt{h} N (M_G^2 R^{(3)} + 2K) + \frac{M_G^2}{2} \int d^4x \sqrt{h} N^{-1} (E_{ij}E^{ij} - E^2) + \cdots$$

where $E_{ij} \equiv \frac{1}{2}(\dot{h}_{ij} - N_{i|j} - N_{j|i}), \quad E \equiv \mathrm{Tr} E$

Total derivative terms not affecting field eqs

☆ We set $N = 1 + \alpha, \; N_i = \partial_i \psi$ to analyze linear scalar perturbations.

○ perturbation variables

☆ The Hamiltonian constraint obtained by differentiation w.r.t. N reads

$$R^{(3)} + 2\frac{K}{M_G^2} - 4X\frac{K_X}{M_G^2} - \frac{1}{N^2}(E_{ij}E^{ij} - E^2) = 0$$

$$\frac{H}{a^2}\partial^2\psi = -\frac{1}{a^2}\partial^2\mathcal{R} + \Sigma\alpha, \quad M_G^2\Sigma \equiv XK_X + 2X^2K_{XX}$$

☆ The momentum constraint obtained by differentiation w.r.t. N^i reads

$$\left[\frac{1}{N}(E_i^j - E\delta_i^j)\right]_{|j} = 2H\alpha_{,i} - 2\dot{\mathcal{R}}_{,i} = 0 \quad \Rightarrow \quad \alpha = \dot{\mathcal{R}}/H$$

☆ Now that both α and ψ have been expressed by $\mathcal{R}$ we can obtain the second order action for $\mathcal{R}$ as

$$S_2 = M_G^2 \int dt d^3x a^3 \left[\frac{\Sigma}{H^2}\dot{\mathcal{R}}^2 - \varepsilon_H \frac{(\partial\mathcal{R})^2}{a^2}\right], \quad \varepsilon_H \equiv -\frac{\dot{H}}{H^2}$$

☆ Introducing new variables, $z \equiv \frac{a\sqrt{2\Sigma}}{H} = \frac{a\sqrt{2\varepsilon_H}}{c_s}$, and $v \equiv M_G z\mathcal{R}$, the action is expressed with the conformal time η as

$$S_2 = \frac{1}{2}\int \mathrm{d}\eta \mathrm{d}^3x \left[v'^2 - c_s^2(\partial v)^2 + \frac{z''}{z}v^2\right]$$

which is equivalent to an action of a free scalar field with a time-dependent

mass squared $\frac{z''}{z} = (aH)^2\left[(2 - \varepsilon_H - s + \frac{\eta_H}{2})(1 - s + \frac{\eta_H}{2}) - \frac{\dot{s}}{H} + \frac{\dot{\eta}_H}{2H}\right]$

$$s \equiv \frac{\dot{c}_s}{Hc_s}, \quad \eta_H \equiv \frac{\dot{\varepsilon}_H}{H\varepsilon_H}$$

$$S_2 = \frac{1}{2}\int \mathrm{d}\eta \mathrm{d}^3x \left[v'^2 - c_s^2(\partial v)^2 + \frac{z''}{z}v^2\right] \qquad s \equiv \frac{\dot{c}_s}{Hc_s}, \quad \eta_H \equiv \frac{\dot{\varepsilon}_H}{H\varepsilon_H}$$

$$\frac{z''}{z} = (aH)^2 \left[(2 \underbrace{- \varepsilon_H - s + \frac{\eta_H}{2})(1 - s + \frac{\eta_H}{2}) - \frac{\dot{s}}{H} + \frac{\dot{\eta}_H}{2H}}_{\text{small slow-variation parameters}}\right] \equiv (aH)^2(2+q)$$

★ Using the de Sitter scale factor $a = -\frac{1}{H\eta}$, the normalized mode function reads

$$v_k = \left(-\frac{\pi\eta}{4}\right)^{1/2} H_\nu^{(1)}(-kc_s\eta) \cong \frac{1}{\sqrt{2kc_s}}\left(1 - \frac{i}{kc_s\eta}\right)e^{-ikc_s\eta} \quad \nu = \frac{3}{2}\left(1+\frac{4}{9}q\right)^{\frac{1}{2}} \cong \frac{3}{2}$$

★ It behaves similarly to a massless scalar field in de Sitter background, so that long-wave nearly scale-invariant fluctuations will be generated.

$$\mathcal{P}_{\mathcal{R}}(k) \equiv \frac{4\pi k^3}{(2\pi)^3}|\mathcal{R}_k|^2 = \frac{4\pi k^3}{(2\pi)^3}\left|\frac{v_k}{z}\right|^2 = \frac{H^2}{8\pi^2 M_G^2 c_s \varepsilon_H}$$

evaluated at the sound horizon crossing $-kc_s\eta = 1$

$$\mathcal{P}_{\mathcal{R}}(k) \equiv \frac{4\pi k^3}{(2\pi)^3}|\mathcal{R}_k|^2 = \frac{4\pi k^3}{(2\pi)^3}\left|\frac{v_k}{z}\right|^2 = \frac{H^2}{8\pi^2 M_G^2 c_s \varepsilon_H}$$

★ The spectral index of the curvature perturbation is given by

$$\boxed{n_s - 1 \equiv \frac{d\ln \mathcal{P}_{\mathcal{R}}(k)}{d\ln k} = -2\varepsilon_H - \eta_H - s} \qquad \varepsilon_H \equiv -\frac{\dot{H}}{H^2} \qquad s \equiv \frac{\dot{c}_s}{Hc_s}, \quad \eta_H \equiv \frac{\dot{\varepsilon}_H}{H\varepsilon_H}$$

★ In the canonical slow-roll inflation, using the slow-roll equations we find

$$\varepsilon_H = -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{M_G^2 H^2} = \frac{3\dot{\phi}^2}{2V} \underset{\uparrow}{=} \frac{M_G^2}{2}\left(\frac{V'}{V}\right)^2 \equiv \varepsilon_V \qquad \eta_V \equiv M_G^2 \frac{V''}{V} \qquad \eta_H = -2\eta_V + 4\varepsilon_V$$

$3H\dot{\phi} = -V'$

so

$$\boxed{n_s - 1 = -6\varepsilon_V + 2\eta_V}$$

★ The scale dependence of the spectral index, “Running”

$$\boxed{\frac{dn_s}{d\ln k} = 16\varepsilon_V\eta_V - 24\varepsilon_V^2 - 2\xi_V} \qquad \xi_V \equiv M_G^4 \frac{V'V'''}{V^2}$$

These are important observable quantities!

Tensor perturbation from inflation

- ☆ We derive a second-order action for the tensor perturbation $h_{\mu\nu}$.
  It is wise to make use of the known results on GW in the Minkowski space.
  So we first study perturbation around $\eta_{\mu\nu}$ taking metric as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$.

  Taking the TT gauge $h_{00} = h_{0i} = 0,\ h^{\alpha}_{\alpha} = h^{i}_{i} = 0,\ h_{ij}^{;j} = 0$,
  the Ricci scalar reads up to the second order

$$R = h^{ij} h_{ij,\mu}^{;\mu} + \frac{3}{4} h^{ij,\mu} h_{ij,\mu} - \frac{1}{2} h^{ij,l} h_{jl,i}$$

- ☆ The transformation from $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
  to $ds^2 = a^2(\eta)\left[-d\eta^2 + (\delta_{ij} + h_{ij}) dx^i dx^j\right] \equiv \tilde{g}_{\mu\nu} dx^\mu dx^\nu$ can be done
  by the conformal transformation $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$.

- ☆ The Ricci tensors in two conformal metrices are related as

$$\begin{aligned}\tilde{R}_{\mu\nu} = R_{\mu\nu} &- 2\nabla_\mu \nabla_\nu \ln\Omega - g_{\mu\nu} g^{\sigma\tau} \nabla_\sigma \nabla_\tau \ln\Omega \\ &+ 2\nabla_\mu \ln\Omega \nabla_\nu \ln\Omega - g_{\mu\nu} g^{\sigma\tau} \nabla_\sigma \ln\Omega \nabla_\tau \ln\Omega\end{aligned}$$

★ Putting $\Omega = a(\eta)$, the Ricci scalar reads up to the second order

$$\tilde{R} = a^{-2}\left(R + 6\frac{a''}{a} - 3\frac{a'}{a}h^{ij}h'_{ij}\right)$$

★ Since we are interested in tensor perturbations in the inflaitonary Universe let us introduce a cosmological constant to drive inflation, to consider

$$\begin{aligned} S_2 &= \frac{M_G^2}{2}\int(\tilde{R} - 2\Lambda)\sqrt{-\tilde{g}}d^4x\Big| \\ &= \frac{M_G^2}{8}\int d\eta d^3x a^2(h_j^{i\prime}h_i^{j\prime} - h^i_{j,l}h_i^{j,l}) \end{aligned}$$

2nd order

using the background eq. $\Lambda a^4 = 2aa'' - a'^2$

★ Introducing new variables $z_T \equiv a/2, u_{ij} \equiv M_G z_T h_{ij}$ the action reads

$$S_T = \frac{1}{2}\int d\eta d^3x\left[u'^2_{ij} - (\nabla u_{ij})^2 + \frac{a''}{a}u^2_{ij}\right]$$

which is equivalent to the action of a massless scalar field.

In the de Sitter background $a = -1/(H\eta)$, we find

$$u^A_{ij} = \left(-\frac{\pi\eta}{4}\right)^{1/2} H^{(1)}_{3/2}(-k\eta)e^A_{ij}(\boldsymbol{k}), \quad A = +, \times$$

as before.

- ⋆ Introducing new variables $z_T \equiv a/2, u_{ij} \equiv M_G z_T h_{ij}$ the action reads

$$S_T = \frac{1}{2}\int d\eta d^3x \left[u_{ij}'^2 - (\nabla u_{ij})^2 + \frac{a''}{a}u_{ij}^2\right]$$

which is equivalent to the action of a massless scalar field.

In the de Sitter background $a = -1/(H\eta)$, we find

$$u_{ij}^A = \left(-\frac{\pi\eta}{4}\right)^{1/2} H_{3/2}^{(1)}(-k\eta)e_{ij}^A(\boldsymbol{k}), \quad A = +, \times$$

as before.

- ⋆ In case $\varepsilon_H \neq 0$, we can express $a = -\frac{1}{H_*\eta_*}\frac{1}{1-\varepsilon}\left(\frac{-\eta}{-\eta_*}\right)^{\frac{1}{2}-\nu_T}, \quad \nu_T = \frac{3}{2}\frac{1-\varepsilon/3}{1-\varepsilon}$

$$u_{ij}^A = \left(-\frac{\pi\eta}{4}\right)^{1/2} H_{\nu_T}^{(1)}(-k\eta)e_{ij}^A(\boldsymbol{k}), \quad A = +, \times$$

$e_{ij}^A(\boldsymbol{k})$ is the polarization tensor with $\langle e_{ij}^A(\boldsymbol{k})e^{*ijB}(\boldsymbol{k})\rangle = \delta^{AB}$

- ⋆ The power spectrum reads

$$\mathcal{P}_T(k) \equiv \frac{4\pi k^3}{(2\pi)^3}h_{ij}h^{*ij} = \frac{4\pi k^3}{(2\pi)^3}\frac{u_{ij}^A u^{*ijA}}{M_G^2 z_T^2} = \frac{2H^2}{\pi^2 M_G^2}$$

★ Tensor spectral index

$$\boxed{n_t \equiv \frac{d\ln \mathcal{P}_T(k)}{d\ln k} = -2\varepsilon_H}$$

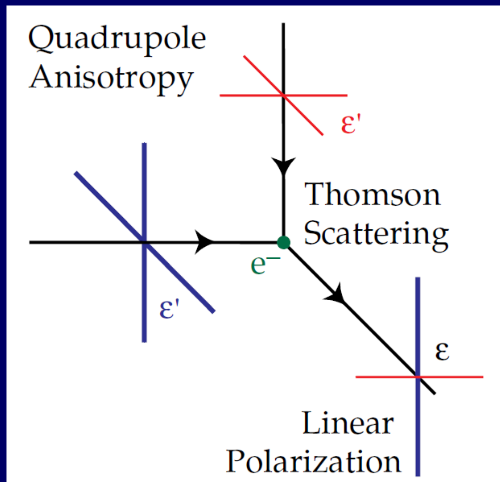
★ The tensor-to-scalar ratio

$$\boxed{r \equiv \frac{\mathcal{P}_T(k)}{\mathcal{P}_{\mathcal{R}}(k)}} \qquad \frac{\mathcal{P}_T(k) = \dfrac{2H^2}{\pi^2 M_G^2}}{\mathcal{P}_{\mathcal{R}}(k) = \dfrac{H^2}{8\pi^2 M_G^2 c_s \varepsilon_H}}$$

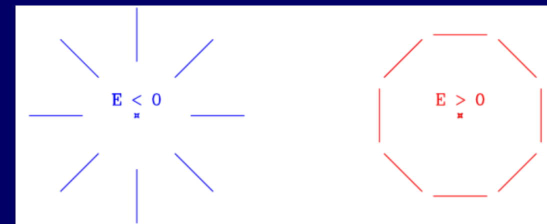
$$\boxed{r = 16 c_s \varepsilon_H = -8 c_s n_t}$$

consistency relation

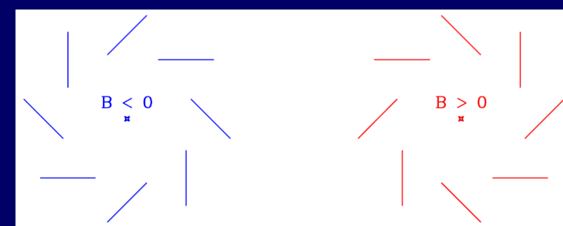
Low frequency components may be observed by B-mode polarization of CMB anisotropy



E mode



B mode



- Polarization is generated by quadrupole temperature anisotropy.
- E-mode from both scalar (density) and tensor perturbations.
- B-mode only from tensor perturbations.

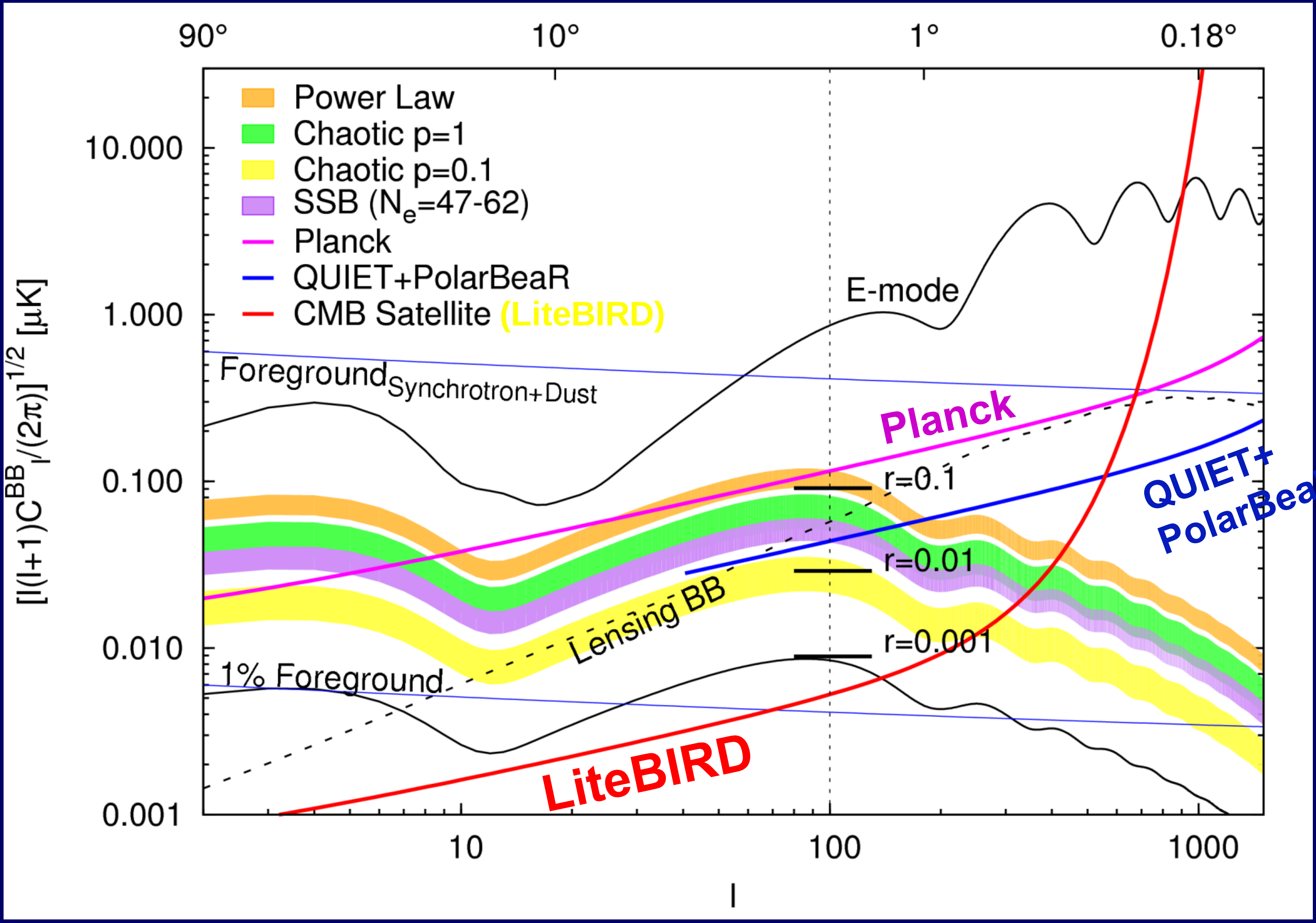
Planned or ongoing experiments and their expected sensitivity

PLANCK	$r \sim 0.1$	QUIET+PolarBear	$r \sim 0.01$
EPIC	$r \sim 0.001$	LiteBIRD	$r \sim 0.001$
CMB-POL	$r \sim 0.001$		

WMAP7 $r < 0.25$

Just one summary plot !

by M. Hazumi



Some of the recent particle physics models of inflation

★ KK … 3)

★ DE … l extra dimension

(Silverstein & Tong 04)

… possible.

…ohyama & Kinoshita 07)

★ Ra … o, Kallosh, Linde &Quevedo 04)

…uning of parameters

★ Ax …

…vity

Newer models of "stringy" inflation

- ★ Monodoromy model

  $r = 0.03$ (Silverstein & Westphal 08)

- ★ Fibre inflation

  $r = 0.005$ (Cicoli, Burgers, & Quevedo 08)

- ★ Warped Wilsonline DBI inflation

  $r \sim 0.1$ (Avgoustidis & Zavala 08)

etc

★ Higgsflation (Kaloper, Sorbo & JY 08)

$r = 0.003$ detectable tensor perturbation even if variation in φ is small

★ Chaotic inflation in supergravity with shift symmetry

(Kawasaki, Yamaguchi & Yanagida 00)

$r = 0.15$ allows large variation of φ thanks to the shift symmetry

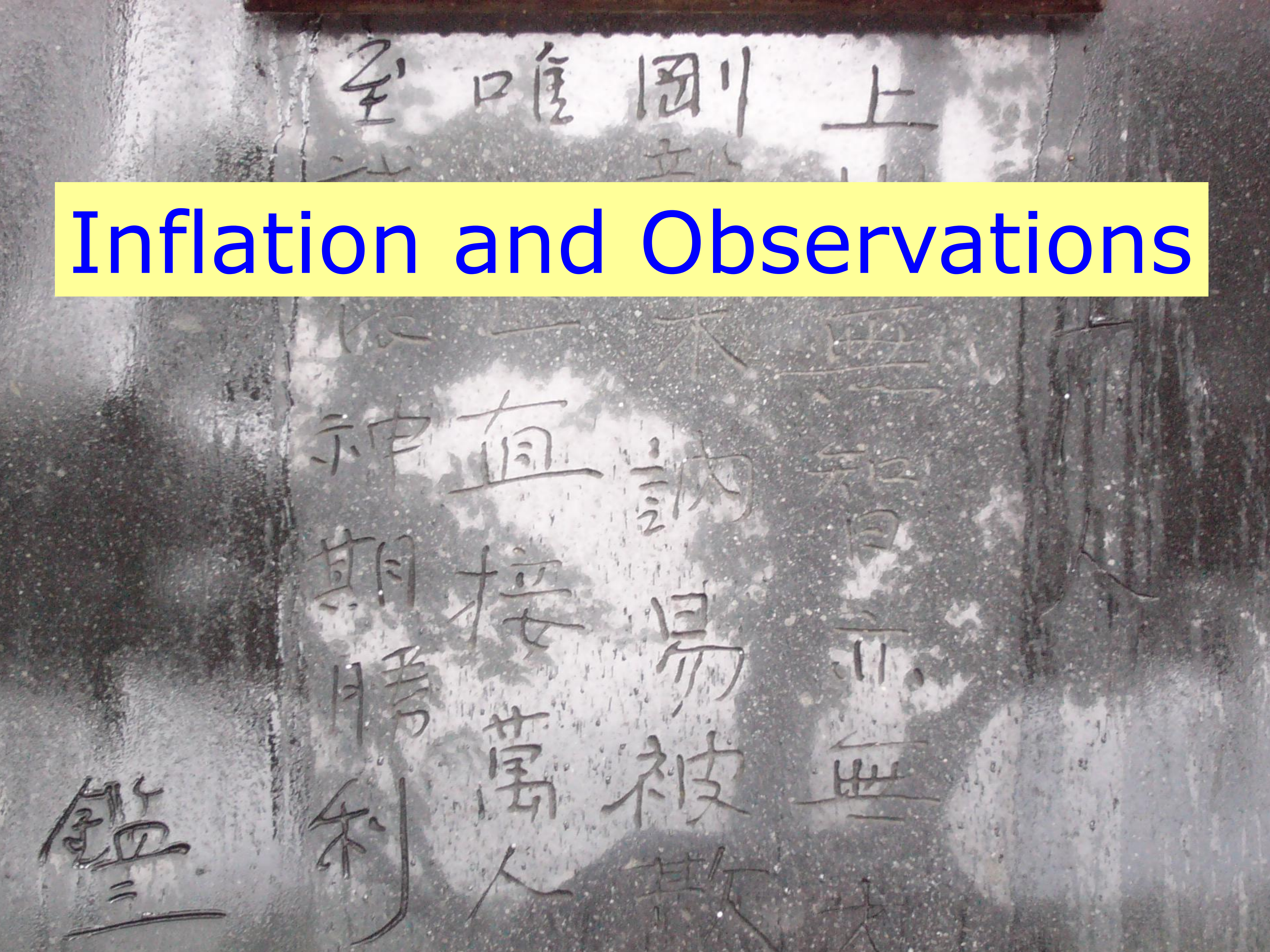
★ MSSM inflation (Allahverdi, Enqvist, Garcia-Bellido & Mazumdar 06)

$r \sim 10^{-25}$

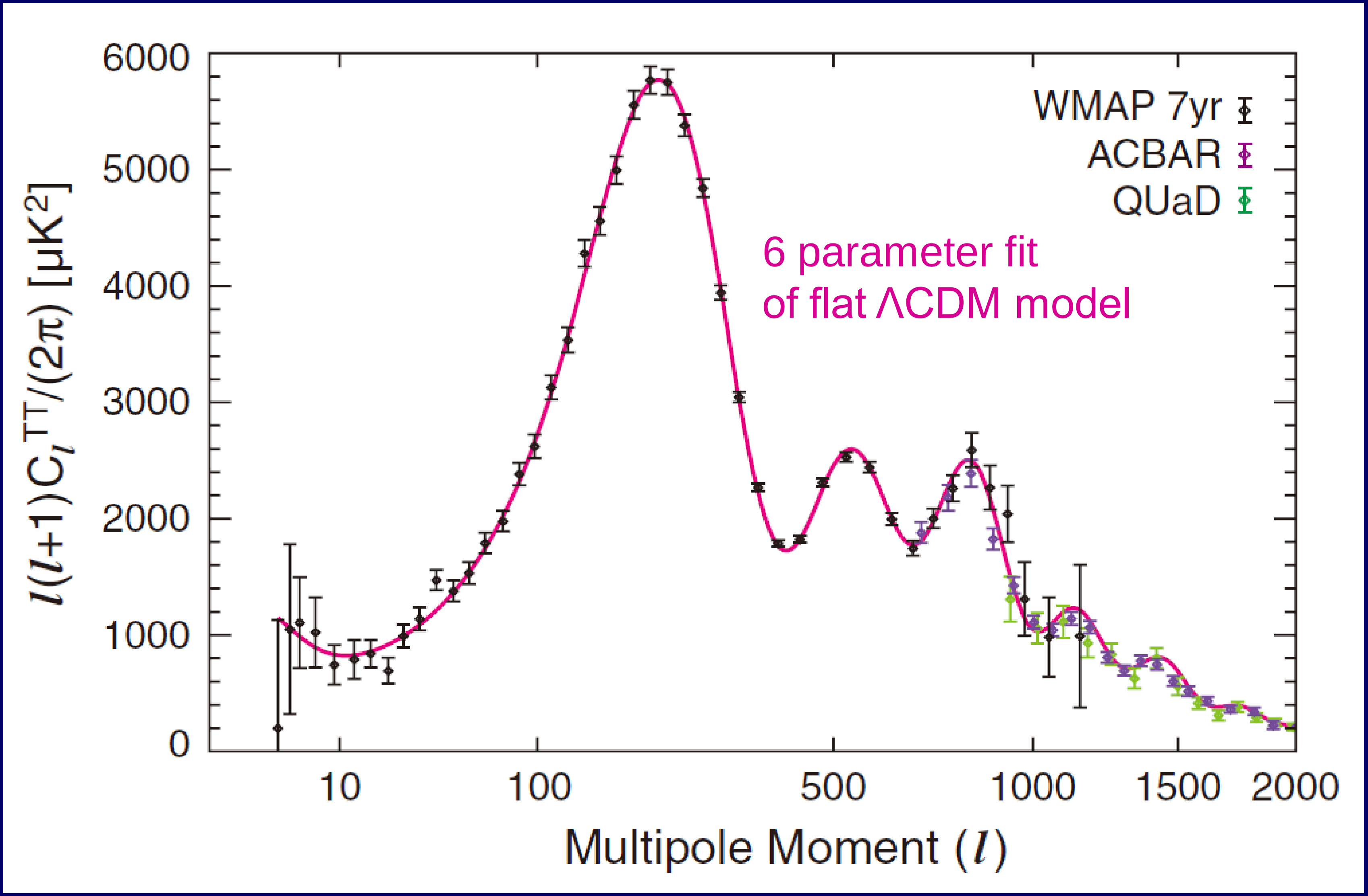
The inflaton can be identified in MSSM w/ fine-tuned parameters.

It can occur only if previous inflation with a slightly higher energy scale is realized with a sufficiently low reheat temperature, very contrived. (Kamada & JY 09)

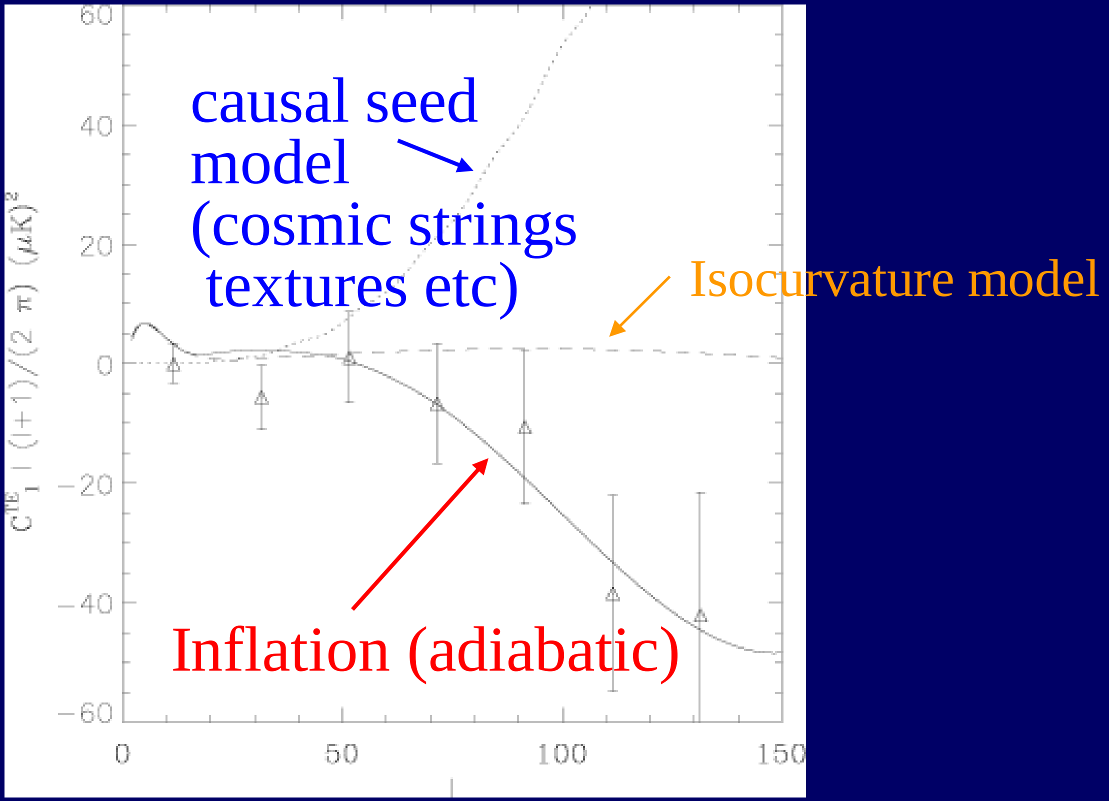
Inflation and Observations



Theory and observations basically agree.



WMAP observed negative correlation between temperature anisotropy and E-mode polarization which is predicted by super-Hubble adiabatic fluctuations produced during inflation.



Spatial curvature

$$-0.0133 < \Omega_{K0} \equiv \overset{1-\Omega_{tot0}}{-\frac{K}{a_0^2 H_0^2}} < 0.0084 \quad (95\%\mathrm{CL}),$$

Amplitude and spectral index of curvature perturbation

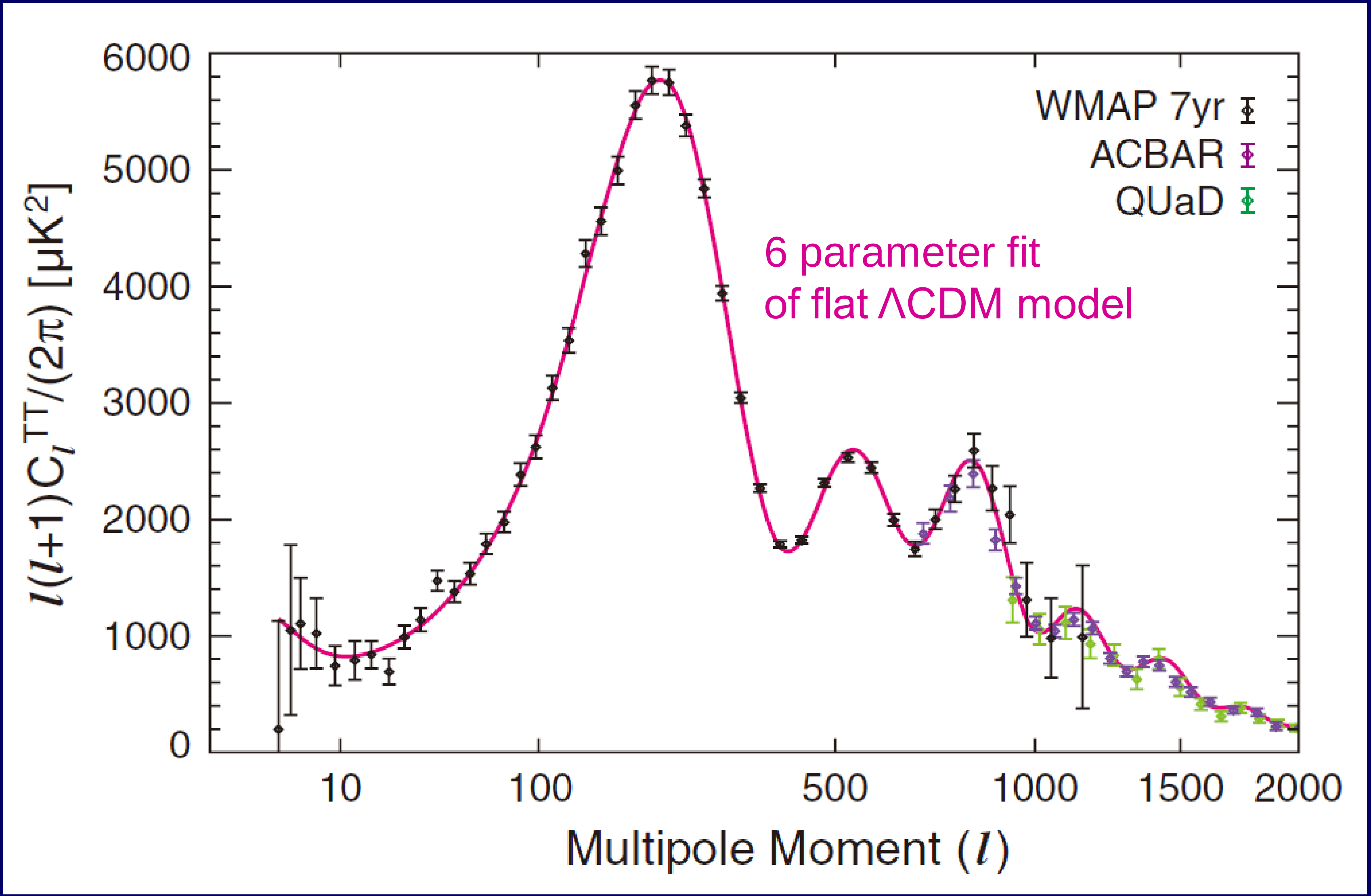
$$\mathcal{P}_{\mathcal{R}}(k_0) = (2.430 \pm 0.091) \times 10^{-9} \qquad k_0 = 0.002\mathrm{Mpc}^{-1}$$

$$n_s = 0.968 \pm 0.012$$

(7year WMAP+BAO+HST)

Based on Markov-Chain-Monte-Carlo method
Values of these parameter change depending on which parameters to fit and which dataset we use.

Theory and observations basically agree.



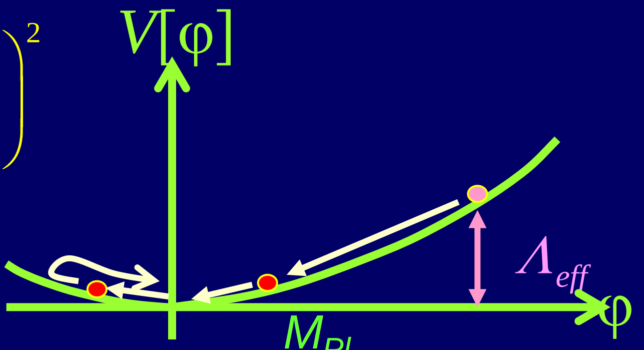
We wish to proceed model selection of inflation...

Observables: Large-field model

$$V[\phi] = \frac{1}{2} m^2 \phi^2$$

- Slow-roll parameters $\varepsilon_V = \eta_V = 2\left(\frac{M_G}{\phi}\right)^2 \quad \xi_V = 0$
- Number of e-folds from $\phi = \phi_N$ to the end of inflation

$$N = \int H dt = \int H \frac{d\phi}{\dot{\phi}} = \int \frac{3H^2}{V'[\phi]} d\phi = \int \frac{V[\phi] d\phi}{M_G^2 V'[\phi]} = \frac{1}{4}\left(\frac{\phi_N}{M_G}\right)^2$$



Chaotic inflation

- Amplitude of fluctuations

$$\mathcal{P}_{\mathcal{R}}(k_0) = \frac{H^2}{8\pi^2 M_G^2 \varepsilon_V} = \frac{1}{6\pi^2}\left(\frac{mN}{M_G}\right)^2 = 2.4 \times 10^{-9}$$

$$\Rightarrow m = 1.6 \times 10^{13}\,\mathrm{GeV}, \ \ \lambda < 8 \times 10^{-13} \ \text{for} \ \frac{\lambda}{4}\phi^4. \qquad @\, N \simeq 55$$

- The coupling between the inflaton and other fields must be small.

e.g. Yukawa coupling $h < 10^{-3}$; decay width $\Gamma_\phi = \frac{h^2}{8\pi} m < 6 \times 10^5\,\mathrm{GeV}$

$$T_R = 0.1 \left(\frac{200}{g_*}\right)^{1/4} \sqrt{M_{Pl} \Gamma_\phi} \simeq 10^{11} \left(\frac{200}{g_*}\right)^{1/4} \left(\frac{\Gamma_\phi}{10^5\,\mathrm{GeV}}\right)^{1/2} \mathrm{GeV}$$

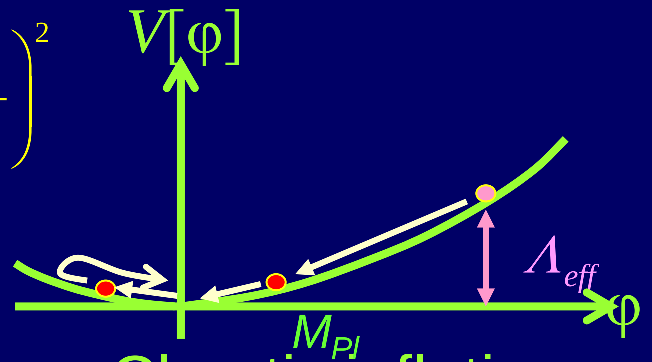
Observables: Large–field model

$$V[\phi] = \frac{1}{2} m^2 \phi^2$$

- ★ Slow-roll parameters $\varepsilon_V = \eta_V = 2\left(\frac{M_G}{\phi}\right)^2 \quad \xi_V = 0$

- ★ Number of e-folds from $\phi = \phi_N$ to the end of inflation

$$N = \int H dt = \int H \frac{d\phi}{\dot{\phi}} = \int \frac{3H^2}{V'[\phi]} d\phi = \int \frac{V[\phi] d\phi}{M_G^2 V'[\phi]} = \frac{1}{4}\left(\frac{\phi_N}{M_G}\right)^2$$



Chaotic inflation

- ★ Spectral index and its scale dependence

$$n_s = 1 - \frac{2}{N} = 0.964, \qquad \frac{dn_s}{d\ln k} = -6.6 \times 10^{-4}.$$

$$@\, N \simeq 55$$

- ★ Tensor-to-scalar ratio

$$r = 16\varepsilon_V = 0.15.$$

Observable by Planck!

Observables: Small–field model

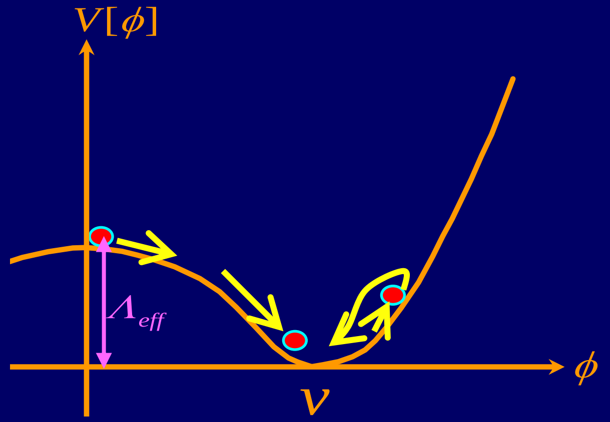
$$V[\phi] = \frac{\lambda}{4}(\phi^2 - v^2)^2 \qquad v \equiv \beta M_G$$

- Slow-roll parameters

$$\varepsilon_V = \frac{8M_G^2\phi^2}{(\phi^2 - v^2)^2}, \quad \eta_V = \frac{4M_G^2(3\phi^2 - v^2)}{(\phi^2 - v^2)^2}, \quad \xi_V = \frac{96M_G^4\phi^2}{(\phi^2 - v^2)^3}$$

- Inflation ends when $\varepsilon_H = \varepsilon_V = 1$ at the field value

$$\phi_f^2 = v^2 + 4M_G^2 - \sqrt{16M_G^2 + 8M_G^2 v^2}$$
$$\cong (\beta^2 - 2\sqrt{2}\beta)M_G^2$$

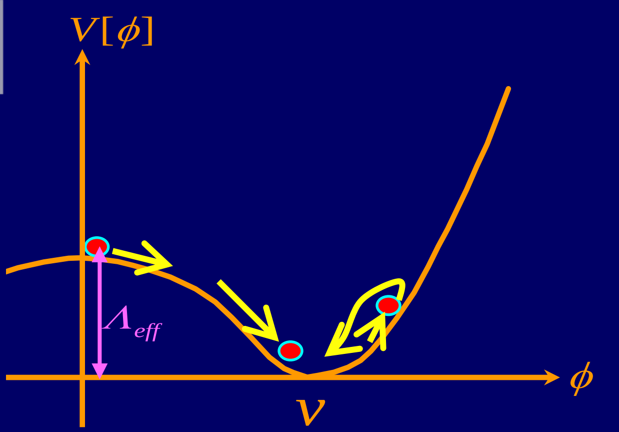


- Number of e-folds from $\phi = \phi_N$ to the end of inflation

$$N = \int_{\phi_N}^{\phi_f} H\frac{d\phi}{\dot{\phi}} = \frac{\beta^2}{4}\ln\frac{\phi_f}{\phi_N} - \frac{1}{8M_G^2}(\phi_f^2 - \phi_N^2) \simeq \frac{\beta^2}{4}\left(\ln\frac{\phi_f}{\phi_N} - \frac{1}{2}\right)$$

Observables: Small–field model

$$V[\phi] = \frac{\lambda}{4}(\phi^2 - v^2)^2 \qquad v \equiv \beta M_G$$



- Curvature perturbation

$$\mathcal{P}_{\mathcal{R}}(k_0) = \frac{\lambda(\phi_N^2 - v^2)^4}{768\pi^2 M_G^6 \phi_N^2} \simeq \frac{\lambda\beta^8}{768\pi^2}\left(\frac{M_G}{\phi_N}\right)^2 \simeq \frac{\lambda\beta^8}{768\pi^2(\beta-\sqrt{2})^2}\exp\left(\frac{8N}{\beta^2}+1\right)$$

Taking $N = 55$, $\beta = 15$, the normalization gives $\lambda = 7\times10^{-14}$.

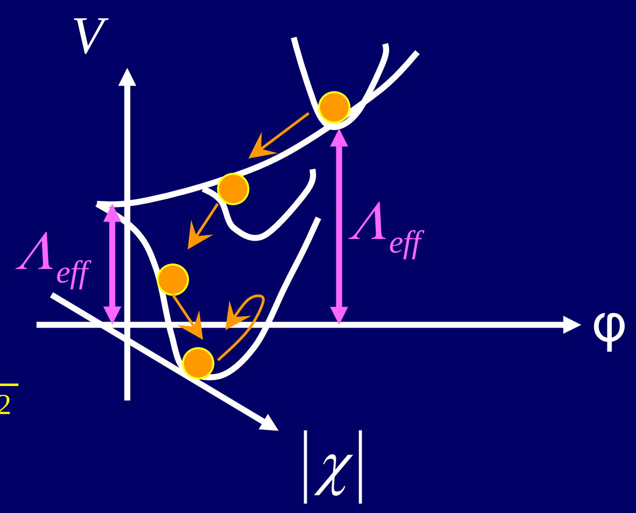
- Spectral index and its scale dependence

$$n_s - 1 = -\frac{8(3\phi_N^2 + v^2)M_G^2}{(\phi_N^2 - v^2)^2} \simeq -\frac{8}{\beta^2}, \qquad 0.964$$

$$\frac{dn_s}{d\ln k} = -\frac{(320v^2\phi_N^2 + 192\phi_N^4)M_G^4}{(\phi_N^2 - v^2)^4} \simeq -\frac{320}{\beta^6}\left(\frac{\phi_N}{M_G}\right)^2, \qquad \text{tiny}$$

Observables: Hybrid inflation model

$$V[\phi] = V_0 + \frac{m^2}{2}\phi^2$$ near the origin



- Consider false-vacuum dominated case

$$\varepsilon_V = \frac{M_G{}^2}{2}\left(\frac{m^2\phi}{V_0}\right)^2 = \frac{1}{18}\left(\frac{m}{H}\right)^4\left(\frac{\phi}{M_G}\right)^2 \qquad \eta_V = \frac{M_G^2 m^2}{V_0} = \frac{m^2}{3H^2}$$

- Spectral index and its scale dependence

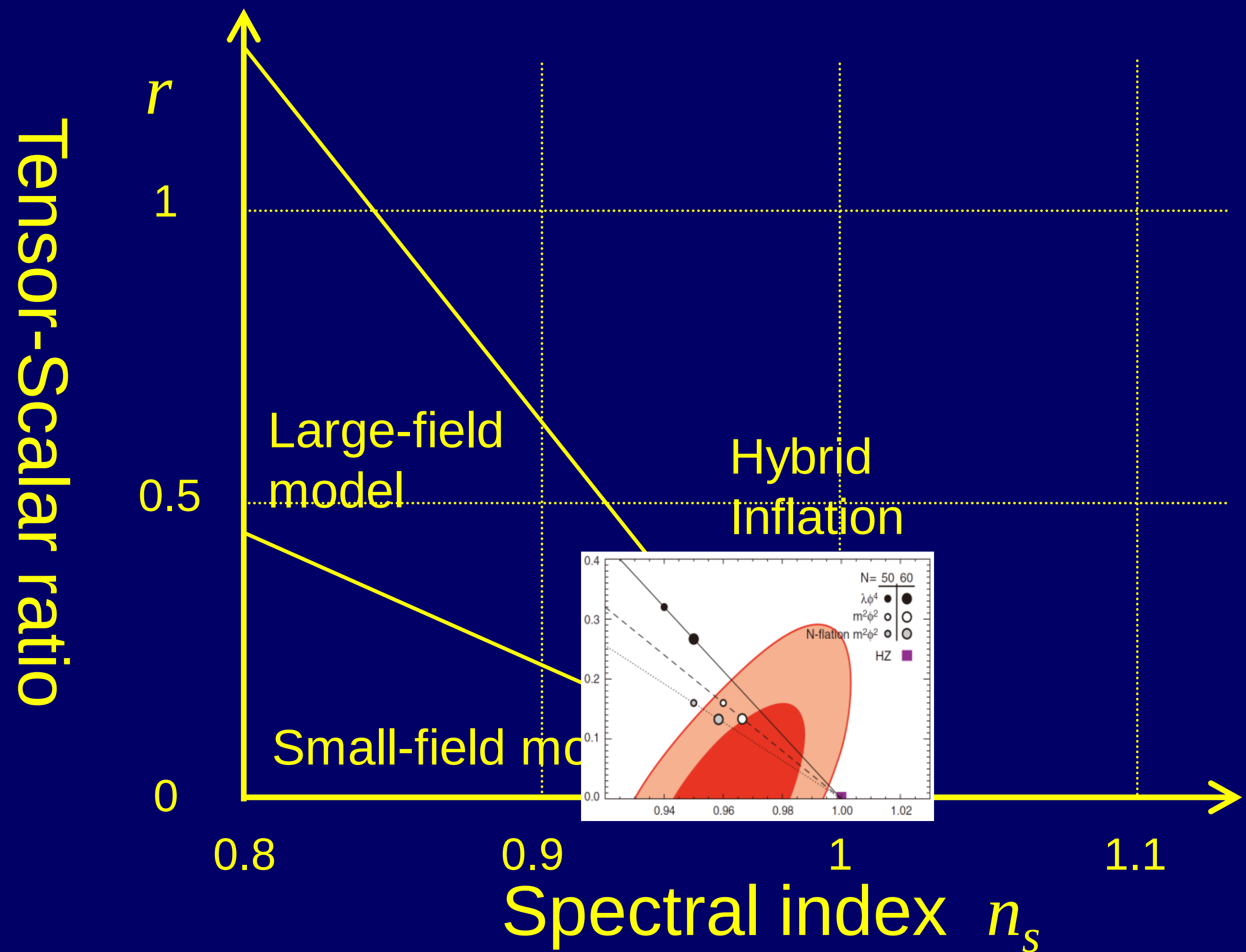
$$n_s - 1 \cong 2\eta_V = \frac{2m^2}{3H^2}$$

$$\frac{dn_s}{d\ln k} \cong 16\varepsilon\eta = \left(\frac{2m^2}{3H^2}\right)^3\left(\frac{\phi}{M_G}\right)^2 = (n-1)^3\left(\frac{\phi}{M_G}\right)^2 \sim 10^{-4}\left(\frac{\phi}{M_G}\right)^2$$

- Tensor-to-scalar ratio

$$r = 2\left(\frac{2m^2}{3H^2}\right)^2\left(\frac{\phi}{M_G}\right)^2 = (n-1)^2\left(\frac{\phi}{M_G}\right)^2 \sim 0.005\left(\frac{\phi}{M_G}\right)^2$$

Inflation models may be distinguished by observations.



$WMAP+\mathrm{BAO}+H_0$

(Dodelson, Kinney, Kolb 97)

Deviation from Gaussian: NonGaussianity of fluctuations

may also help distinguish models.

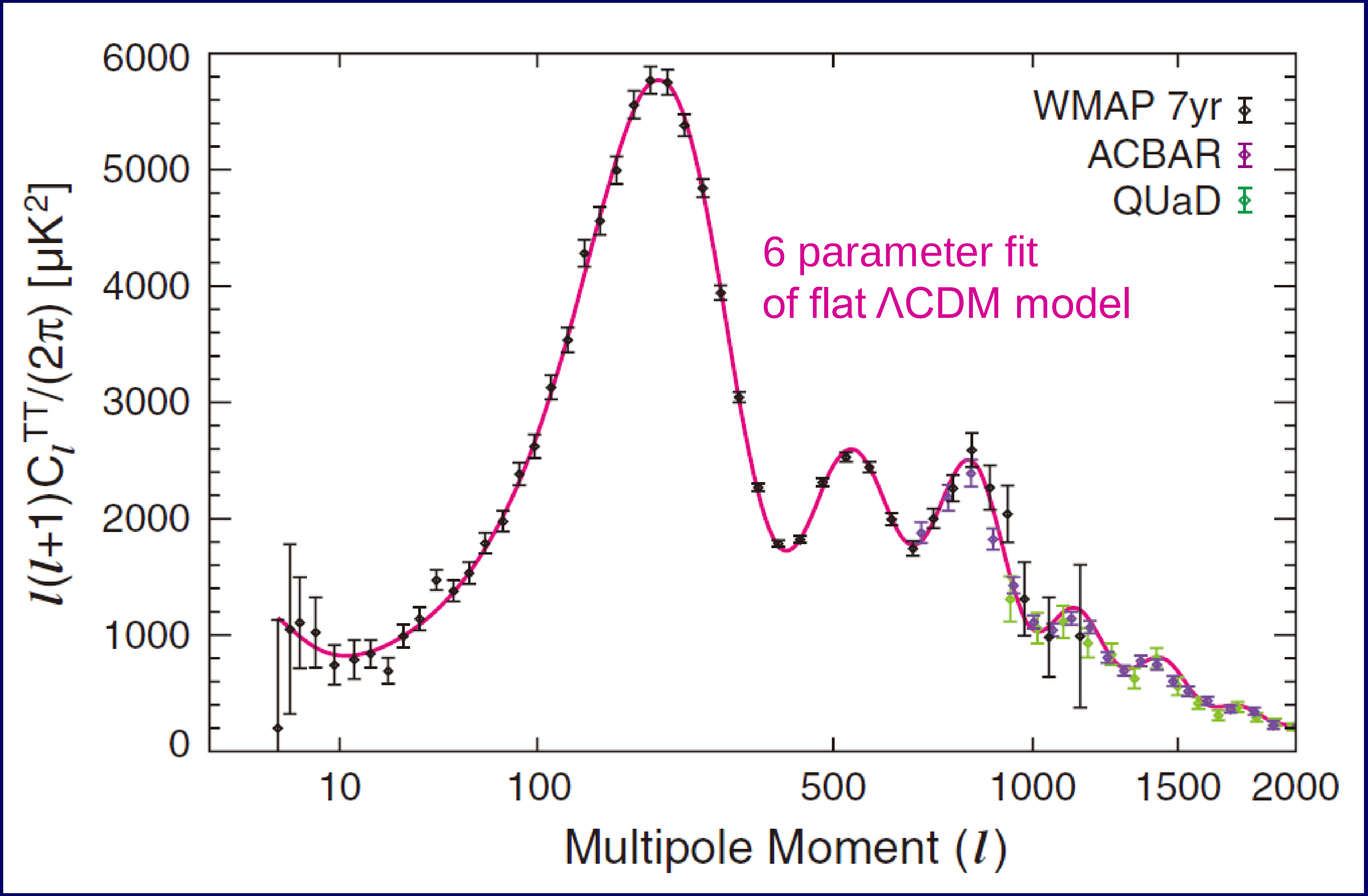
- Potential-driven slow-roll models
  NonGaussianity is small, because the inflaton is very weakly coupled with other fields as we have seen.

- k-inflation, G-inflation,…
  NonGaussianity can be large.

- Beyond the single-field inflation

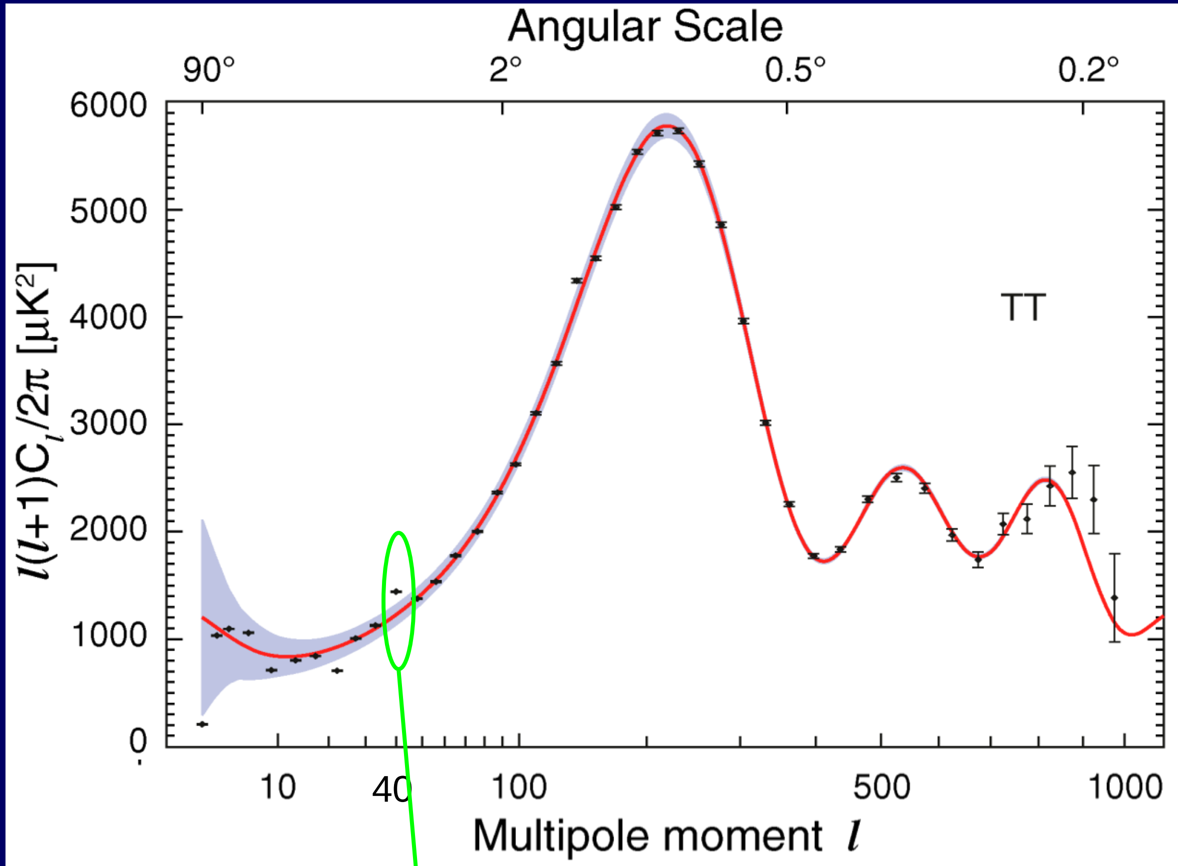
NO detection yet

Theory and observations basically agree.

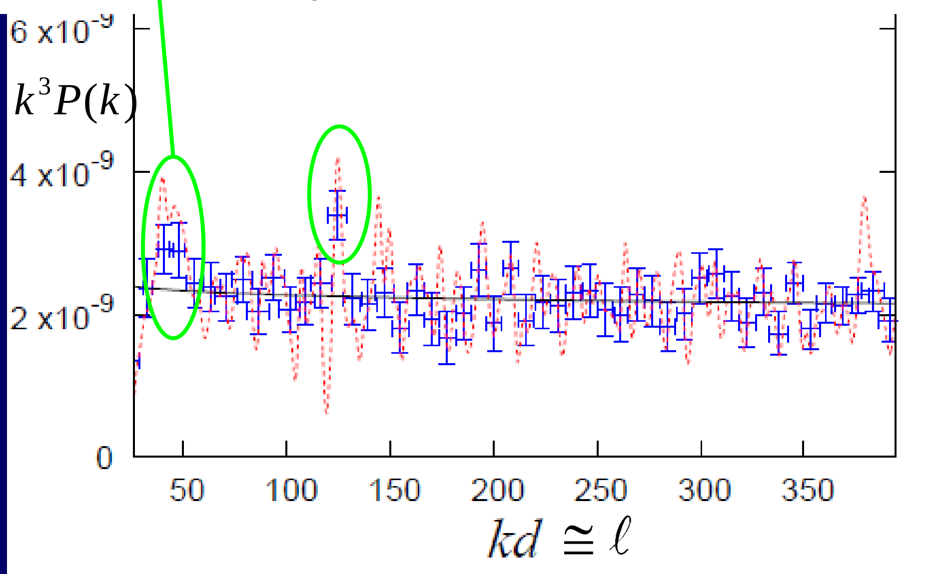


If you look at it closer in detail...

Reconstruction of primordial power spectrum from CMB



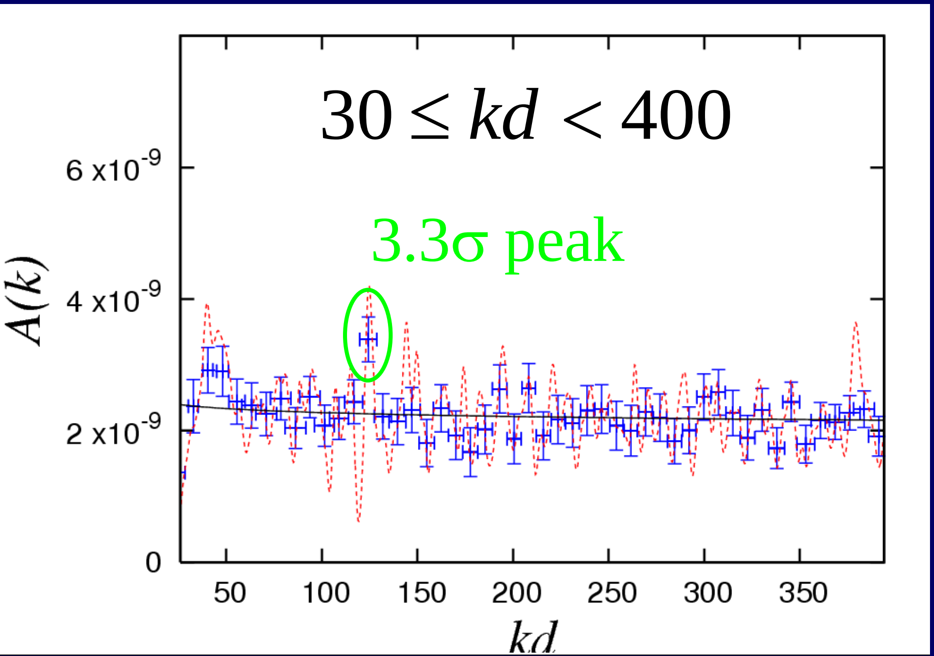
Deviation around $kd \approx \ell \approx 40$ can be seen even in the binned C_ℓ but those at 125 can not be seen there.

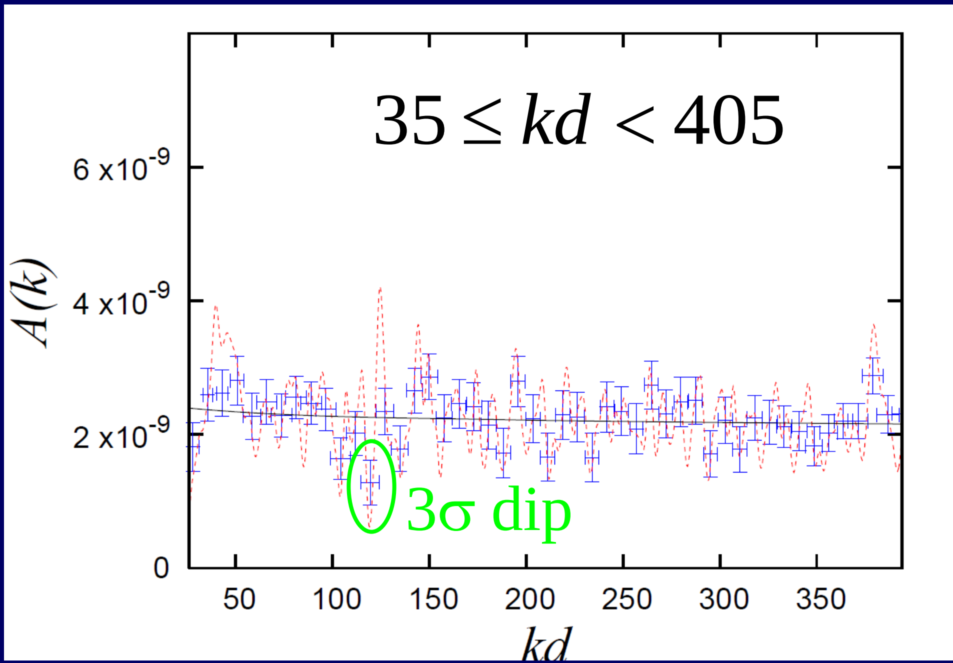


curvature perturbation

(Nagata & JY 08)

d=1.3x10^{4}Mpc distance to the last scattering surface





In fact, if we change the wavenumber domain of decomposition slightly, we obtain a dip rather than an excess even for the band power analysis.

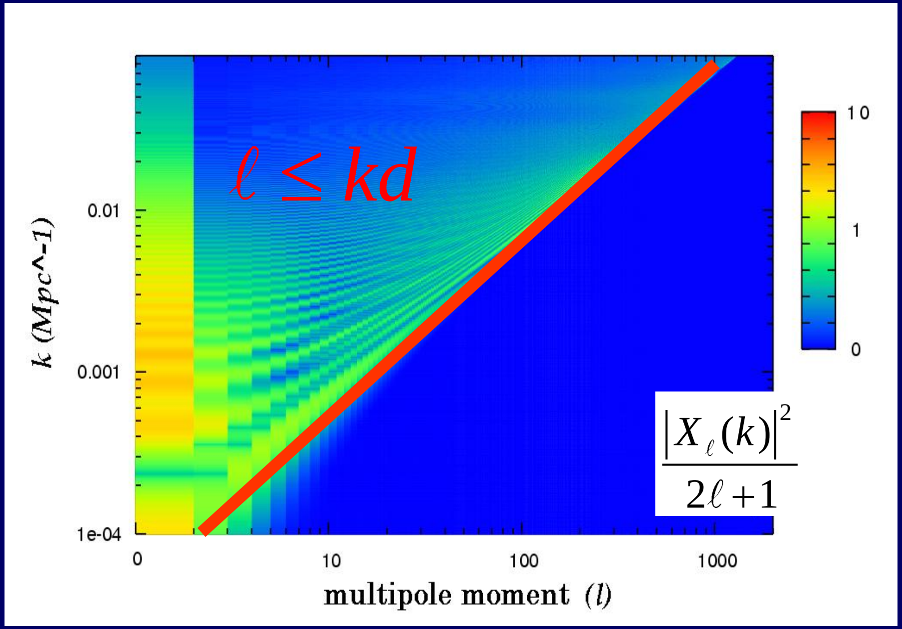
Forward Analysis

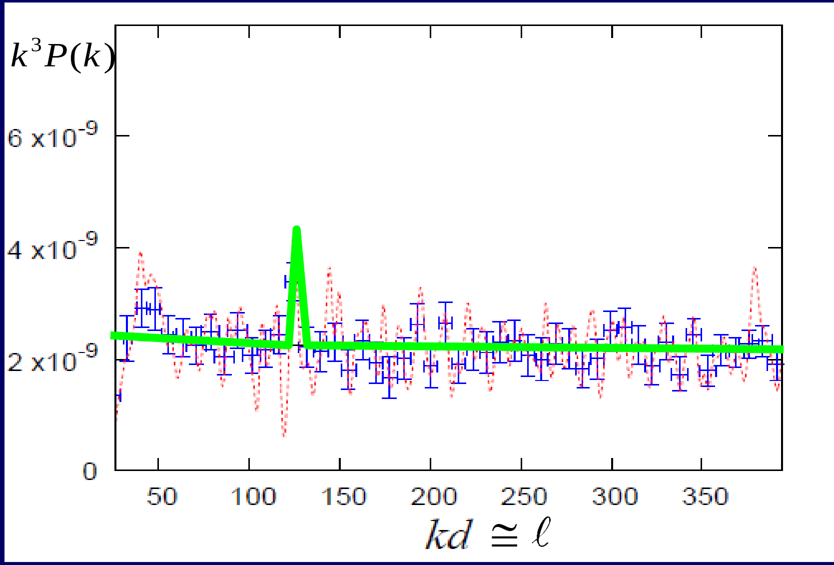
- Assume various shapes of modified power spectrum $P(k)$ with three additional parameters in addition to the standard power-law.

- Perform Markov-Chain Monte Carlo analysis with CosmoMC with these three additional parameters in addition to the standard 6 parameter ΛCDM model.

Transfer function shows that C_ℓ depends on $P(k)$ with $kd \geq \ell$.

$$C_\ell = \int \frac{d^3k}{(2\pi)^3} \frac{4\pi}{(2\ell+1)^2} |X_\ell(k)|^2 P(k)$$



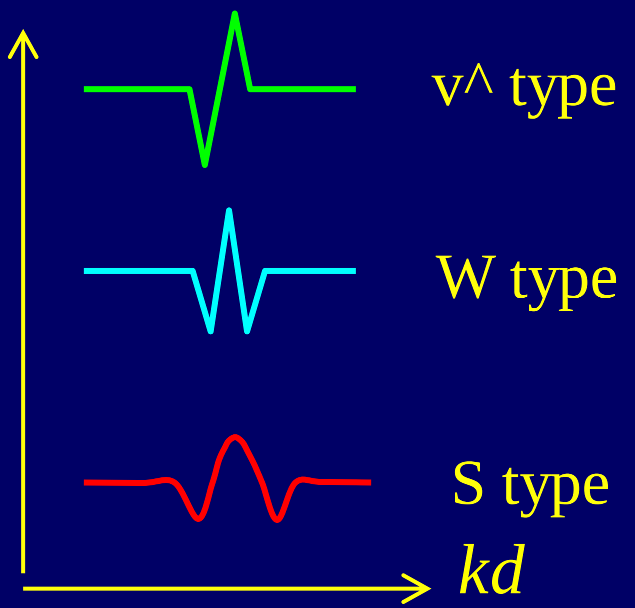


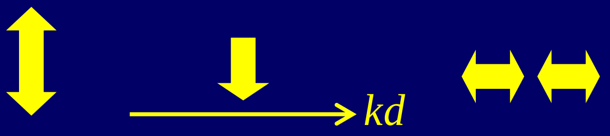
If we add some extra power on $P(k)$ at $kd \approx 125$, it would modify all C_ℓ's with $\ell \leq kd \approx 125$.

Simply adding an extra power around $kd \approx 125$ does not much improve the likelihood, because it modifies the successful fit of power-law model at smaller ℓ's.

Consider power spectra which change C_ℓ's only locally.

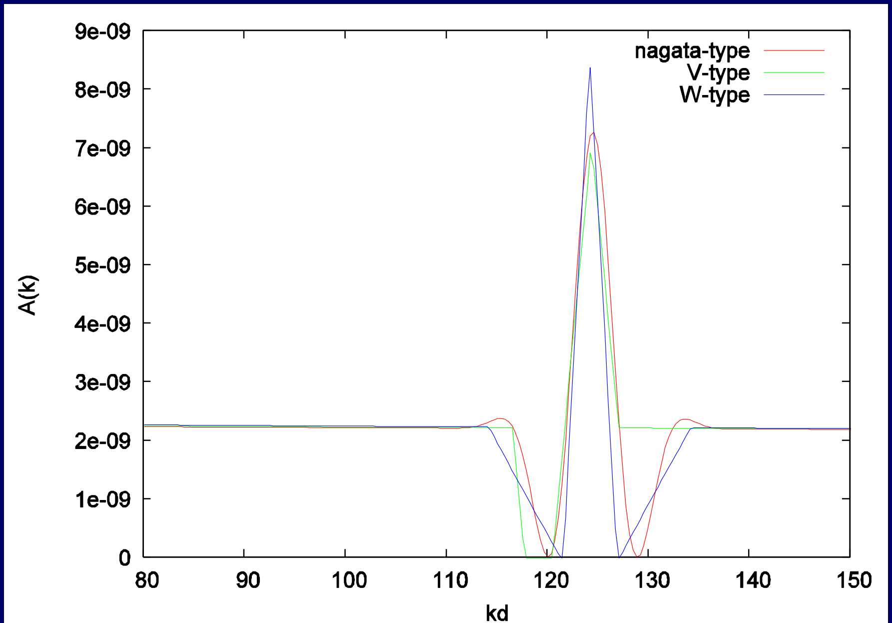
$A(k) \equiv k^3 P(k)$





Height, location, & width of the peak are 3 additional parameters.

$$k^3 P(k) = A\left(\frac{k}{k_0}\right)^{n-1} + B\left(\frac{k}{k_0}\right)^{n-1} \exp\left(-\frac{(k-k_*)^2}{\kappa^2}\right) \cos\left(\pi \frac{k-k_*}{\kappa}\right)$$



χ^2_{eff} improves as much as 22 by introducing 3 additional parameters.

	Power law	Λ-type	V^{Λ}-type	S-type	W-type
Ω_b	0.0438	0.0441	0.0443	0.0441	0.0444
Ω_m	0.256	0.256	0.260	0.257	0.262
Ω_Λ	0.744	0.744	0.740	0.743	0.738
H_0	72.1	72.1	71.7	72.0	71.6
$10^{10}A$	23.88	23.24	23.51	23.34	23.90
n_s	0.964	0.975	0.969	0.970	0.964
τ	0.0864	0.0879	0.0846	0.0835	0.0845
$\Delta\chi^2_{\rm eff}$	0	-6.5	-19	-22	-16
k_*d		124.5	124.4	124.5	$\cdots$
$10^{10}B$		23.80	47.26	55.66	37.95
$-2\ln\mathcal{L}$	2658.1	2651.6	2639.1	2636.2	2641.8
D.o.F.		3	3	3	3

(Ichiki, Nagata, JY, 08)

If χ^2 improves by 2 or more, it is worth introducing a new parameter, according to Akaike's information criteria (AIC).

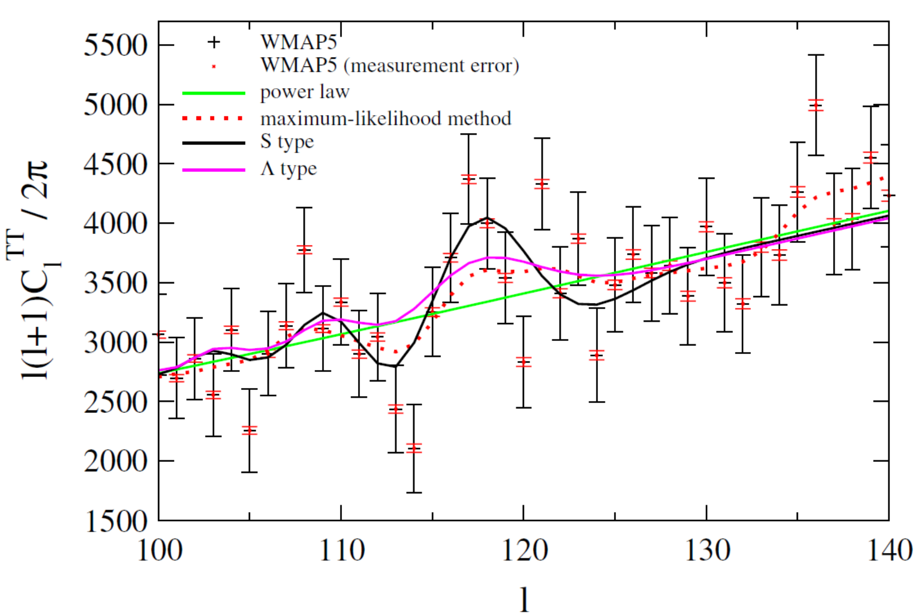
Unlike our reconstruction methods, MCMC calculations use not only TT data but also TE data.

$\Delta\chi^2_{eff}$ due to improvement of TT fit = -12.5

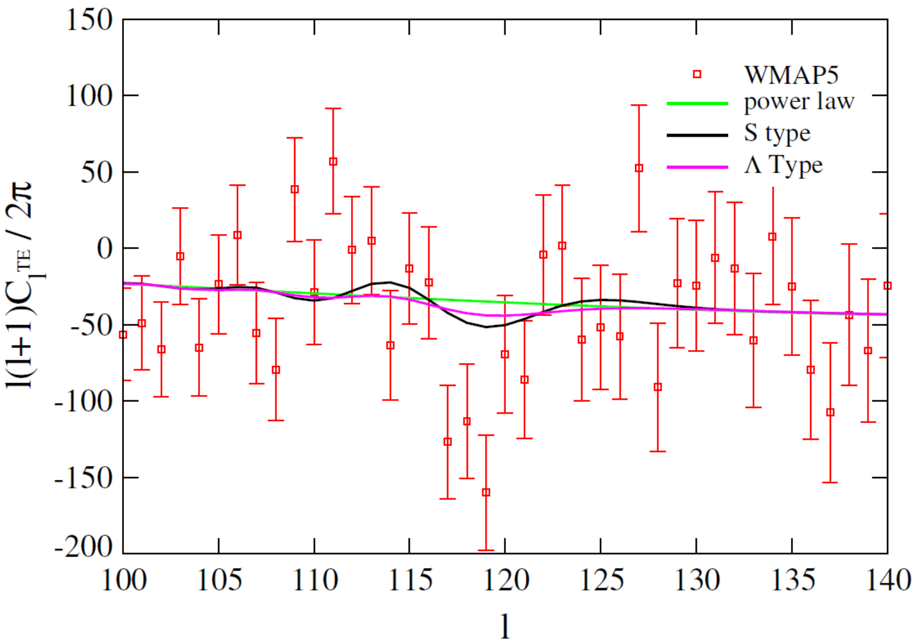
$\Delta\chi^2_{eff}$ due to improvement of TE fit = -8.5

It is intriguing that our modified spectra improve TE fit significantly even if we only used TT data in the beginning.

TT(temp-temp) data and model



TE(temp-Epol) data and model



★ Posterior probability to find vanishingly small deviation from a power-law.

$$P(B < 1 \times 10^{-10}) = 4.8 \times 10^{-5}.$$

based on a local analysis in the range $\Delta kd = 20$.

★ Posterior probability to find vanishingly small deviation from a power-law at any observed wavenumber domain.

$$P(B < 1 \times 10^{-10}) \sim 8 \times 10^{-4} .$$

based on a global analysis in the range $40 < kd < 380$.

★ This may or may not be so by chance.
In either case, however,…

The presence of such a fine structure changes the estimate of other cosmological parameters at an appreciable level by Planck.

Maximum of the difference from the power law

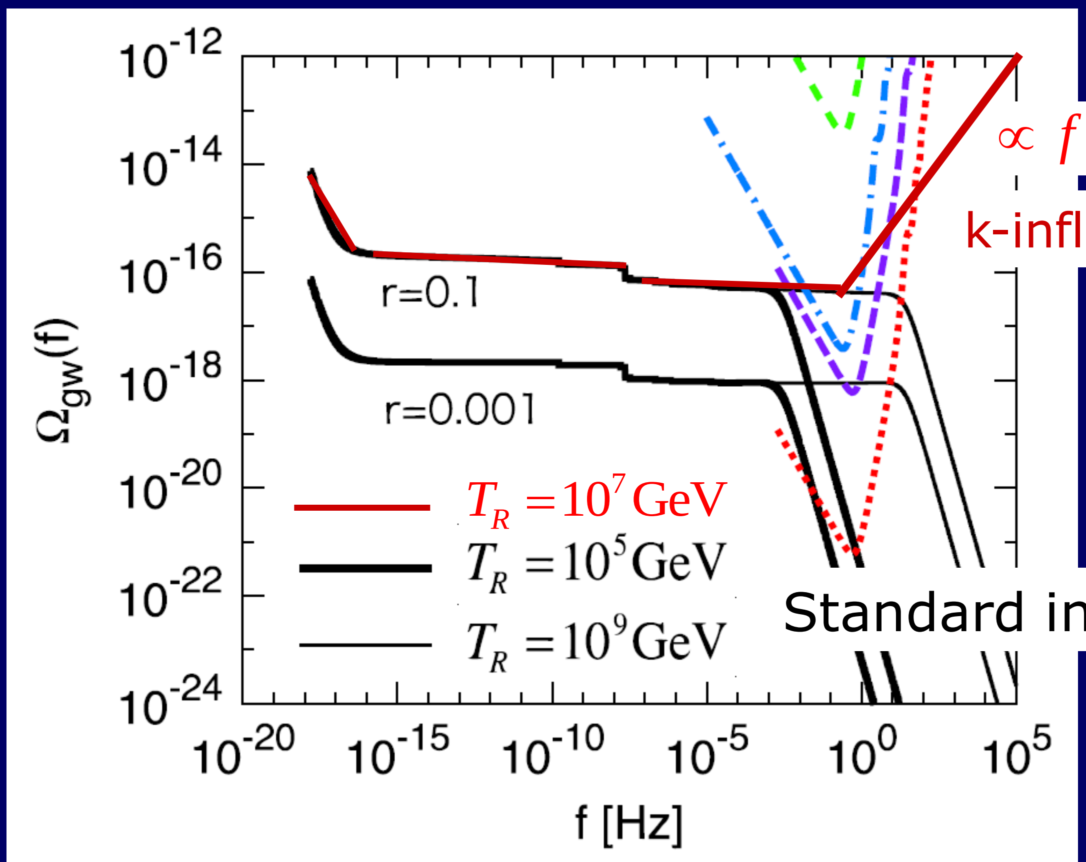
	Power law	Λ-type	V^{Λ}-type	S-type	W-type	$\Delta_{\max}$	WMAP5	Planck
Ω_b	0.0438	0.0441	0.0443	0.0441	0.0444	0.0006	0.0030	0.0003
Ω_m	0.256	0.256	0.260	0.257	0.262	0.006	0.027	
Ω_Λ	0.744	0.744	0.740	0.743	0.738	0.006	0.015	0.009
H_0	72.1	72.1	71.7	72.0	71.6	0.5	2.7	2.7
$10^{10}A$	23.88	23.24	23.51	23.34	23.90	0.54	1.12	
n_s	0.964	0.975	0.969	0.970	0.964	0.006	0.015	0.0045
τ	0.0864	0.0879	0.0846	0.0835	0.0845	0.0029	0.017	0.005
$\Delta\chi^2_{\rm eff}$	0	-6.5	-19	-22	-16			
$k_* d$		124.5	124.4	124.5	$\cdots$			
$10^{10}B$		23.80	47.26	55.66	37.95			
$-2\ln\mathcal{L}$	2658.1	2651.6	2639.1	2636.2	2641.8			
D.o.F.		3	3	3	3			

Expected Errors by PLANCK

Higher frequency tensor perturbation

Its spectrum can be used to probe post-inflationary thermal history of the early Universe.

sensitivity curves of DECIGO



$w = 1$ between inflation and reheating

$w = 0$ between inflation and reheating

Conclusion

The precision cosmology is entering a new era with even higher precision.

Hopefully we will be able to know which if any is the correct inflation model that occurred in the early Universe.

上州無智亦無才
剛毅木訥易被欺
唯以正直接萬人
至誠依神期勝利

鑑三