Nuclear Symmetry Energy in holographic QCD

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Based on arXiv:1011.0868

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Def: Symmetry Energy

- Liquid Drop Model's Empirical formula:

$$E_{\mathrm{B}} = a_{\mathrm{v}} A - a_a (N-Z)^2/A - a_c Z^2/A^{1/3} - a_s A^{2/3} \pm a_\delta / A^{3/4} .$$

Turn off e&m

N=#(n), Z=#(p)

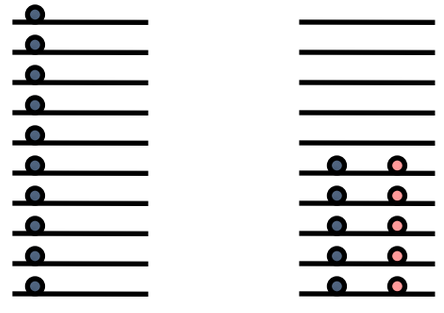
It is the loss in E_{B} when we go off N=Z.

- no linear term? Due to Isospin Invariance
-

$$E(\rho, \tilde{\alpha}) \simeq E(\rho, 0) + S_2(\rho)\tilde{\alpha}^2$$

$$\tilde{\alpha} \equiv (N-Z)/A$$

Physics of Es



1. Es(N-Z)^2 term is the consequence of Pauli principle. See the figure.

2.For Es→infinity: N=P

For Es→0, pure neutron star is possible.

Importance of Es

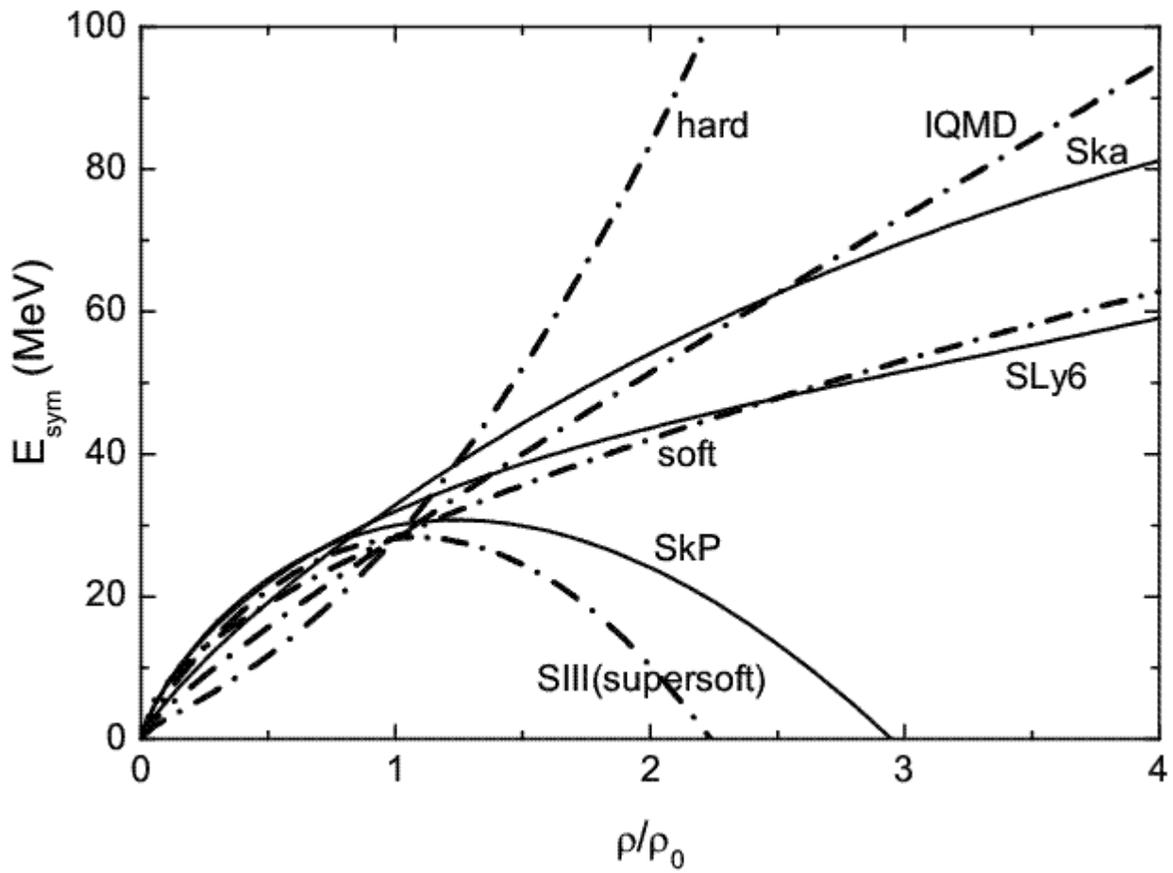
- Structure of Neutron Star

  The slope of the nuclear symmetry energy at saturation density is a crucial quantity to determine the mass and width of neutron-star crusts.

- Nucleo-Systhesis during the supernova explosion.

What is known for E_s ?

- Little is known. not Exp. nor theoretical.



Why difficult?

1. Strongly interacting.
   No good calculational tool in this regime.
2. Density effect: Even lattice qcd does not help much.

   It is problem of tool. not imagination.

String theory idea:

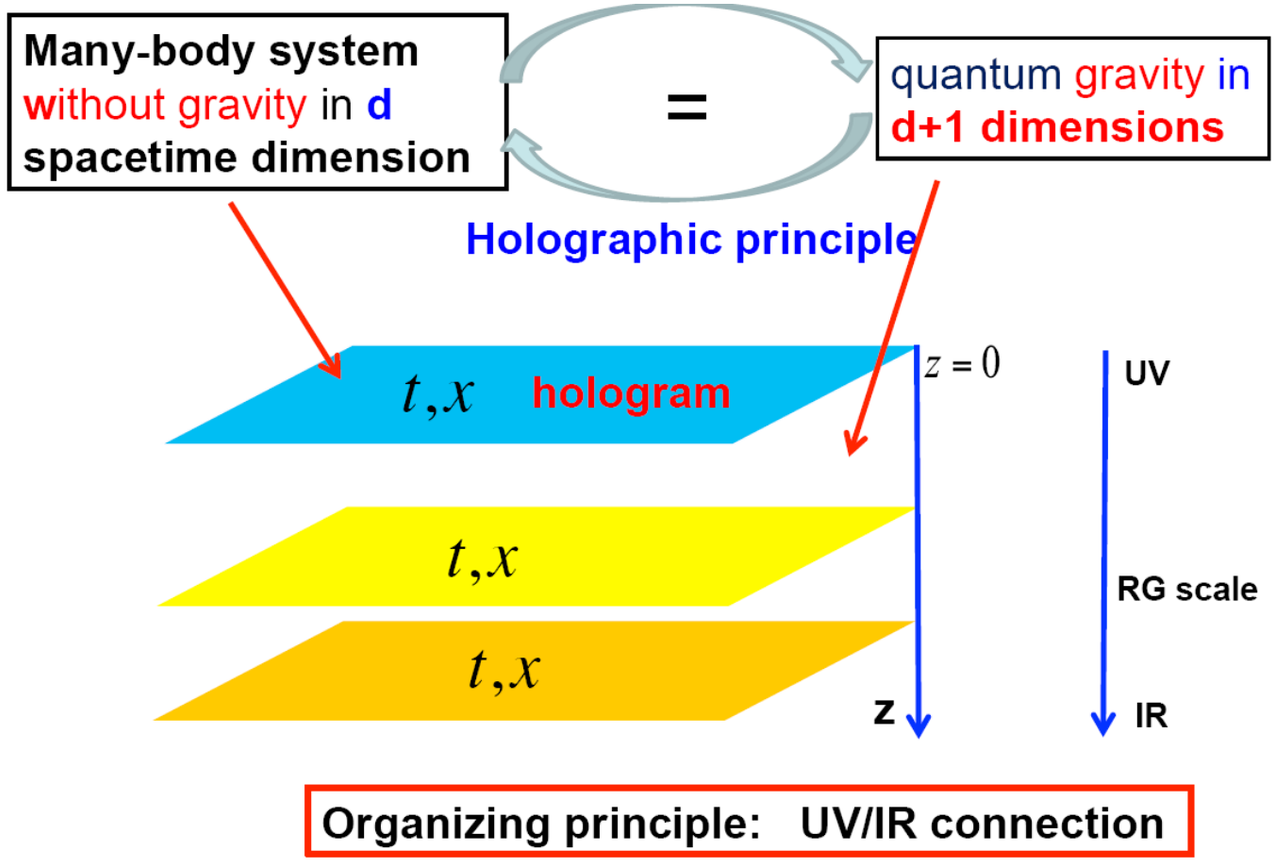
- Replace Nuclear force by classical gravity

- Called ads/cft
  by Maldacena, Witten, GKP

Character of AdS/CFT I

- Holographic: 5d gravity theory for 4d QCD

  origin of +1 dim? Scale

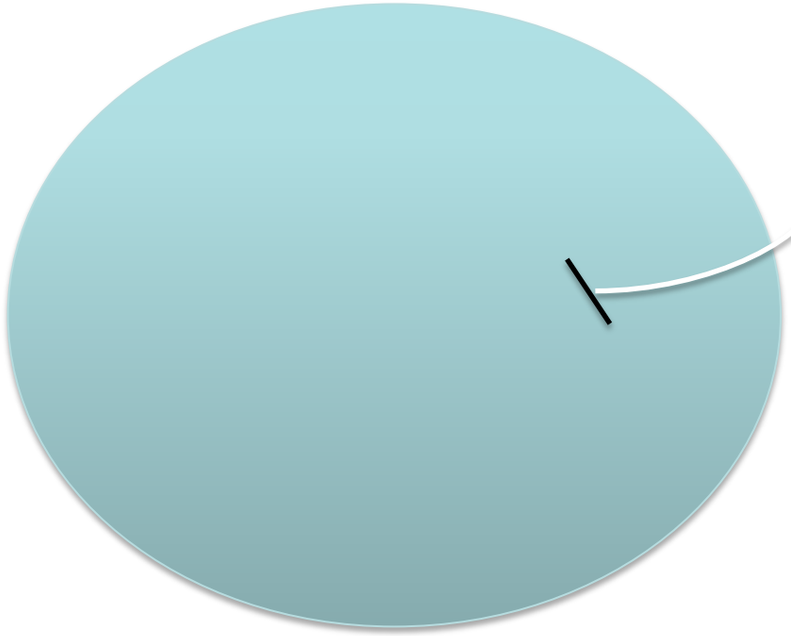


z direction is warped! → ads

Stolen from Hong Liu

Character of AdS/CFT II

- Within the validity,
  Do not need loop calculation.



1. ls<<Rads
2. gs<<1

$$\lambda = g^2 N_c >> 1$$

Inherit Large N theory's
Good and Bad

Character of AdS/CFT III

- Super-symmetry
  Original version is N=4 SUSY.
  SUSY can be broken by BC. T. d. etc

- Higher dim.
  Trade vibrational mode with KK mode.

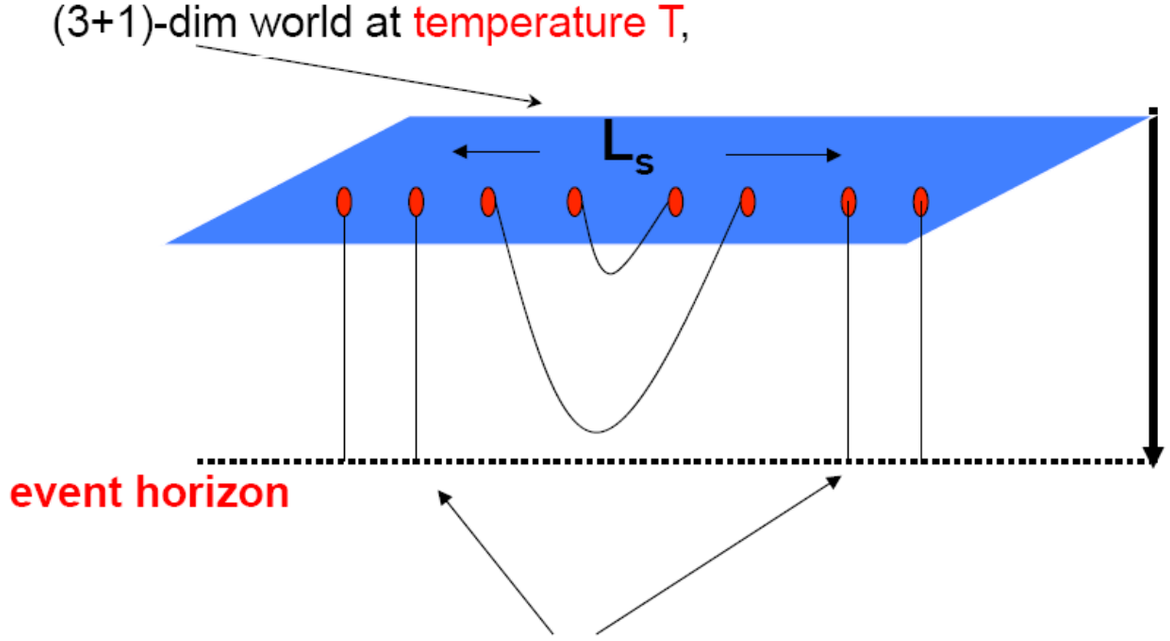
- Is it QCD?
  Hopefully some properties will be universal. Some results are too good to be irrelevant ……
  [eta/s, glueon mass, ads/qcd, SS]

Gluon dynamics→ Geometry.

Confinement or deconfinement depends on geometry.

Goemetry with BH
→deconfinement

Screening of quarks in a QGP

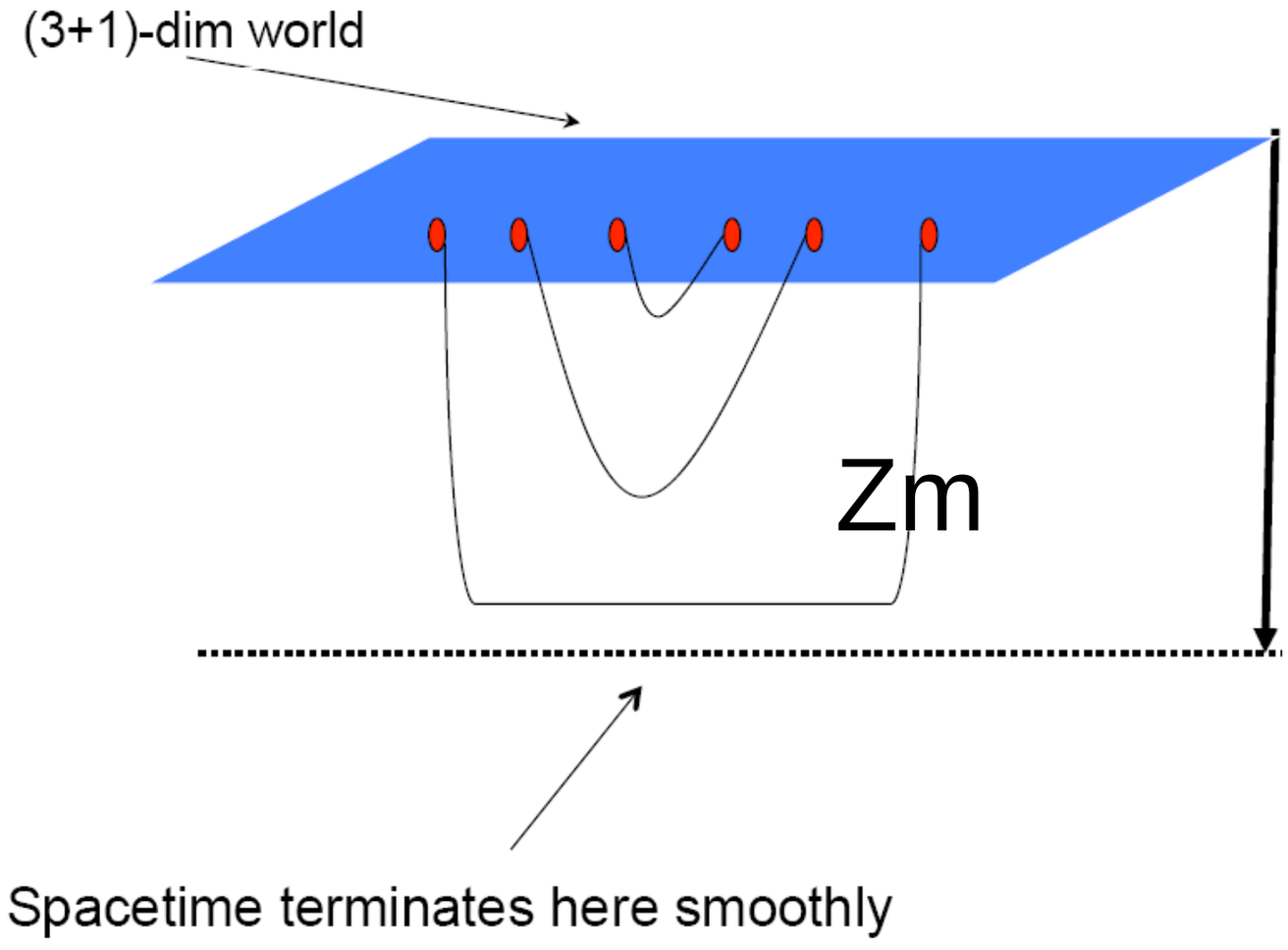


N=4 : $L_S = 0.277/T$, QCD (2 flavor): $L_S \sim 0.5/T$

(lattice)

Geometry with repulsive gravity→

Confinement



Bottom up model

- linear sigma model in ads5.
  with gauged chiral symmetry
  SU(Nf)xSU(Nf)

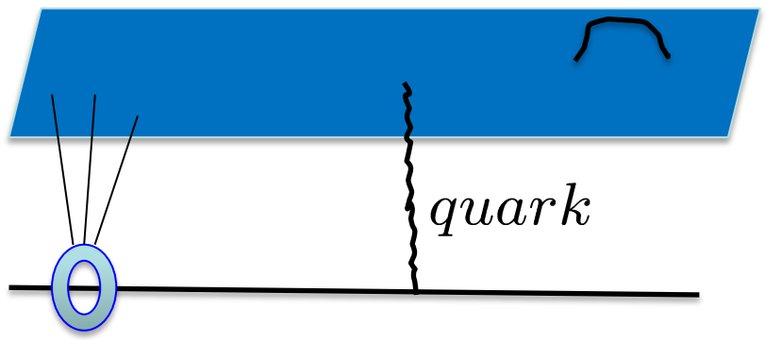
$$S = \int d^5x\, \sqrt{g}\ \mathrm{Tr}\Big\{|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2)\Big\}$$

$$X_0(z) = \frac{1}{2}Mz + \frac{1}{2}\Sigma z^3, \qquad \Sigma^{\alpha\beta} = \langle \bar{q}^\alpha q^\beta \rangle.$$

- No potential.// BC. Instead
  15% error with 3 parameter fit.

Top down model

- Quark: Bifundamental,



- Meson: adjoint
  Dynamics: Dirac-Born-Infeld action

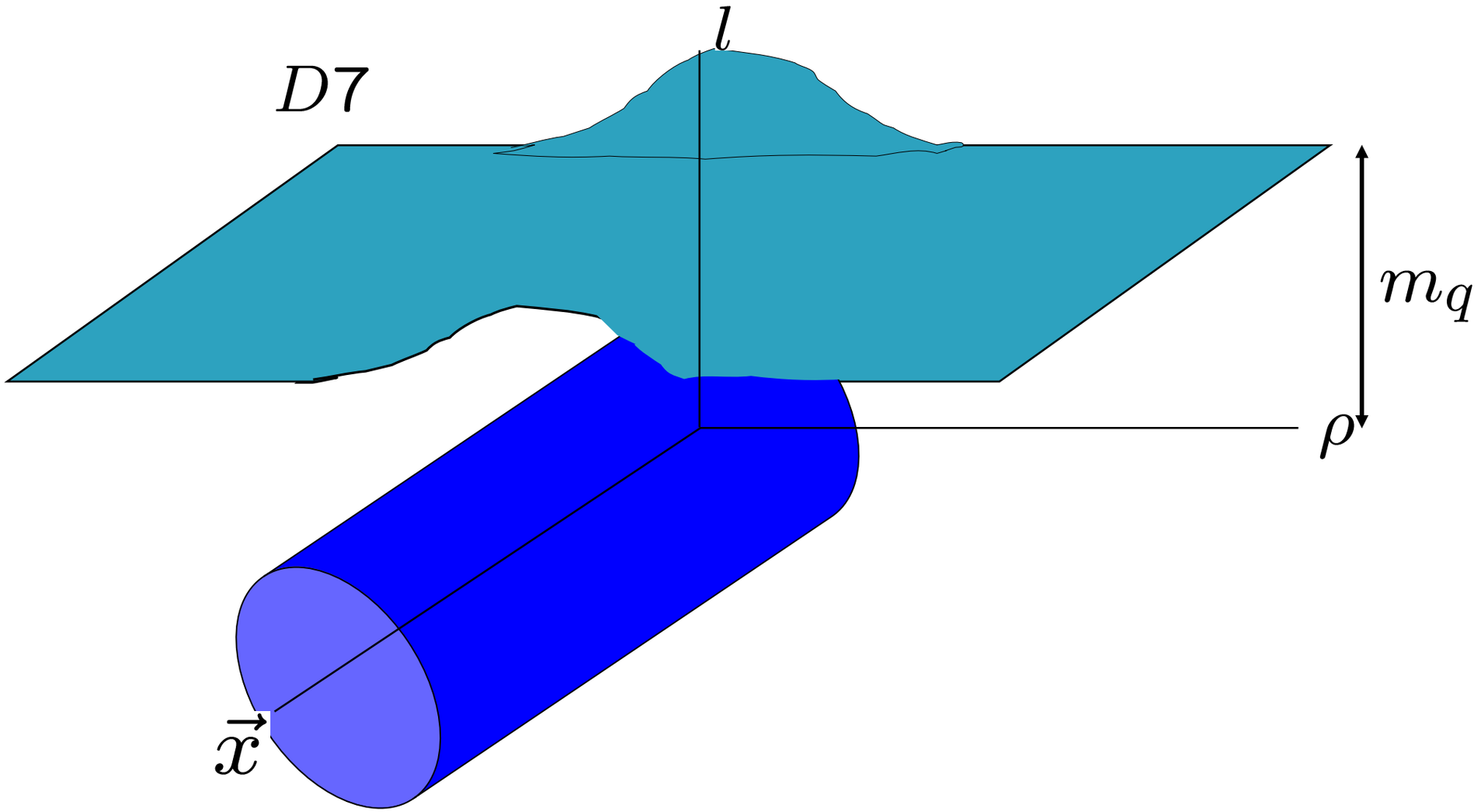
$$S_{D4} = -\mu_4 \int e^{-\phi} \sqrt{\det(g + 2\pi\alpha' F)} + \mu_4 \int A_{(1)} \wedge G_{(4)}$$

- Baryon: compact D5

Model: D4/D6 +cD4

- Nc D4 provide Gluonic gravity background with confinement. One compactification x4.
- D4 baryon vertex.
- 2 flavor probe brane.

Repulsive gravity for confinement



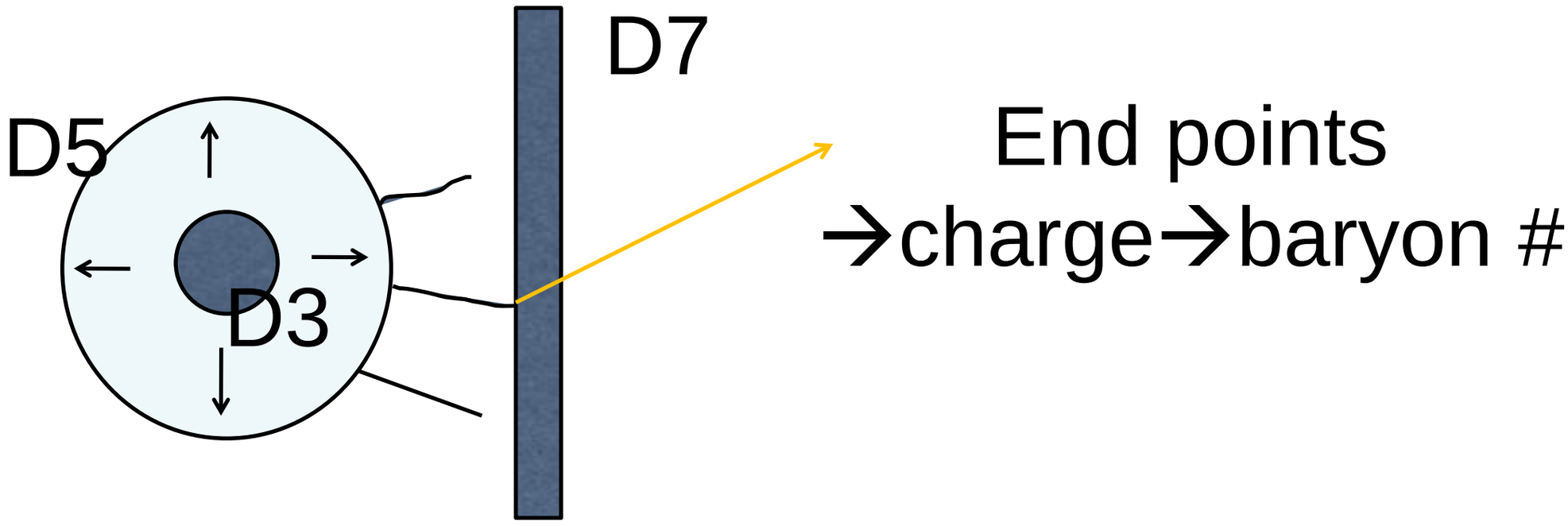
- set At=0 at rho=0. Then mu and Q will be related. (BH)
- mu=dH/dQ for DBI action.

Single Baryon

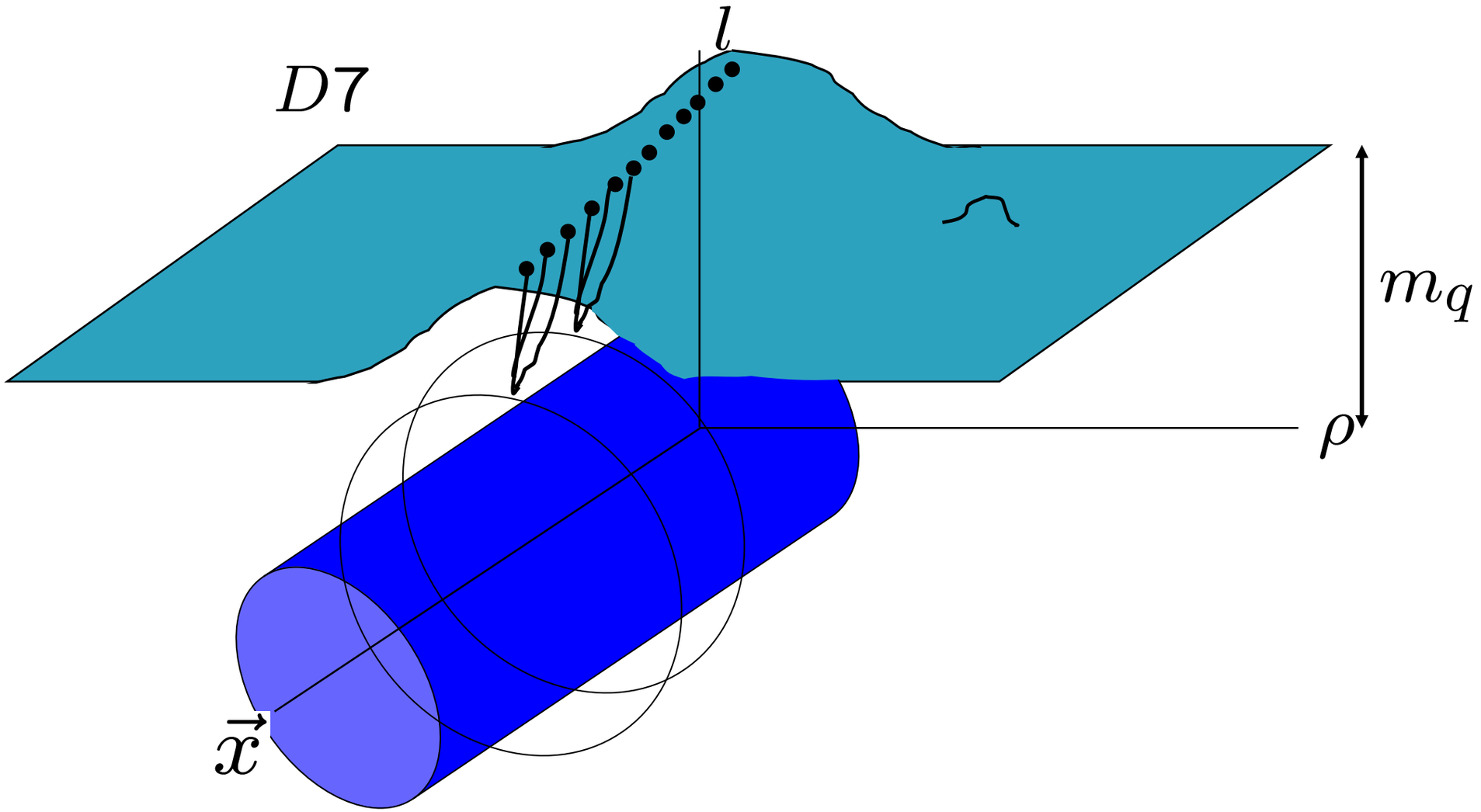
Compact D-brane that wraps the transverse dimension.
For D3, compact D5. for D4, compact D4 etc.

Ted pole Anomally cancellation requests that

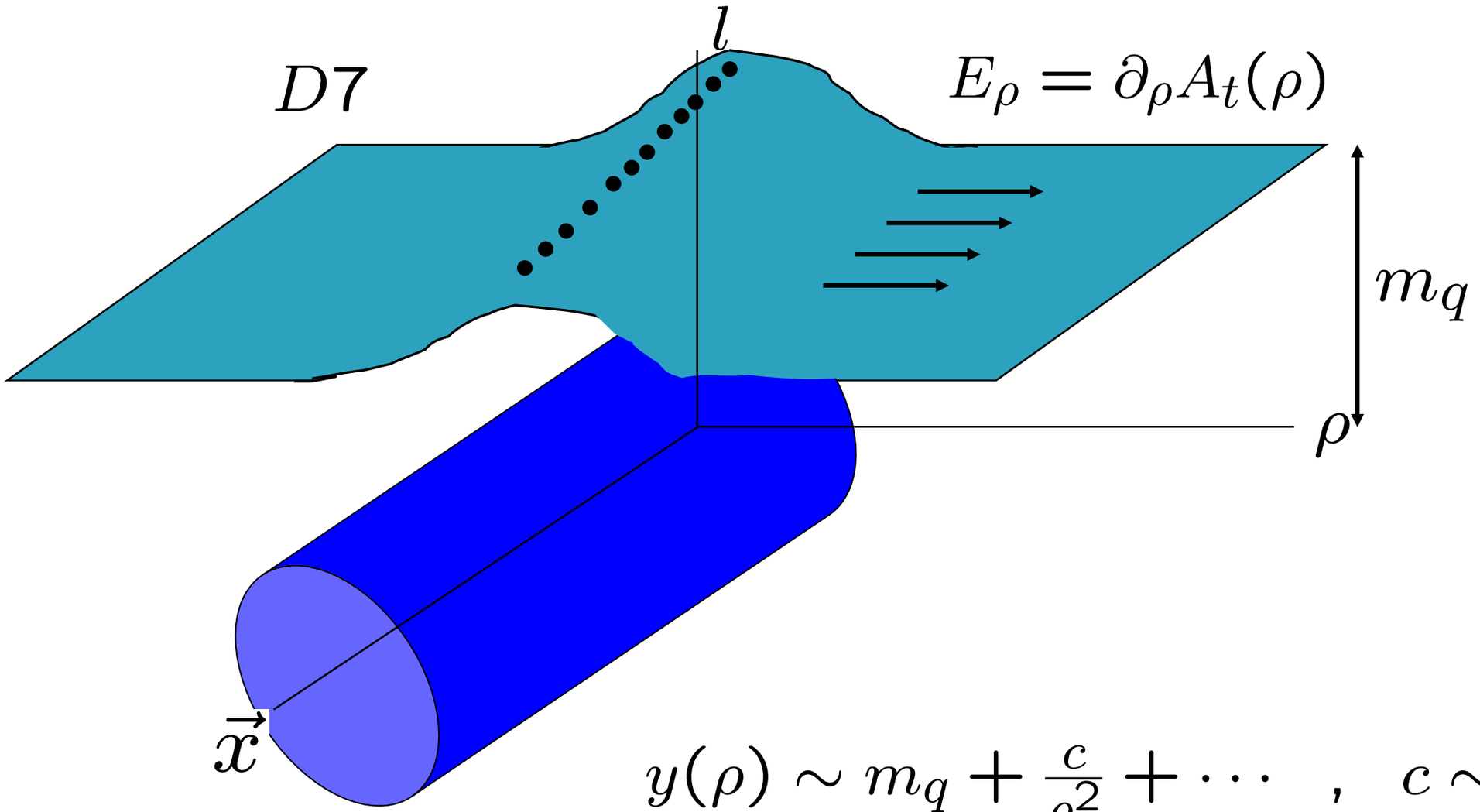
Nc - F1 is attached to the probe brane



Dense matter with baryon vertex



Chemical potential



$$y(\rho) \sim m_q + \frac{c}{\rho^2} + \cdots \quad , \quad c \sim < \bar{\psi}\psi >$$

$$A_t(\rho) \sim \mu + \frac{Q}{\rho^2} + \cdots \quad , \quad Q \sim < \psi^\dagger \psi >$$

- set At=0 at rho=0. Then mu and Q will be related. (BH)
- mu=dH/dQ for DBI action.

Confining metric

$$ds^2 = \left(\frac{U}{R}\right)^{3/2} \left(-dt^2 + d\vec{x}^2 + f(U)dx_4^2\right) + \left(\frac{R}{U}\right)^{3/2} \left(\frac{U}{\xi}\right)^2 \left(d\xi^2 + \xi^2 d\Omega_4^2\right),$$

where $f(U) = 1 - (U_{KK}/U)^3$ and $(U/U_{KK})^{3/2} = (\xi^{3/2} + \xi^{-3/2})/2 \equiv \xi^{3/2}\omega_+/2$.

Dynamics of probe barne

- DBI action: S= $\int \mathrm{tr}(\det[\eta_{\mu\nu} + (2\pi\alpha')F_{\mu\nu}])^{1/2}$

Density→charge of F,
fixed charge→Legendre transformation.

DBI to Hamiltonian

- Hamiltonian of baryon vertex D4

$$\mathcal{H}_{D4} = \tau_4 \int d\theta \sqrt{\omega_+^{4/3}(\xi^2 + \xi'^2)} \sqrt{D(\theta)^2 + \sin^6\theta}\,,$$

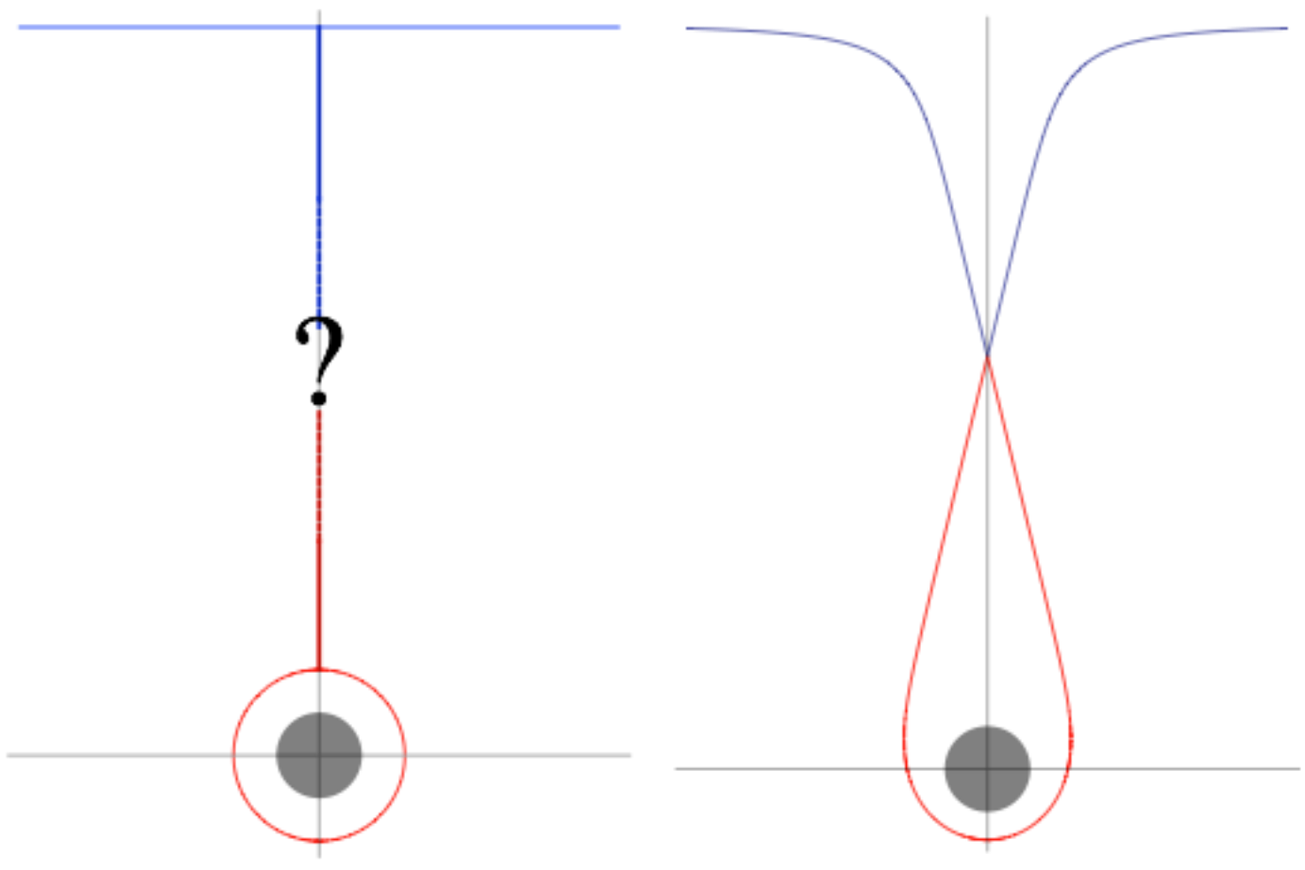
$$\tau_4 = \tfrac{1}{2^{2/3}}\mu_4 \Omega_3 g_s^{-1} R^3 U_{KK},\ D(\theta) = -2 + 3\cos\theta - \quad \cos^3\theta$$

- Hamiltonian of baryon vertex D4

$$\mathcal{H}_{D6} = \tau_6 \int d\rho \sqrt{1+\dot{y}^2} \sqrt{\omega_+^{4/3}\left(\tilde{Q}^2 + \rho^4 \omega_+^{8/3}\right)},$$

where $\tau_6 = \frac{1}{4}\mu_6 V_3 \Omega_2 g_s^{-1} U_{KK}^3$. $\tilde{Q}$ is dimensionless related to the number of fundamental strings Q, $\frac{U_{KK} Q}{2 \cdot 2^{2/3} \pi \alpha' \tau_6}$

Force balance between D4 D6



Main idea:
asymmetry in Z-N → that in Q1-Q2

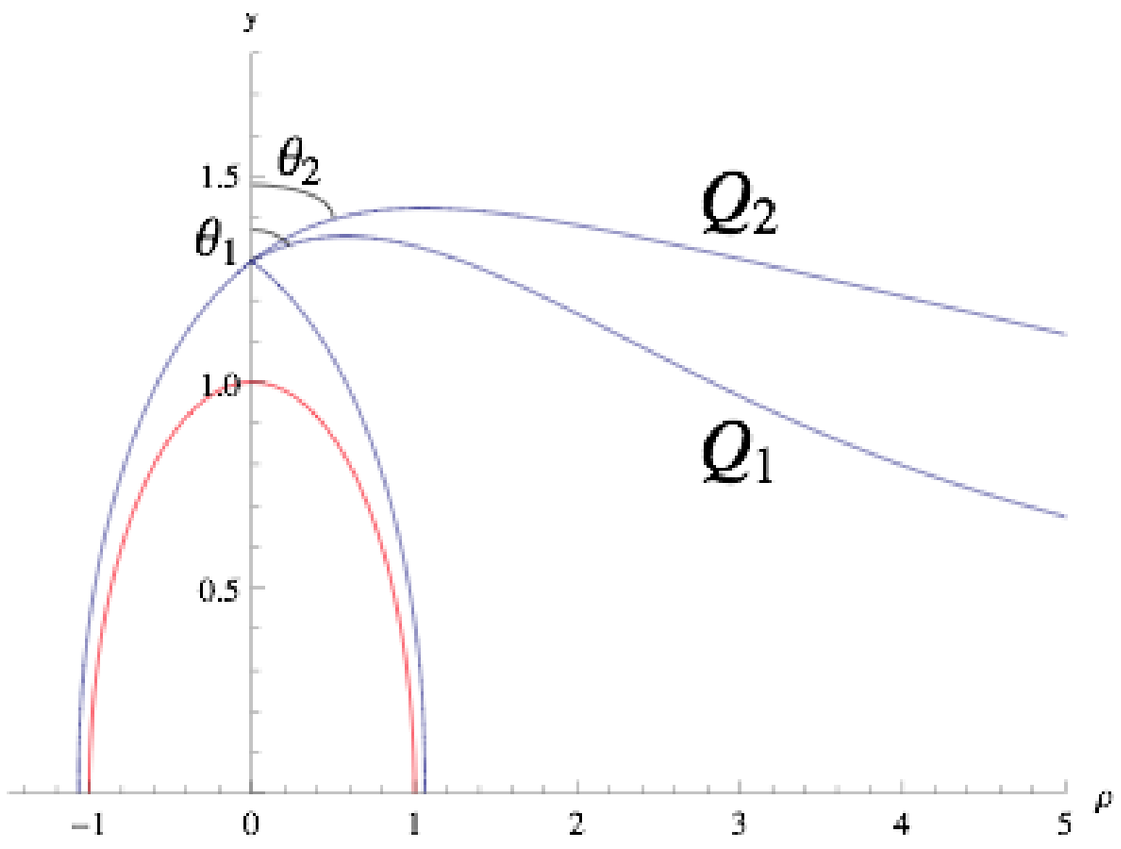
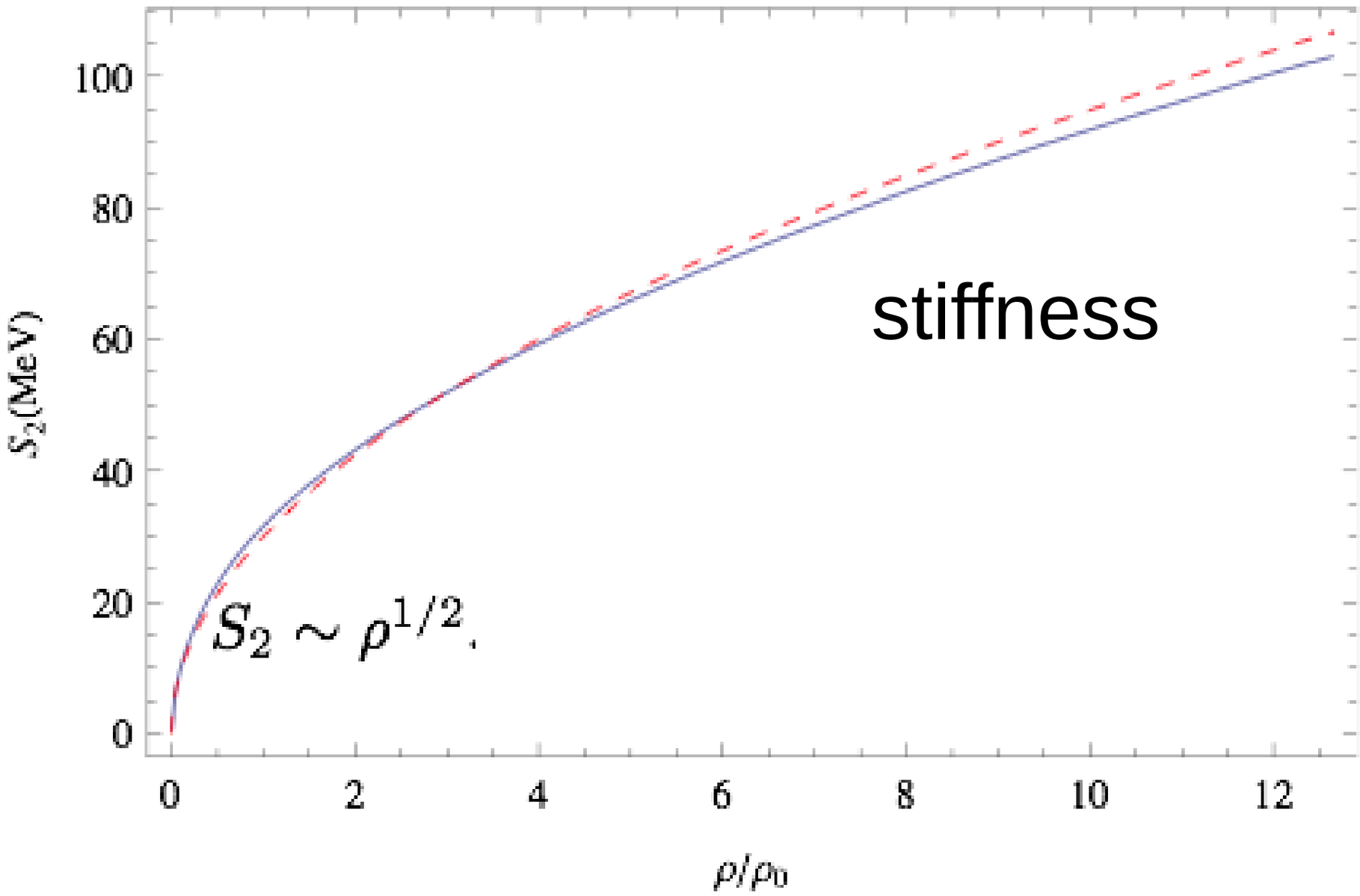


FIG. 3. (Color online) Embedding of D-branes with $\alpha \neq 0.5$. The asymptotic heights of two branes are the same ($m_1 = m_2 = 0.1$). Red curve denotes to the position of U_{KK}.

Numerical Result



Understanding $S_2 \sim \rho^{1/2}$.

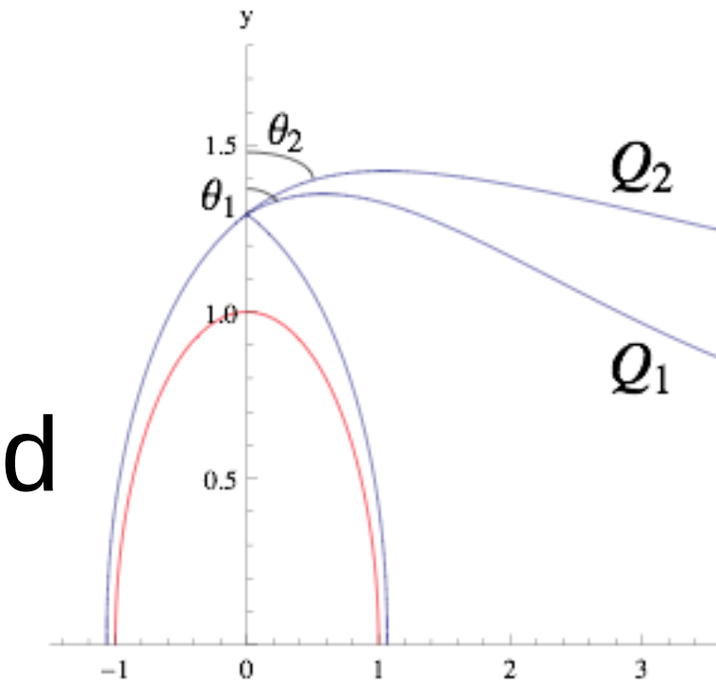
$$S_2 = \frac{2\tau_6}{N_B}\int d\rho \frac{\sqrt{1+\dot{y}^2}\tilde{Q}^2\omega_+^{10/3}\rho^4}{(\tilde{Q}^2+4\omega_+^{8/3}\rho^4)^{3/2}},$$

- For the flat embedding we can evaluate.

$$S_2 = \left(\Gamma(\frac{5}{4})\right)^2\sqrt{\frac{\lambda\rho_0}{2M_{KK}}}\sqrt{\frac{\rho}{\rho_0}}.$$

Stiffness

- For the balance, each D6 should balance the half the force.
- Coulomb energy



$$Q_1 \cos\theta_1 \sim Q_2 \cos\theta_2.$$

$$\sim (Q_1^2 + Q_2^2).$$

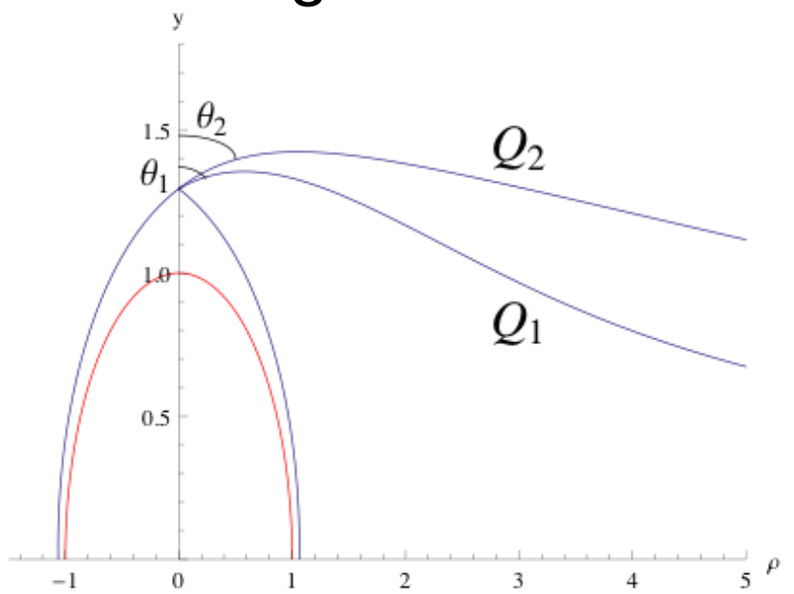
Minimum energy requests $Q_1 = Q_2 = Q/2$.

As the number of attached strings increases, the brane gets stiffer and the maintaining the angle difference costs more and more energy.

Pauli principle in hQCD.

- two puzzles
  in 4d: Driving force of Z=N is Pauli principle. Ads/cft is counting QM by classical dynamics. So how to count Pauli force.

  In 5d: coulomb force by charge. But charge is dual to baryon number, which is global charge. So what is dual of the coulomb force?



$$\mathcal{H}_{D6} = \tau_6 \int d\rho \sqrt{1+\dot{y}^2} \sqrt{\omega_+^{4/3}\left(\tilde{Q}^2 + \rho^4 \omega_+^{8/3}\right)},$$

quark matter vs nuclear matter?

- From the point of view coulomb repulsion, we do not see much difference.
- Gluon dynamics: BH vs core bubble.
- Can we count the long range force of coulombic force? NOT YET!

Conclusion

- Symmetry Energy can be calculated using the holographic principle.
- Our method is the only one that can count the baryon density.

Ads/cft Dictionary

- Let O(x) is an color singlet operator with dimension Δ and spin p
  A(x) is an source of it.
- Then AdS/CFT says:
  Extend it to d+1(=5) dim by

$$A(x,z) = A(x) z^{d-p-\Delta} - < O > z^{\Delta - p} + ...$$

- If we know action and BC, it can be calculated CLASSICALLY.
- So is all correlation functions and its corollaries.

Chiral condensation and mass

- Mass op. $\psi\psi$ is dual to a scalar σ
- Extend it to 5 dim by

$$\sigma(x,z) = m_q z - c z^3$$

$$c = < \bar{\psi}\psi >$$

Density and chemical potential

- In 4d, Source of baryon number op $J_0 = \bar{\psi}\gamma_0\psi$ is A_0
- Extend it to 5dim by

$$A_0 = \mu z^{3-\Delta} - Q z^{\Delta-1}$$

$\Delta =$ dimesion of Operator $=3$

$Q = <\bar{\psi}\gamma_0\psi>$

$\mu = chemical\ potentia$

Gluon condensation and dilaton

- Tr(F^2) is dual to scalar ϕ
- Extend it to 5dim by

$$\phi(x,z) = \phi_0 z^{4-\Delta} - c z^{\Delta}$$

$$\phi_0 = 0 \qquad c = < TrF^2 >$$

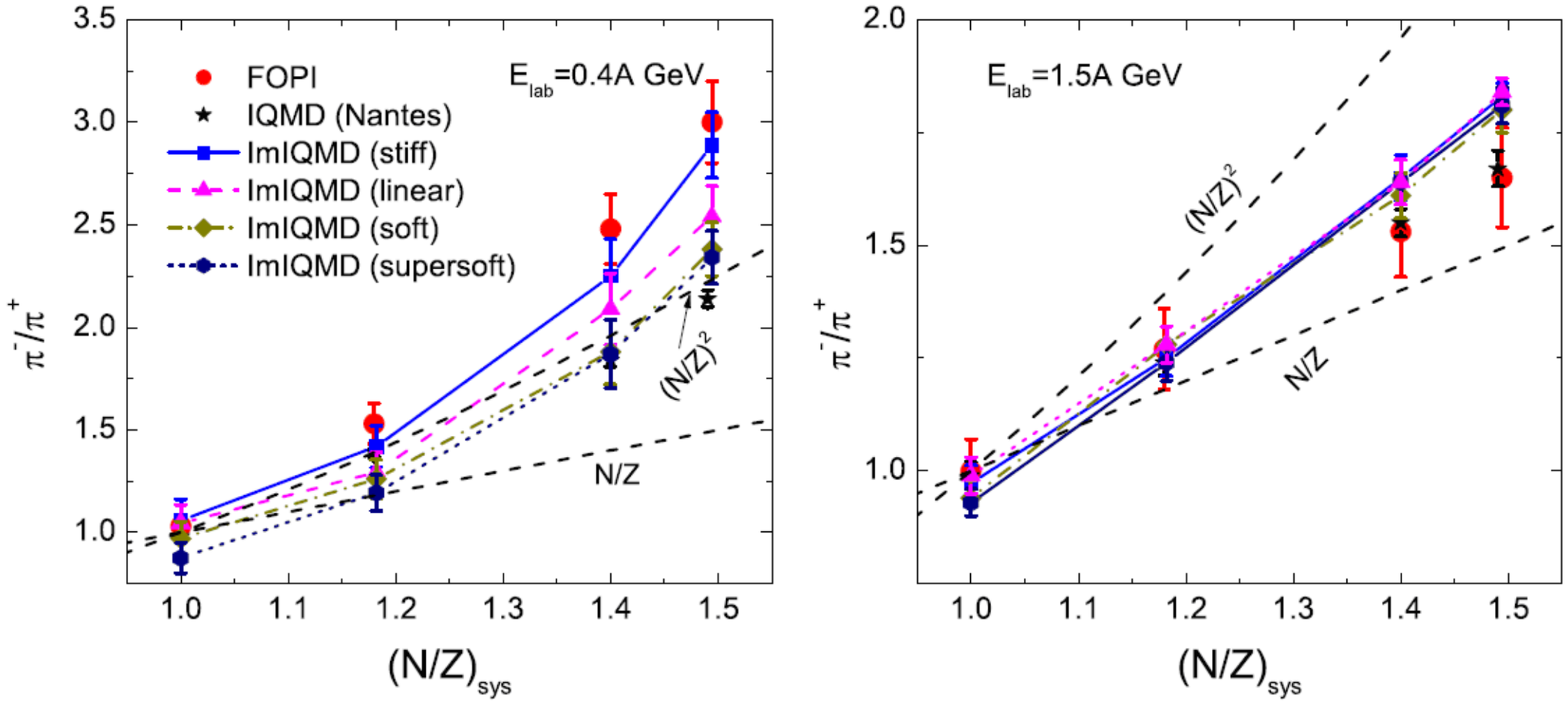
finally, How to detect Es ?

- Asymmetry in N-P is
  ~ that in π- π+

- π-/π+ yields are sensitive to
  the stiffness of the symmetry energy
  near threshold energy.

Experiment

Z.-Q. Feng, G.-M. Jin / Physics Letters B 683 (2010) 140–144



^{40}Ca+^{40}Ca, ^{96}Ru+ ^{96}Ru, ^{96}Zr+^{96}Zr and ^{197}Au+^{197}Au, and also plotted the ratios of N/Z and $(N/Z)^2$ as a function of N/Z at incident energy 0.4A GeV and 1.5A GeV, respectively.

Meson in terms of quarks