微視的クラスター模型を用いた $^{2}H(d,\gamma)^{4}He$ と $^{2}H(d,p)^{3}H$ におけるテンソル力の役割

Nagoka Nat. Coll. Tech.　Koji Arai

Niigata Univ.　S. Aoyama
Niigata Univ.　Y. Suzuki
ULB　P. Descouvemont
ULB　D. Baye

Introduction

- **Conventional microscopic cluster model (RGM)**

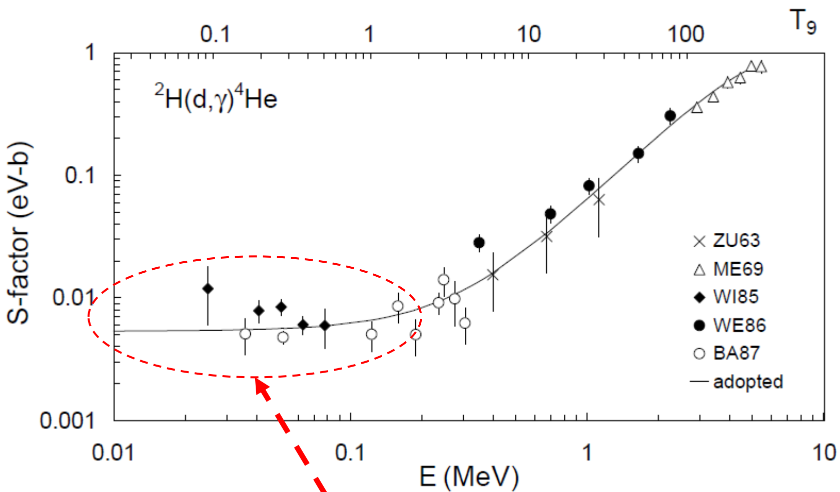
⇨ Simple cluster wave function
  (*S*-wave w.fs. for the α, ^{3}H, ^{3}He, d)
  Effective N-N interaction
  (central + LS, no tensor, e.g. Minnesota)

- **Extension of the microscopic cluster model**

⇨ Precious few-body w.fs. for the clusters
  Realistic N-N interaction
  (e.g. ^{3}He+p, *Arai et al. PRC81(2010)037301*
  d+d, *Aoyama et al submitted in FBS)*

➡ *Ab initio* calculation

• Role of the tensor force

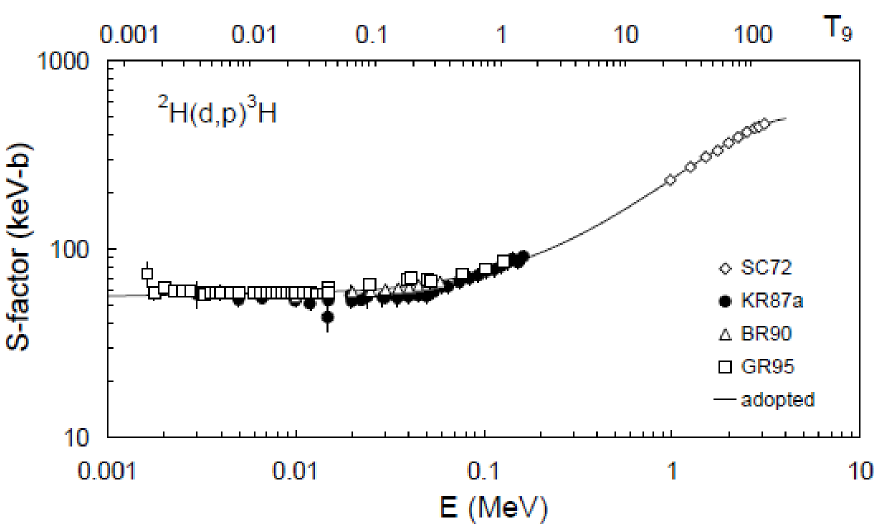


$^2H(d,\gamma)^4He$ Astrophysical S-factor

Nacre compilation
(C.Angulo et al. NPA656('99)p.3)

d+d S-wave → D-state in the 0^+ g.s. of ^4He

H. J. Assenbaum and K. Langanke, PRC36('87)p.17



$^2H(d,p)^3H$, 2H(*d*, *n*)3He

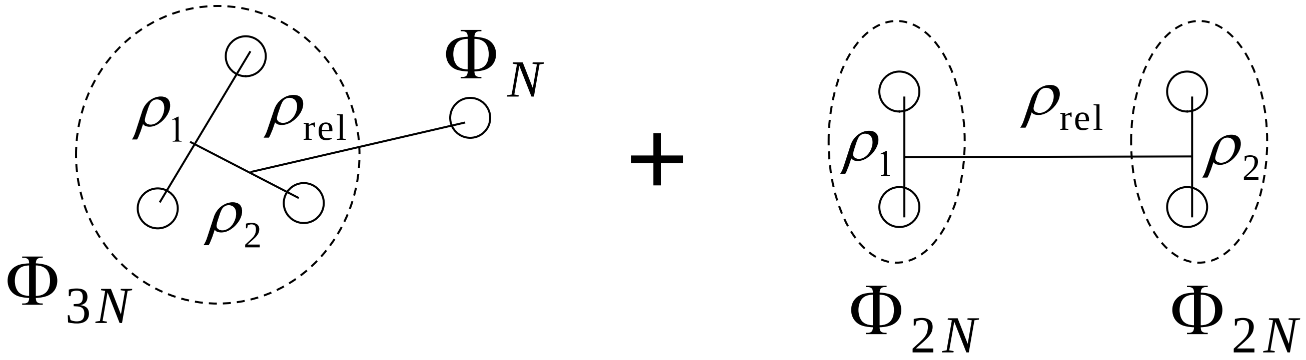
Role of the tensor force?

Microscopic Cluster Model (RGM)

[^{3}H(1/2$^+$)+p] + [^{3}He+n] + [d+d] + [d(0$^+$) + d(0$^+$)]
+ [2p(0$^+$) + 2n(0$^+$)] two-cluster model

Total wave function

$$\Psi = A\{\Phi_{3N}(\rho_1,\rho_2)\,\Phi_N\ \chi(\rho_{\rm rel})\} + A\{\Phi_{2N}(\rho_1)\,\Phi_{2N}(\rho_2)\,\chi(\rho_{\rm rel})\}$$



$$\ell_1,\ \ell_2,\ \ell_{\rm rel} \le 2$$

- $\chi(\rho_{\mathrm{rel}})$: Cluster relative wave function

  Microscopic R-matrix method (Baye, Descouvemont)

  a : channel raidus

  $$\begin{cases} \rho_{\mathrm{rel}} < a \text{ --- Gaussian expansion} \\ \rho_{\mathrm{rel}} > a \text{ --- Exact Coulomb function} \end{cases}$$

- N-N inteaction

  - Realistic N-N pot. → AV8' or G3RS + 3BF (Hiyama, PRC70('04))
  - Effective N-N pot. → Minnesota pot. (Central + coulomb)

Microscopic *R*-matrix method

(D.Baye, et al. NPA291 ('77)230)

- Schrodinger e.q. $\left(\hat{H}+\hat{L}-E\right)\Psi^{int}=\hat{L}\,\Psi^{ext}$
- Bloch operator $\hat{L}(E)=\left(\hbar^2/2\mu r\right)\delta(r-a)\left[\frac{d}{dr}r-b\right]$

$$\begin{cases} b=0 & \text{for open channel} \\ b=2kaW'(2ka)/W(2ka) & \text{for closed channel} \end{cases}$$

- W.F ($r<a$)

Gaussian expansion

$$\Psi^{int}=\sum_{\alpha k} f_{\alpha k} u_{\alpha k}\varphi_{\alpha}$$

$$\begin{cases} \alpha:(\ell, I) \\ u_{\alpha k}: \text{Gaussian basis function} \\ \varphi_{\alpha}: \text{Cluster internal function} \end{cases}$$

$$\Longrightarrow \quad \sum_{\alpha k} f_{\alpha k} \langle u_{\alpha' k'} \varphi_{k'} | \hat{H} + \hat{L} - E | u_{\alpha k} \varphi_k \rangle = \langle u_{\alpha' k'} \varphi_{k'} | \hat{L} | \Psi^{ext} \rangle$$

$$C_{\alpha' k', \alpha k} \equiv \langle u_{\alpha' k'} \varphi_{k'} | \hat{H} + \hat{L} - E | \Psi_{\alpha k} \rangle \qquad W_{\alpha' k'} \equiv \langle u_{\alpha' k'} \varphi_{k'} | \hat{L} | \Psi^{ext} \rangle$$

$$\Longrightarrow \quad \Psi^{int} = \sum_{\alpha k} f_{\alpha k} u_{\alpha k} \varphi_{\alpha} = \sum_{\alpha k \alpha' k'} C^{-1}_{\alpha k, \alpha' k'} W_{\alpha' k'} u_{\alpha k} \varphi_{\alpha}$$

$$\Psi^{ext} = \sum_{\alpha_1} r^{-1}_{\alpha_1} \upsilon^{1/2}_{\alpha_1} C_{\alpha_1} \left\{ I_{\alpha_1} \delta_{\alpha_1 \alpha_0} - U_{\alpha_1 \alpha_0} O_{\alpha_1} \right\} \varphi_{\alpha_1} + \sum_{\alpha_2} C_{\alpha_1} W_{-\eta, \ell+1/2}(2kr)/kr\, \varphi_{\alpha_2}$$

R-matrix

$$R_{\alpha\alpha'} = \hbar^2 a / 2 \left(\mu_{\alpha} \mu_{\alpha'} \right)^{-1/2} \left(k_{\alpha} / k_{\alpha'} \right)^{1/2} \sum_{kk'} u_{\alpha k}(a) C^{-1}_{\alpha k, \alpha' k'} u_{\alpha' k'}(a)$$

S-matrix

$$U = \left(Z^* \right)^{-1} Z \qquad \because \quad \Psi^{ext}(a) = \Psi^{int}(a)$$

$$Z_{\alpha\alpha'} = I_{\alpha} \delta_{\alpha\alpha'} - R_{\alpha\alpha'} (k_{\alpha'} a)\, I'_{\alpha'} (k_{\alpha'} a)$$

Cross section of the capture reaction

$$\sigma_\gamma^{E\lambda}(E) = \frac{2J_f+1}{(2I_1+1)(2I_2+1)}\frac{8\pi}{\hbar}\left(\frac{E_\gamma}{\hbar c}\right)\frac{(\lambda+1)}{\lambda(2\lambda+1)!!^2}$$

$$\times\sum_{J_i I_i \ell_i}\frac{1}{(2\ell_i+1)}\left|\left\langle \Psi^{J_f\pi_f}\left\| M_\lambda^E \right\| \Psi_{\ell_i I_i}^{J_i\pi_i}\right\rangle\right|^2$$

Present cal.： E2 transtion ($2^+ \rightarrow 0^+$ g.s.)

Cross section of the transfer reaction

$$\sigma(E) = \frac{\pi}{k^2}\sum_{J\pi}\frac{2J+1}{(2I_1+1)(2I_2+1)}\sum_{\ell_i\ell_f I_i I_f}\left|U_{i\,\ell_i I_i,\,f\,\ell_f I_f}^{J\pi}(E)\right|^2$$

Presnt cal : $J^\pi = 0^\pm,\ 1^\pm,\ 2^\pm$

- K.Arai, D.Baye, P.Descouvemont, NPA699(02)p.963

- Threshold

AV8' pot.+3BF

d : E = -2.18MeV, P_D = 5.9% E_{exp}= -2.22MeV
t : E = -8.22MeV, P_D = 8.4% E_{exp}= -8.48MeV
h : E = -7.55MeV, P_D= 8.3% E_{exp}= -7.72MeV
α : E = -27.99MeV, P_D=13.8% E_{exp}= -28.30MeV

G3RS pot.+3BF

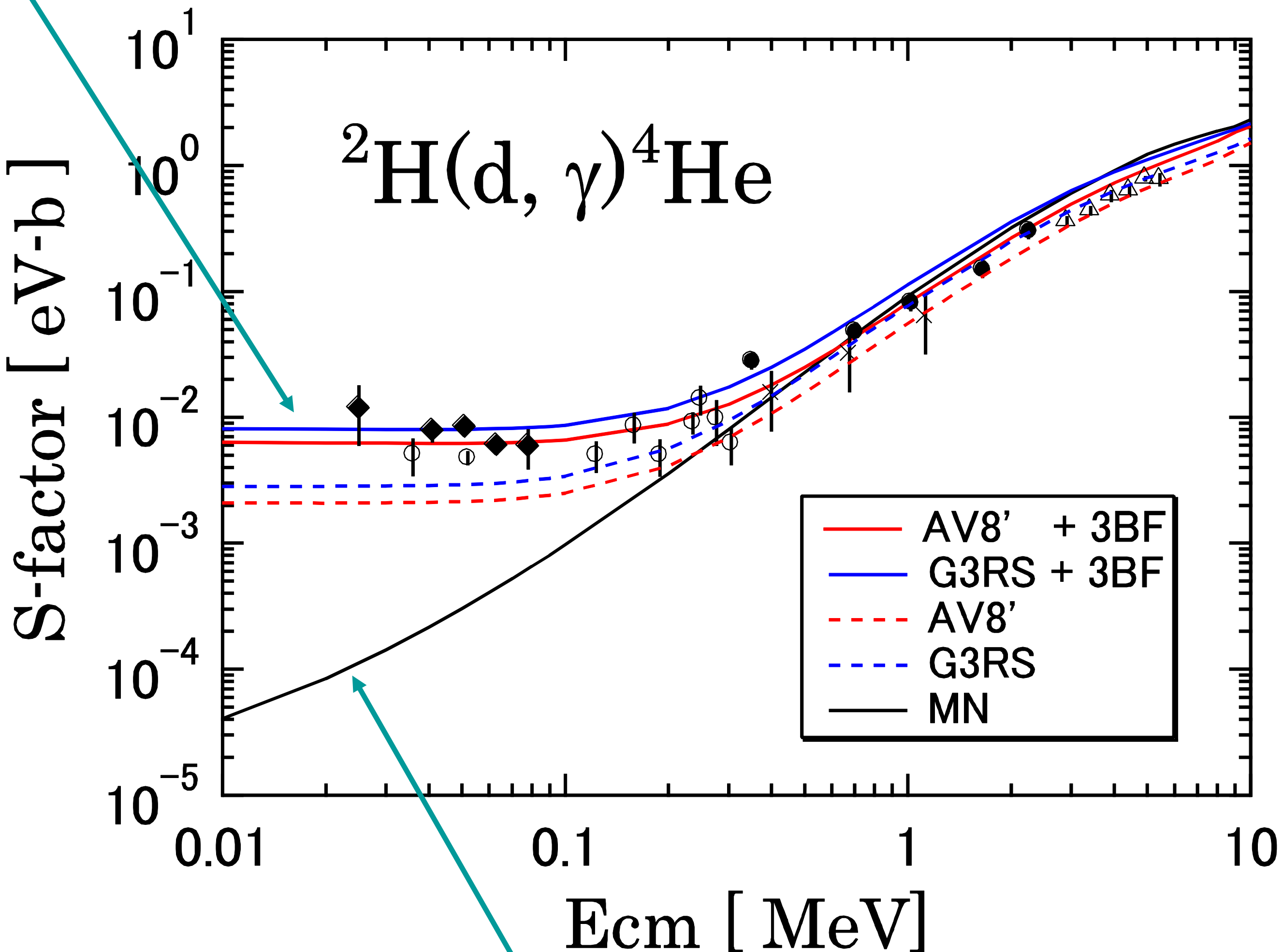
d : E = -2.13MeV, P_D = 5.0% E_{exp}= -2.22MeV
t : E = -8.24MeV, P_D = 6.9% E_{exp}= -8.48MeV
h : E = -7.58MeV, P_D= 6.9% E_{exp}= -7.72MeV
α : E = -27.99MeV, P_D=11.2% E_{exp}= -28.30MeV

MN pot.

d : E = -2.10 MeV,	E_{exp}= -2.22MeV
t : E = -8.38 MeV,	E_{exp}= -8.48MeV
h : E = -7.70 MeV,	E_{exp}= -7.72MeV
α : E = -29.94 MeV,	E_{exp}= -28.30MeV

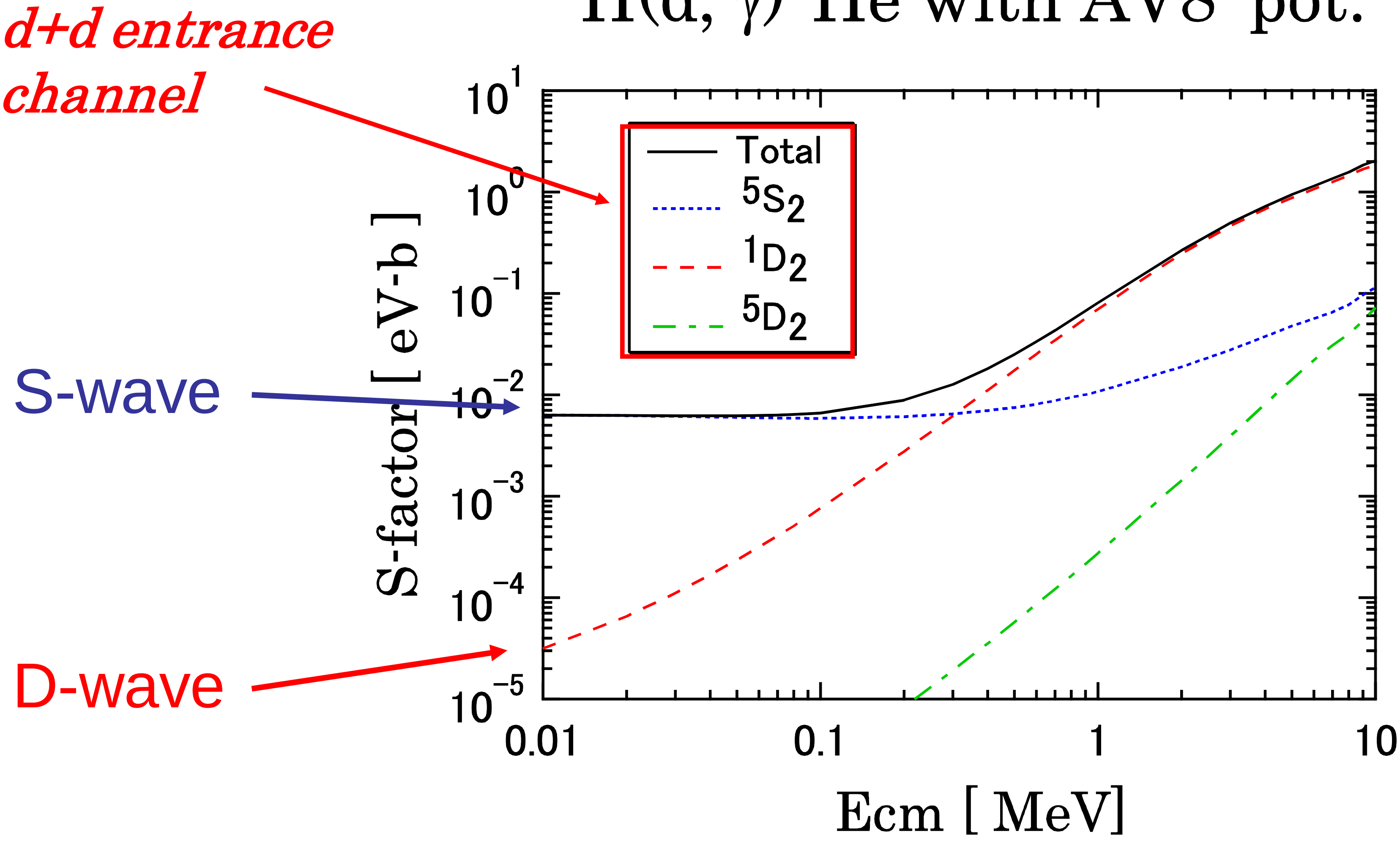
Capture reaction

Realistic force



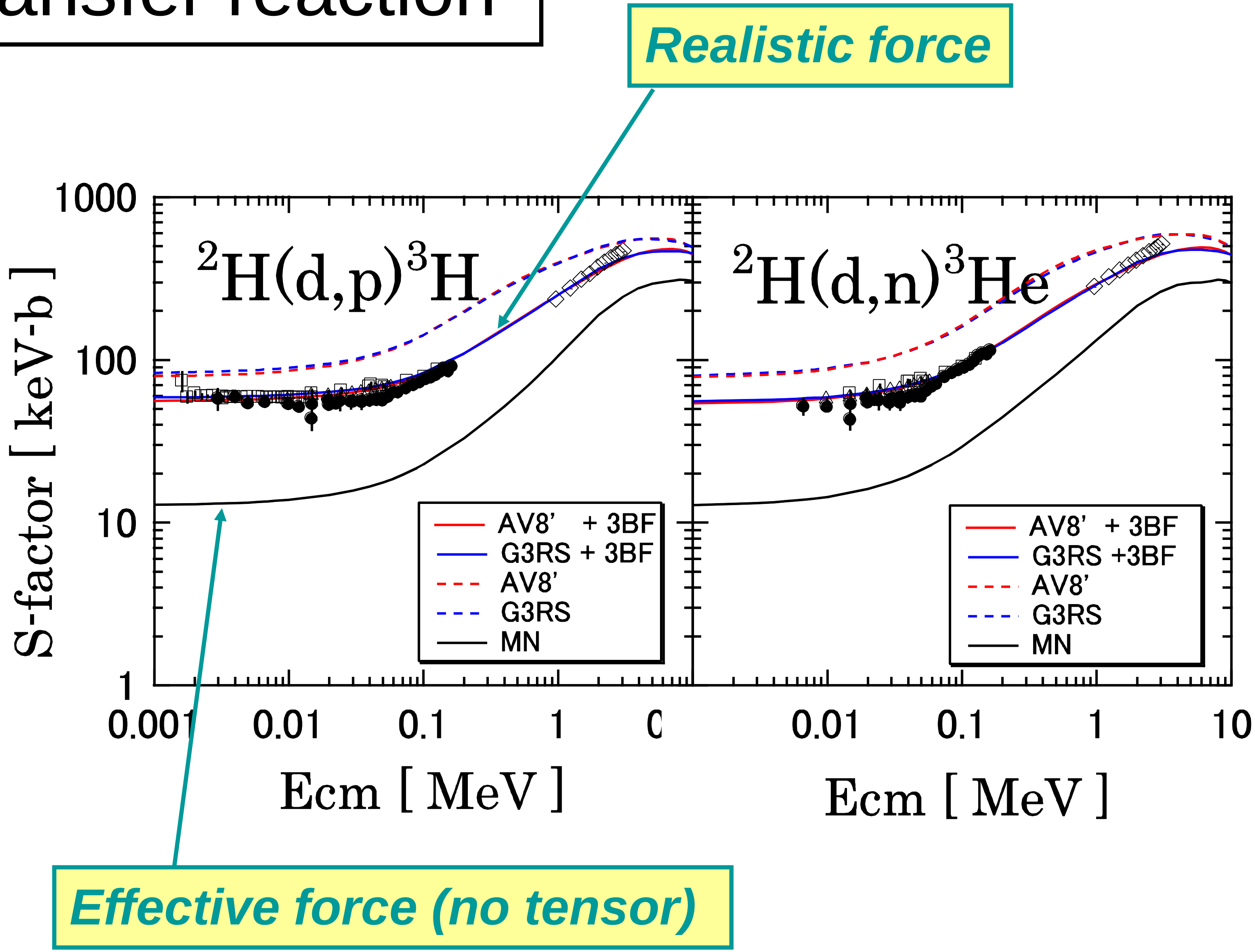
Effective force (no tensor)

$^{2}H(d, \gamma)^{4}He$ with AV8' pot.



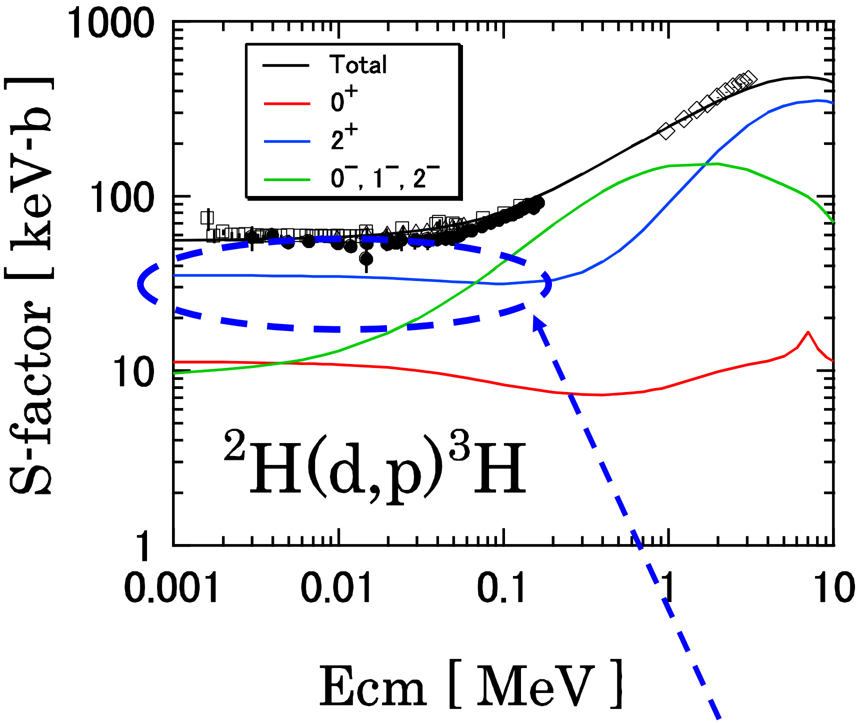
- d+d ***S-wave*** → ^{4}He 0^+ (***L=2, S=2***) ***D-wave*** component
- d+d D-wave → ^{4}He 0^+ (L=0, S=0) S-wave component

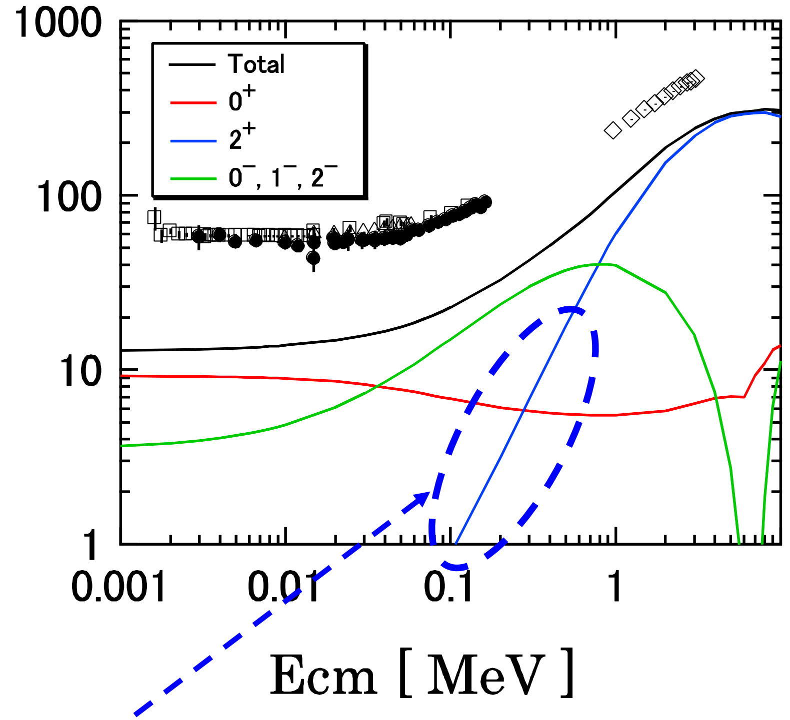
Transfer reaction



Contribution of each spin parity state in $^2H(d,p)^3H$

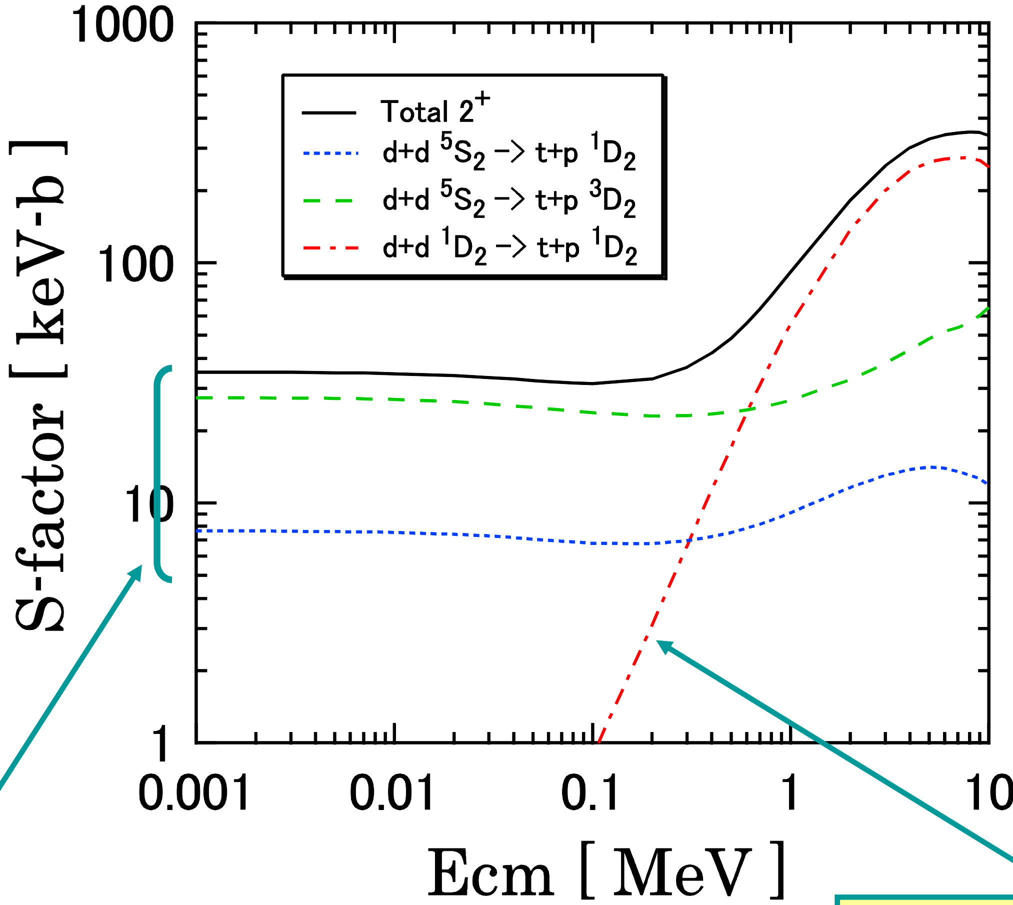
AV8' pot, **MN pot.**





S-factor by the 2^+ state

2^+ contribution in $^2H(d,p)^3H$ is decomposed according to the entrance & exit channel



AV8' pot.

d+d S-wave → t+p D-wave

d+d D-wave → t+p D-wave

Summary

D-wave component

- $^2H(d,\gamma)^4He$

$$\begin{cases} E_{cm} < 0.3\text{MeV} & d+d \;\; S\text{-wave} \rightarrow (\text{L, S}) = (2, 2) \\ E_{cm} > 0.3\text{MeV} & d+d \;\; D\text{-wave} \rightarrow (\text{L, S}) = (0, 0) \end{cases}$$

S-wave component

- $^2H(d, p)^3H$, $^2H(d, n)^3He$

$J^\pi=2^+$ contribution

Coupled by tensor force

$$\begin{cases} E_{cm} < 0.3\text{MeV} & d+d \;\; S\text{-wave} \rightarrow t+p \;\; D\text{-wave} \\ E_{cm} > 0.3\text{MeV} & d+d \;\; D\text{-wave} \rightarrow t+p \;\; D\text{-wave} \end{cases}$$

- Tensor force plays an essential role to reproduce the astrophysical S-factor not only in the capture reaction, $^2H(d,\gamma)^4He$, but also in the transfer reaction, $^2H(d,p)^3H$ and $^2H(d,n)^3He$.