Proton-induced breakup differential cross sections in the *pd* scattering

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2011.8.2 基研研究会

Motivation

nd **and** ***pd*** **scattering based on quark-model baryon-baryon interaction fss2**

Accurate estimation of Coulomb effect is essential to compare with experimental data (5-10% in low energies $E_{\rm lab}$<10 MeV)

If disagree with experiment → which is bad, Coulomb treatment or the nuclear force?

Standard treatment in momentum representation :

"screening and renormalization approach"

E. Alt, W. Sandhas and H. Ziegelmann, Phys. Rev. C17, 1981 (1978) , …

A. Deltuva, A. C. Fonseca and P. U. Sauer, Phys. Rev. C71, 054005 (2005); C72, 054004 (2005)

$$\omega_C^{\rho}(r) = \frac{e^2}{r} e^{-(r/\rho)^n}$$

with n=4, R=20 fm

not easy to solve AGS equation

Taylor's Theorem (screening Coulomb)

Nuovo Cimmento 23B, 318 (1974); M.D. Semon and J. R. Taylor, ibid. 26A, 48 (1975)

$$\omega_C^\rho(r) = \frac{e^2}{r}\alpha^\rho(r) \quad e.g.\ \alpha^\rho(r) = e^{-(r/\rho)^n}, \theta(\rho - r), \ldots$$

sharp cut-off

$0 < \alpha^\rho(r) < 1$: screening function

$\alpha^\rho(r) \to 0$ as $r \to \infty$ (monotonic)

$\alpha^\rho(r) \to 1$ as $\rho \to \infty$

$$\Psi_\ell^{(+)\rho}(k,r) \to \sin(kr - (\pi/2)\ell + \delta_\ell^\rho)$$

: regular solution

$$\square\ \sin(kr - \eta \log 2kr - (\pi/2)\ell + \sigma_\ell + \delta_\ell^N) \text{ as } r \to \infty$$

$$\delta_\ell^\rho \to \sigma_\ell - \eta \log 2k\rho + \delta_\ell^N \quad \text{as } \rho \to \infty \text{ for } \theta(\rho - r)$$

(in distribution)

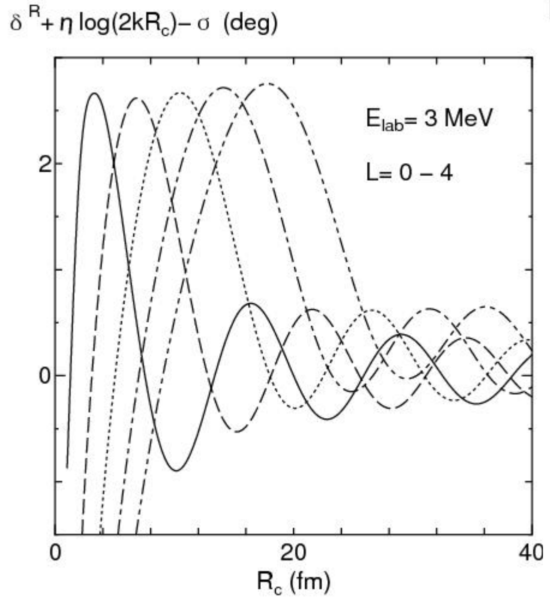
$$\delta_\ell^\rho \to \sigma_\ell - \varsigma(\rho) + \delta_\ell^N \quad \text{as} \quad \rho \to \infty$$

Euler constant

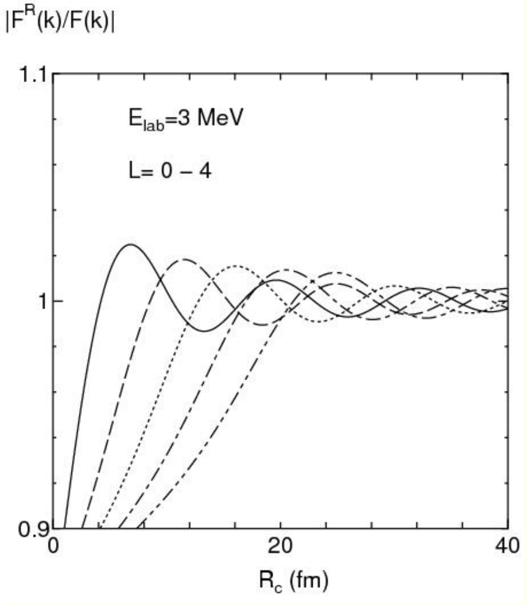
with $$\varsigma(\rho) = \frac{1}{\hbar v}\int_{\frac{1}{2k}}^{\infty} \omega_C^\rho(r)dr = \eta \log 2k\rho - \frac{\gamma}{n} + O(1/\rho)$$

Case of sharp cut-off (*pd* scattering with $\delta_\ell^N = 0$)

phase shift difference



ratio of Jost functions



$$e^{i\varsigma(\rho)}\Psi_\ell^{\rho(+)}(r) \to \Psi_\ell^{(+)}(r)$$

as $\rho \to \infty$

***E_{lab}*=3 MeV**

ℓ = 0 - 4

Convergence is very slow !

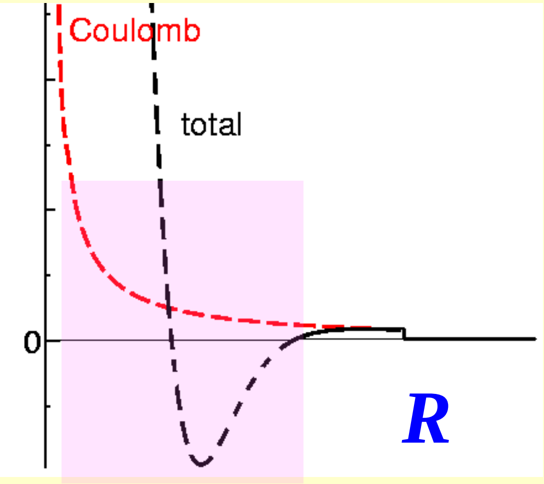
applied to $\pi^{\pm}$ ^{12}C scattering

Vincent-Phatak method (cut-off Coulomb)

Phys. Rev. C10, 391 (1974)

$$V_C^R(r) = \frac{Z_1 Z_2 e^2}{r}\theta(R-r)$$

$$\tan\delta_\ell^N = -\frac{[F_\ell, u_\ell]_R + \tan\delta_\ell^R [F_\ell, v_\ell]_R}{[G_\ell, u_\ell]_R + \tan\delta_\ell^R [G_\ell, v_\ell]_R}$$



u_λ, v_λ: Riccati Bessel and Neumann functions

(advantage) exact for finite *R*

$$\delta_\ell^R \to \sigma_\ell - \eta \log 2kR + \delta_\ell^N \quad \text{as} \quad R \to \infty$$

Screening and renormalization procedure

Alt et al. (1978), …, Deltuva et al. (2005), Witala et al.(2009), Ishikawa (2009), Oryu (2006), …

Ali-Bodmer potential (with folded Coulomb)

Nucl. Phys. 80, 99 (1966)

$$V_{\alpha\alpha}^{ABd}(r) = V_1 e^{-\kappa_1 r^2} + V_2 e^{-\kappa_2 r^2} + \frac{4e^2}{r}\mathrm{erf}(\beta r)$$

with $\mathrm{erf}(x) = (2/\sqrt{\pi})\int_0^x e^{-t^2} dt$

If we use the sharp cut-off Coulomb at the nucleon level the αα direct potential is given by

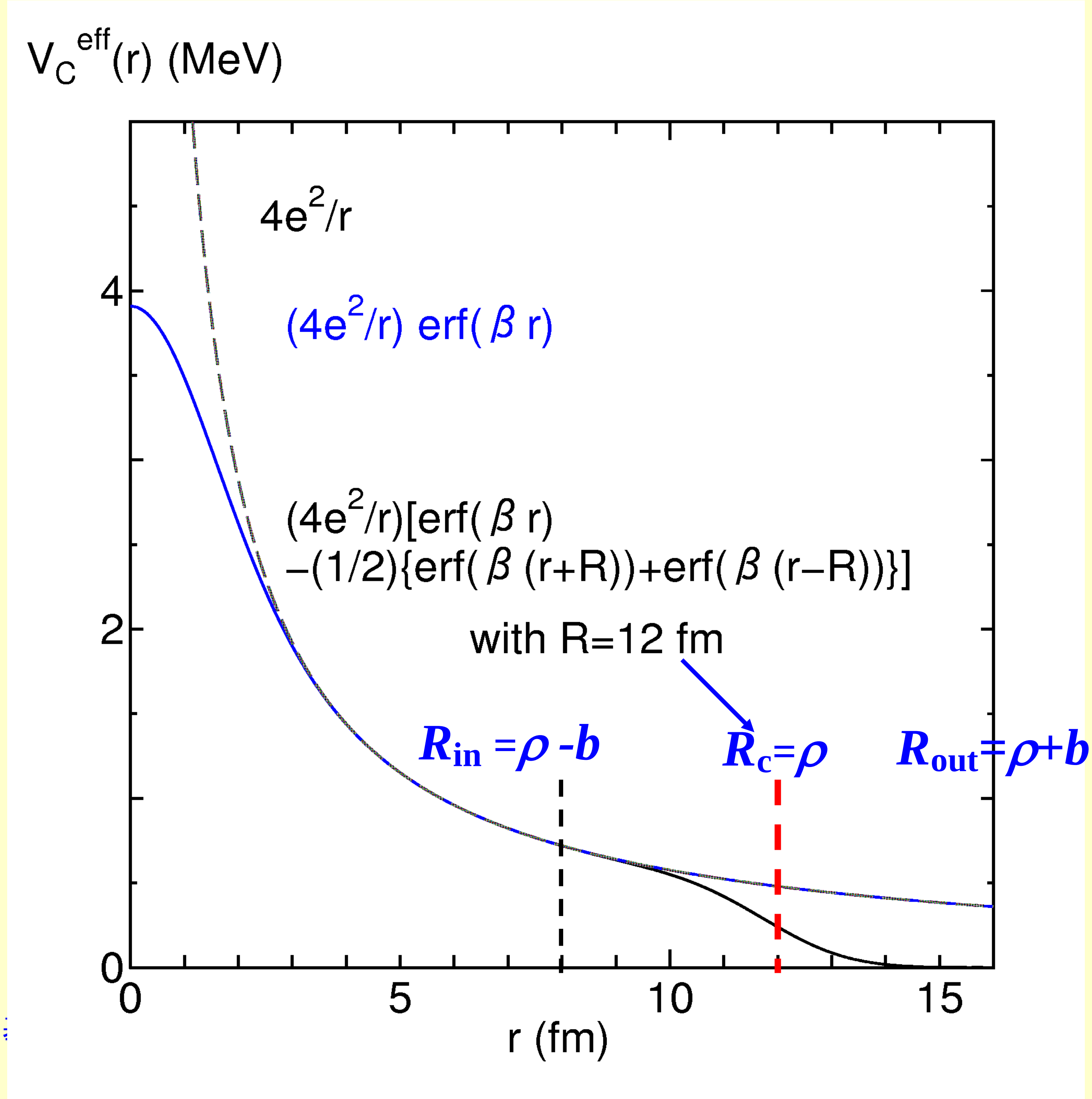
$$V_C^{\rho}(r) = \frac{4e^2}{r}\{\mathrm{erf}(\beta r) - \tfrac{1}{2}[\mathrm{erf}(\beta(r+\rho)) + \mathrm{erf}(\beta(r-\rho))]\}$$

$\upsilon(r)$ short range

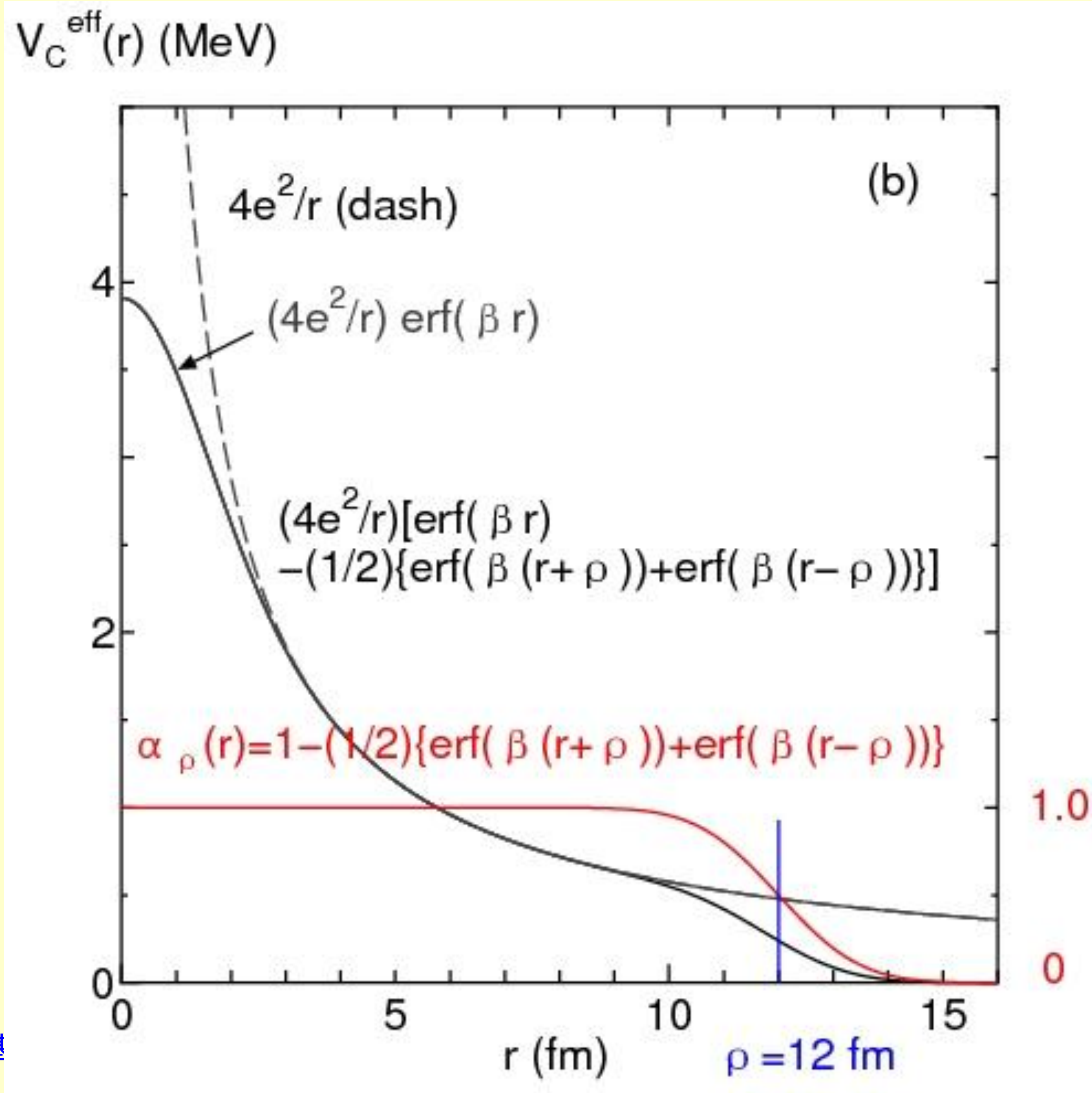
$$= -\frac{4e^2}{r}[1-\mathrm{erf}(\beta r)] + \frac{4e^2}{r}\alpha_\rho(r)$$

$\omega_C^{\rho}(r)$

with $\alpha_\rho(r) = \{1 - \tfrac{1}{2}[\mathrm{erf}(\beta(r+\rho)) + \mathrm{erf}(\beta(r-\rho))]\}$



$V_C^{eff}(r)$ (MeV)
$4e^2/r$
$(4e^2/r)\,\mathrm{erf}(\beta r)$
$(4e^2/r)[\mathrm{erf}(\beta r)$
$-(1/2)\{\mathrm{erf}(\beta(r+R))+\mathrm{erf}(\beta(r-R))\}]$
with R=12 fm
$R_{in}=\rho-b$
$R_c=\rho$
$R_{out}=\rho+b$
4
2
0
0
5
10
15
r (fm)



V_C^eff(r) (MeV)
(b)
4e²/r (dash)
(4e²/r) erf(β r)
(4e²/r)[erf(β r)
−(1/2){erf(β (r+ ρ))+erf(β (r− ρ))}]
α ρ (r)=1−(1/2){erf(β (r+ ρ))+erf(β (r− ρ))}
4
2
0
1.0
0
0
5
10
15
r (fm)
ρ =12 fm

Sharp cut-off Coulomb approach の不十分な点

(現象面) *pd* elastic scattering: low-energy (3 MeV 以下) の
A_y, T_{11}, 部分波の問題 (I_{max}=3 では不十分？)

(原理的な面）

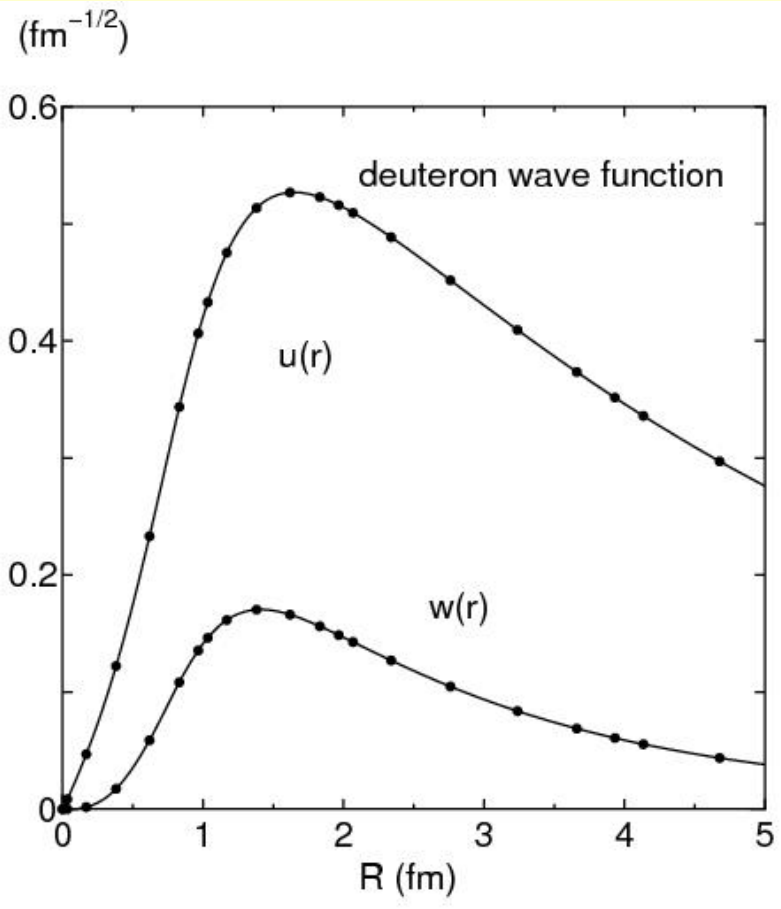
quark level で sharp cut-off: $(1/r)\theta(\rho-r)$

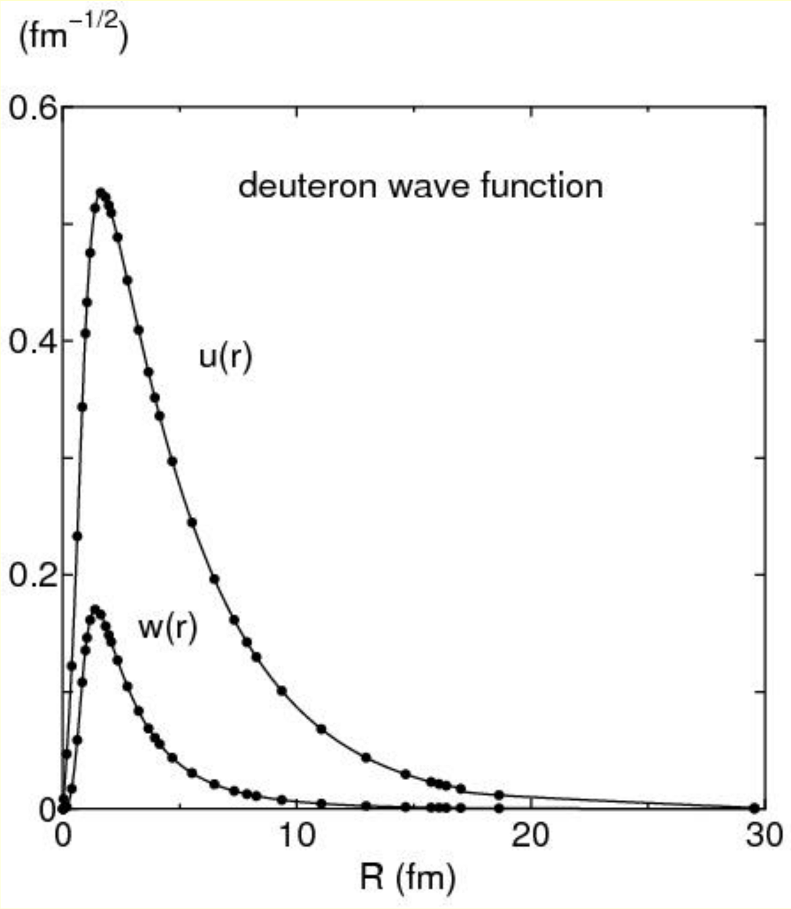
nucleon level で error Coulomb ↕ should be consistent

pd level で sharp cut-off ⟶ folded Coulomb pot.

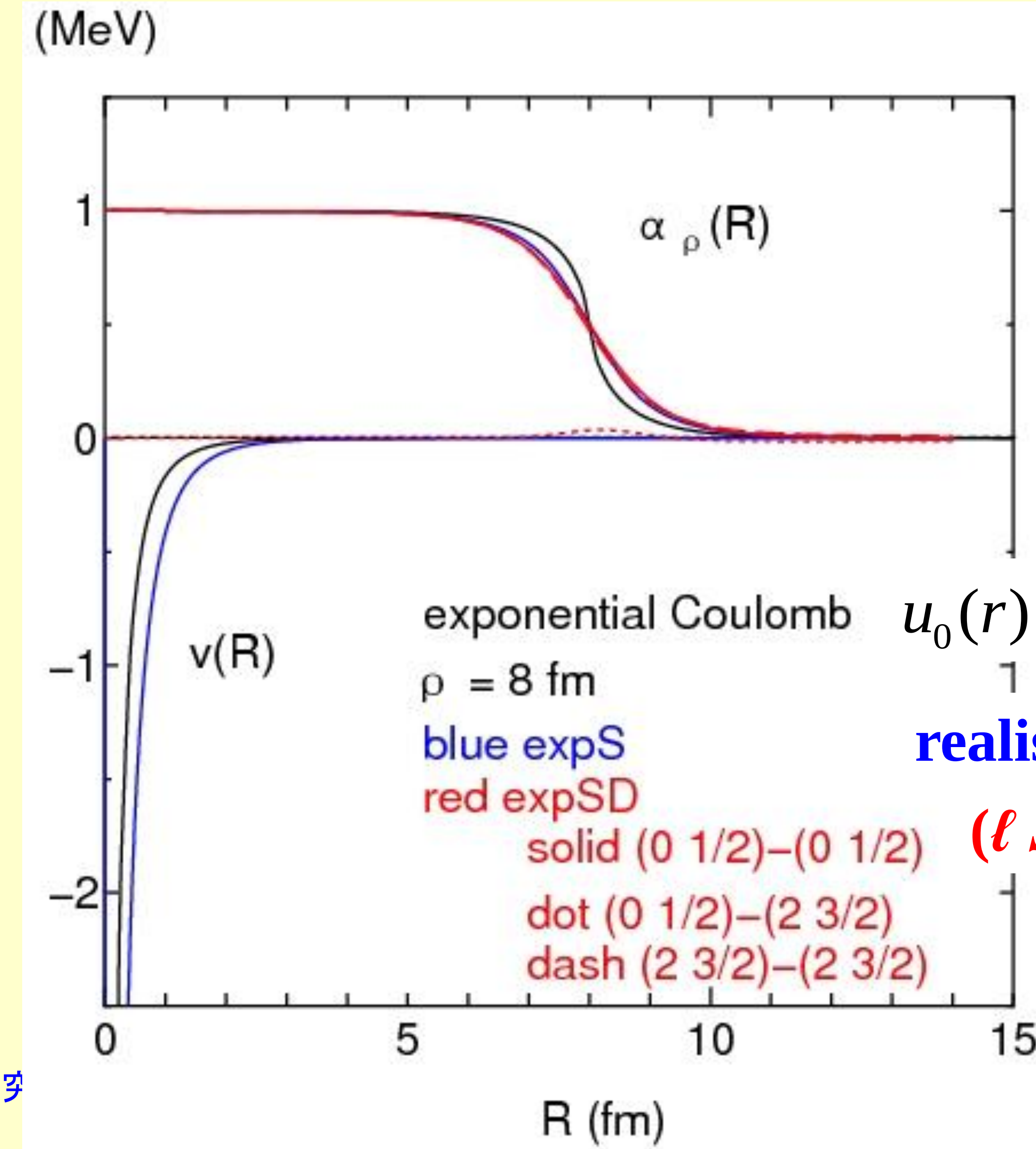
- realistic deuteron wave function で folding すべき
- channel-spin formalism では coupled channel
- model space の truncation による channel dependence
  (部分波が不十分だと nonsense)
- quasi singular nature of screened Coulomb force
  (AGS equation が正確に解けない)

deuteron folding for *pd* elastic scattering





pd folded Coulomb potential and the screening factor



$$u_0(r) = \sqrt{2\gamma}\; e^{-\gamma r}$$

realistic case

$(\ell\ S_c)\ J^{\pi}=1/2^{+}$

A new practical method (in LS-RGM)

1) solve δ_ℓ^ρ in momentum representation

2) solve the screening Coulomb problem

for $\tilde{F}_\lambda(r)$ and $\tilde{G}_\lambda(r)$ from R_{in} to R_{out}

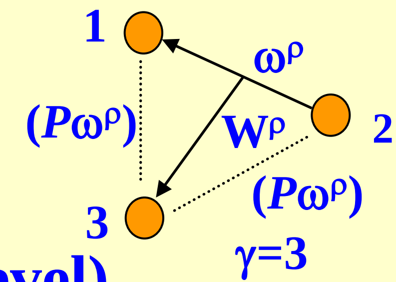
3) calculate δ_ℓ^N through the connection condition

$$\tan\delta_\ell^N = -\frac{[\tilde{F}_\ell, u_\ell]_{R_{\text{out}}} + \tan\delta_\ell^\rho [\tilde{F}_\ell, v_\ell]_{R_{\text{out}}}}{[\tilde{G}_\ell, u_\ell]_{R_{\text{out}}} + \tan\delta_\ell^\rho [\tilde{G}_\ell, v_\ell]_{R_{\text{out}}}}$$

Three different radii are introduced.

$$R_{\text{in}}(=\rho-b) \;\square\; R_{\text{c}}(=\rho) \;\square\; R_{\text{out}}(=\rho+b)$$

3-body Coulomb problemへの応用



Step 1. 2-body *t*-matrix (sharp cut-off at quark level)

$$t^{\rho} = (v_{\mathrm{RGM}} + \omega^{\rho}) + (v_{\mathrm{RGM}} + \omega^{\rho}) G_0 t^{\rho}$$

: error Coulomb

isospin formalism → only for *I*=1 pair with factor 2/3

$$V_C^{(3)\rho} = \Sigma_{\gamma} (\omega^{\rho})_{\gamma} = \omega^{\rho} + [(P\omega^{\rho}) - W^{\rho}] + W^{\rho} = \omega^{\rho} + \mathcal{W} + W^{\rho}$$

$$\mathcal{W} = \lim_{\rho \to \infty} [(P\omega^{\rho}) - W^{\rho}] = [(P\omega) - W]$$

: short range

$$W^{\rho}(R) = \frac{e^2}{R} \alpha_{\rho}(\mathrm{R}) \quad \text{with} \quad \alpha_{\rho}(R) = 1 - \frac{R}{e^2} \langle \psi_d | (P\omega) - (P\omega^{\rho}) | \psi_d \rangle$$

Step 2. AGS (Alt-Glassberger-Sandhas) equation

$$U^{\rho} | \phi \rangle = G_0^{-1} P | \phi \rangle + P t^{\rho} G_0 U^{\rho} | \phi \rangle$$

$$\text{with} \ | \phi \rangle = | \boldsymbol{q}_0, \psi_d \rangle \ \text{and} \ P = P_{(123)} + P_{(123)}{}^2$$

波動函数 (2種類)

$v = v_{\rm RGM} + W$ **: short range force**

$|\Psi^{\rho(+)}\rangle = |\psi^{\rho(+)}\rangle + G^{\rho}(P v_{\rm RGM} + W)|\Psi^{\rho(+)}\rangle$: distorted wave

$|\Psi^{\rho(+)}\rangle = |\phi\rangle + g^{\rho} P(v_{\rm RGM} + \omega^{\rho})|\Psi^{\rho(+)}\rangle$: total wave function

$|\Psi^{\rho}\rangle = |\phi\rangle + g^{\rho}(v_{\rm RGM} + \omega^{\rho})P|\Psi^{\rho}\rangle$: Faddeev component

with $|\Psi^{\rho(+)}\rangle = (1+P)|\Psi^{\rho}\rangle$ and $|\psi^{\rho(+)}\rangle = |\phi\rangle + g^{\rho} W^{\rho}|\psi^{\rho(+)}\rangle$

where

$g^{\rho} = (z - H_0 - v_{\rm RGM} - \omega^{\rho})^{-1}$ and $G^{\rho} = (z - H_0 - v_{\rm RGM} - \omega^{\rho} - W^{\rho})^{-1}$

with $z = E + \varepsilon_d + i0$

左から deuteron 波動函数で内積をとる

$$\langle\psi_d|\Psi^{\rho(+)}\rangle = |q_0\rangle + \bar{g}_0\langle\psi_d|P(v_{\rm RGM} + \omega^{\rho})|\Psi^{\rho(+)}\rangle$$

$$\text{2nd term m.e.} = \langle\psi_d|W|\Psi^{\rho(+)}\rangle + W^{\rho}\langle\psi_d|\Psi^{\rho(+)}\rangle$$

connection condition

Step 3. 2-potential formula

$$\langle\phi|U^{\rho}|\phi\rangle=\langle\phi|T_{C}^{\rho}|\phi\rangle+\langle\psi^{\rho(-)}|\tilde{U}^{\rho}|\psi^{\rho(+)}\rangle$$

renormalize して $\rho\to\infty$ の極限をとる　（2項目：近似）

Step 4. elastic differential cross sections

$$\frac{d\sigma}{d\Omega}=|\langle\phi|U|\phi\rangle|^{2}$$

Step 5. breakup cross sections

$\mathcal{O}^{pp}=(1+\tau_z(1))/2\ (1+\tau_z(2))/2$

$$\langle\boldsymbol{p},\boldsymbol{q}|\Sigma_{\gamma}(v_{RGM}+\omega^{\rho})_{\gamma}|\Psi^{\rho(+)}\rangle=\langle\boldsymbol{p},\boldsymbol{q}|(1+P)t^{\rho}G_{0}U^{\rho}|\phi\rangle=\langle\boldsymbol{p},\boldsymbol{q}|U_{0}^{\rho}|\phi\rangle$$

$$\langle\boldsymbol{p},\boldsymbol{q}|U_{0}|\phi\rangle=\lim_{\rho\to\infty}e^{iz_{\rho}(p)}\langle\boldsymbol{q},\psi_{\boldsymbol{p}}^{\rho(-)}|\tilde{U}_{0}^{\rho}|\psi^{\rho(+)}\rangle e^{i\zeta_{\rho}(q_{0})}=\langle\boldsymbol{q},\psi_{\boldsymbol{p}}^{(-)}|\tilde{U}_{0}|\psi^{(+)}\rangle$$

$$\frac{d^{5}\sigma}{d\Omega_{1}d\Omega_{2}dS}=w|\sum_{\alpha=1}^{3}\langle\boldsymbol{p}_{\alpha},\boldsymbol{q}_{\alpha}|t^{\rho}G_{0}U^{\rho}|\phi\rangle|^{2}$$

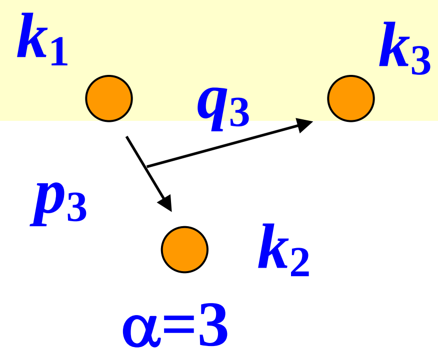
$(\rho\to\infty)$ （有限で近似）

phase space factor

$$E=E_{inc}+\varepsilon_{d}=\frac{\hbar^{2}}{M}\left(\boldsymbol{p}_{\alpha}{}^{2}+\frac{3}{4}\boldsymbol{q}_{\alpha}{}^{2}\right)\ \text{with}\ E_{inc}=\frac{3\hbar^{2}}{4M}\boldsymbol{q}_{0}{}^{2}$$

breakup differential cross sections

(Various breakup configurations)



- **QFS (quasi-free scattering)** $k_\alpha=0$
- **FSI (final-state interaction)** $p_\alpha=0$
- **COLL (collinear)** $q_\alpha=0$
- **SS (standard space star)** 120° perpendicular
- **COP, CST (coplanar star)** 120° coplanar
- **non-standard:** other non-specific configurations

- **13 MeV** $p + d$ (p incident)
  $n + d$ (n incident)
- **16 MeV** $d + p$ (d incident)

Experimental data

nd J. Strate et al., Nucl. Phys. A501 (1989) 51

pd G. Rauplich ey al., Nucl. Phys. A535(1991)313

F.D. Correll et al., Nucl. Phys. A475 (1987) 407

Cf. Three-nucleon force effects in nucleon induced deuteron breakup II. Comparison to data, J. Kuroś-Żolnierczuk et al., Phys. Rev. C66, 024004 (2002)

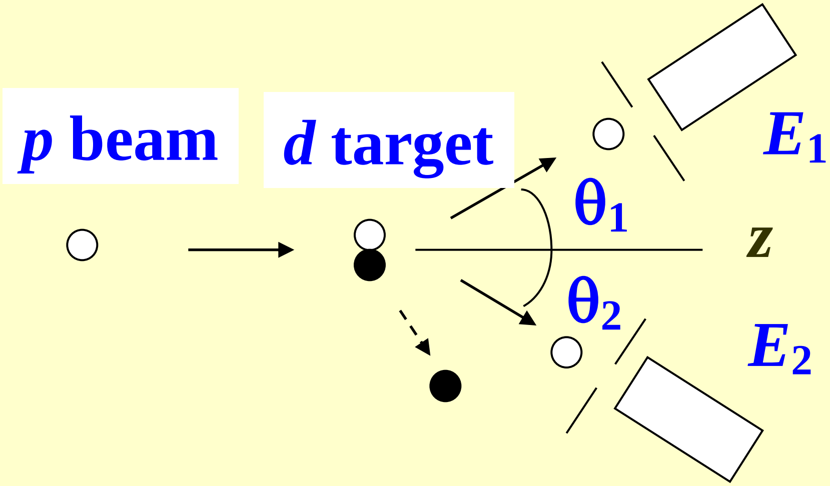
comparison with meson-exchange predictions

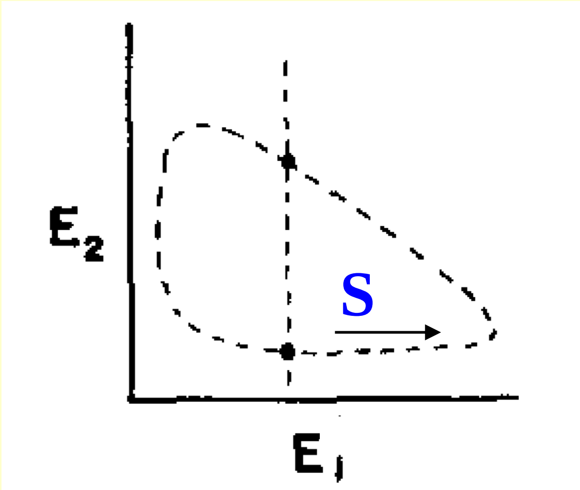
fss2 による *pd* breakup differential cross sections

Y. Fujiwara and K. Fukukawa

Motivation

クォーク模型バリオン間相互作用の非局所性に起因した off-shell 効果が強く現れる。

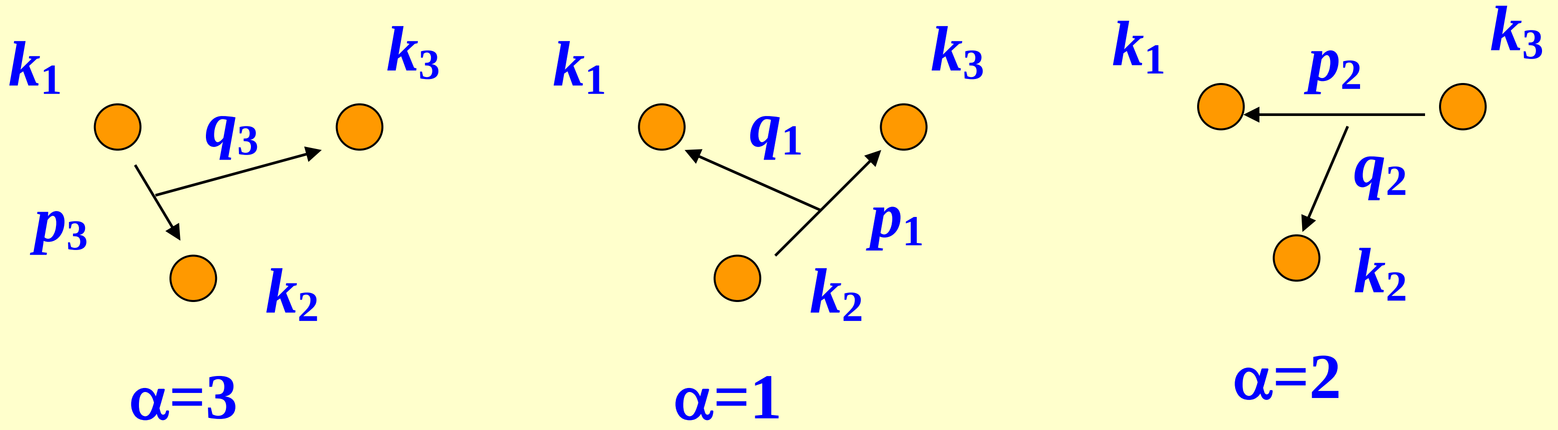




自由度　3×3-(3+1) = 5　：Ω_1 Ω_2 *S*

$$\frac{d^5\sigma}{d\Omega_1 d\Omega_2 dS} = w \left| \sum_{\alpha=1}^{3} f^{(db)}(p_\alpha, q_\alpha) \right|^2$$

w **: phase space factor**



direct breakup amplitude

$$f^{(db)}(p,q) = t\ \tilde{Q}\ f$$

$$\tilde{Q}|\phi\rangle = P|\phi\rangle + P\tilde{G}_0\tilde{t}\tilde{Q}|\phi\rangle$$

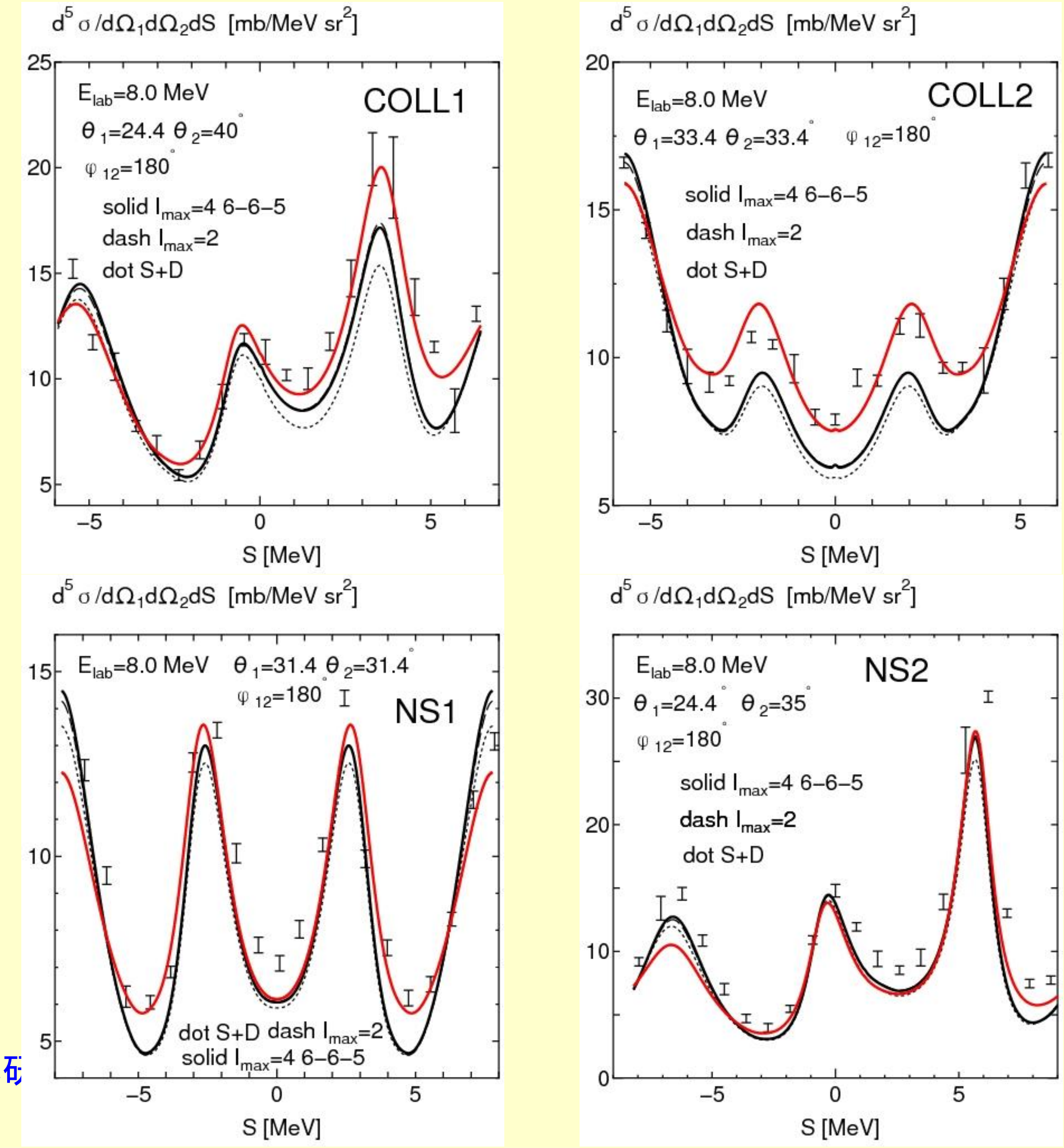
=(2-body half-on shell *t*-matrix) × (solution of AGS equation)
× (elastic scattering amplitude)

$$E = E_{inc} + \varepsilon_d = \frac{\hbar^2}{M}\left(p_\alpha{}^2 + \frac{3}{4}q_\alpha{}^2\right)$$

: 3-body on-shell energy

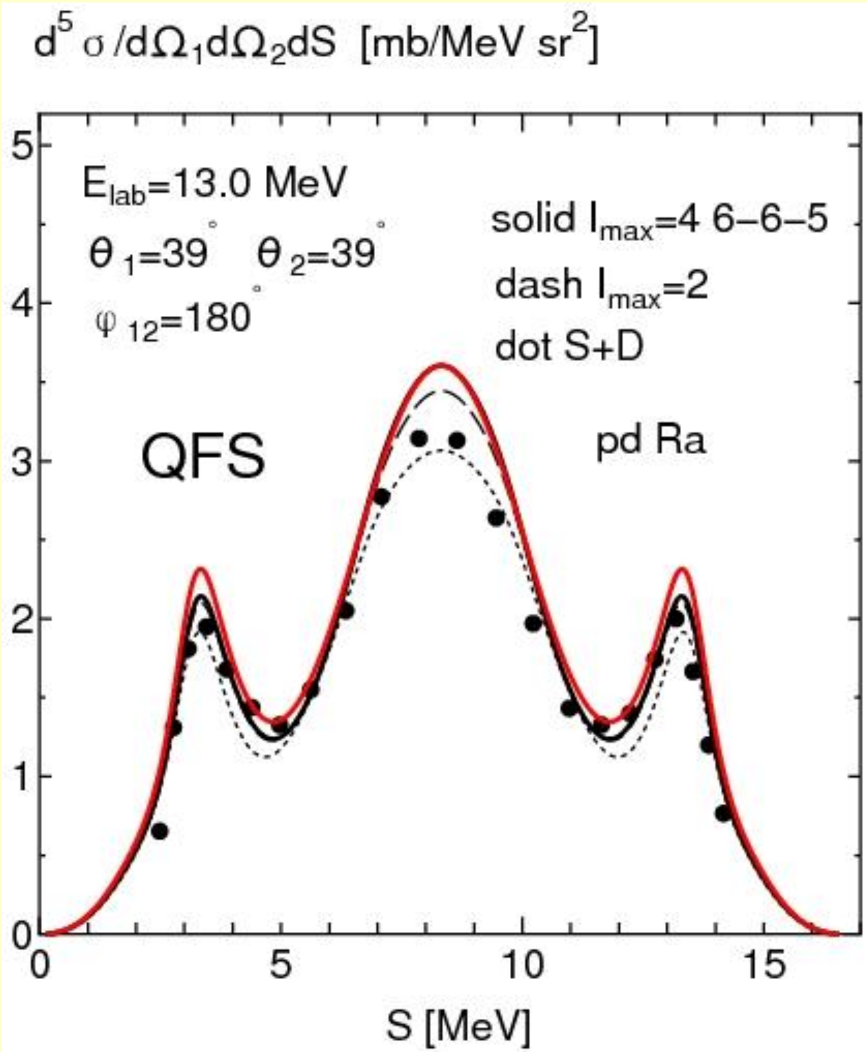
$E_d = 16$ MeV deuteron incident ($E_p = 8$ MeV)

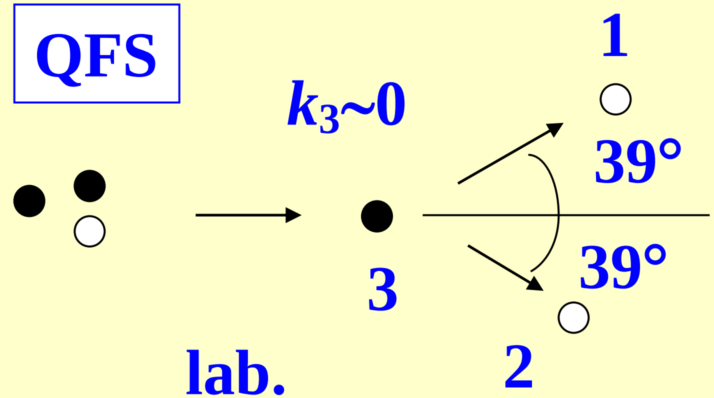
red Coulomb with $\rho = 8$ fm

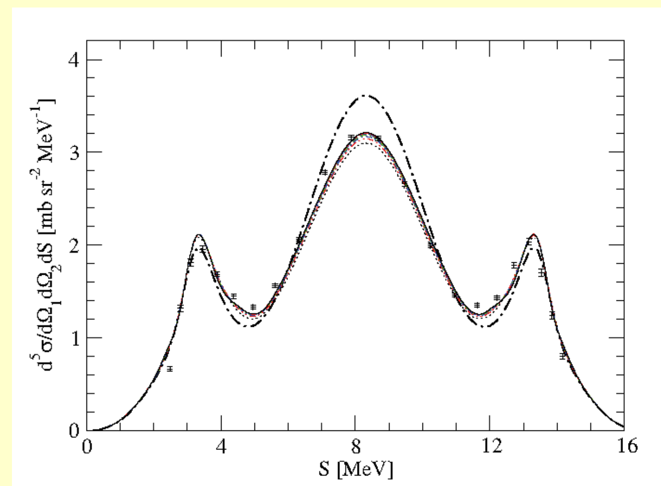


Exp: F.D. Correll et al., Nucl. Phys. A475 (1987) 407

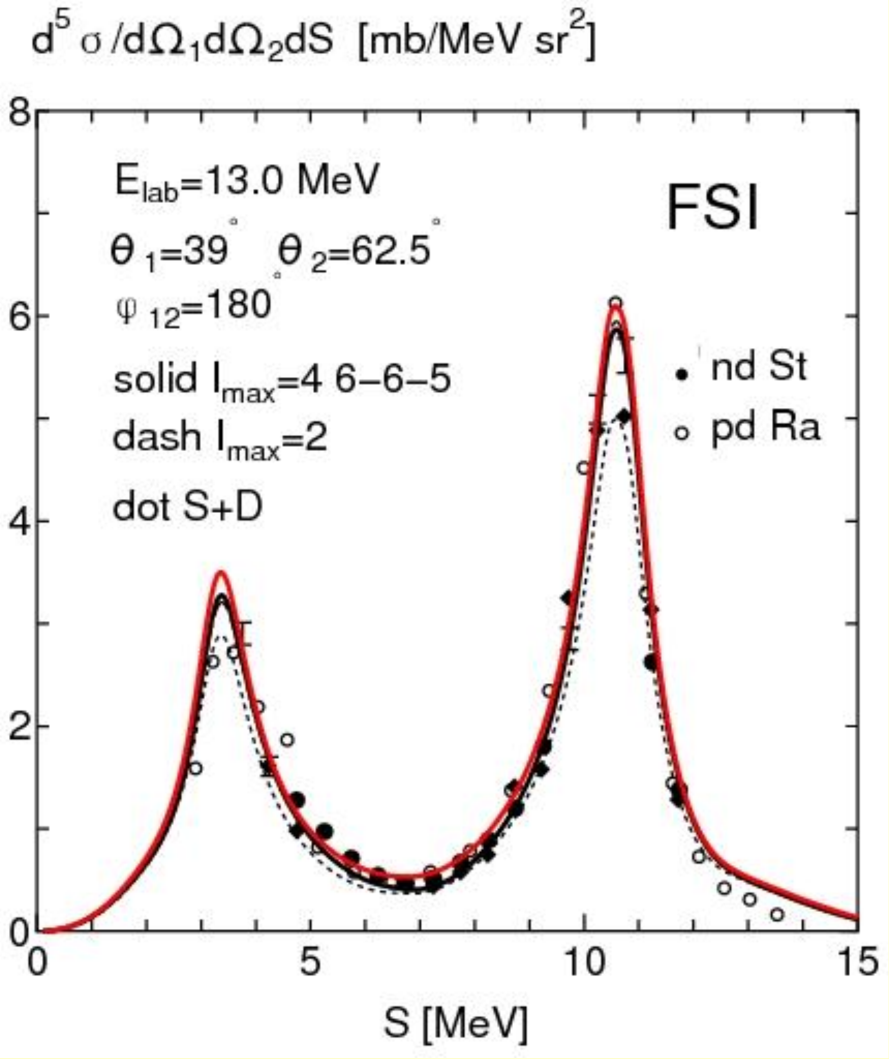
Breakup differential cross sections at E_p=13 MeV



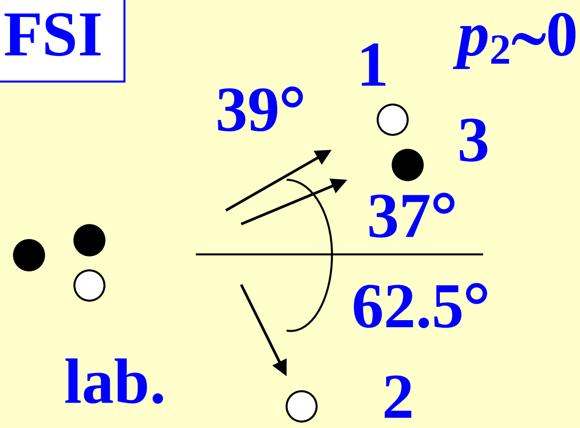


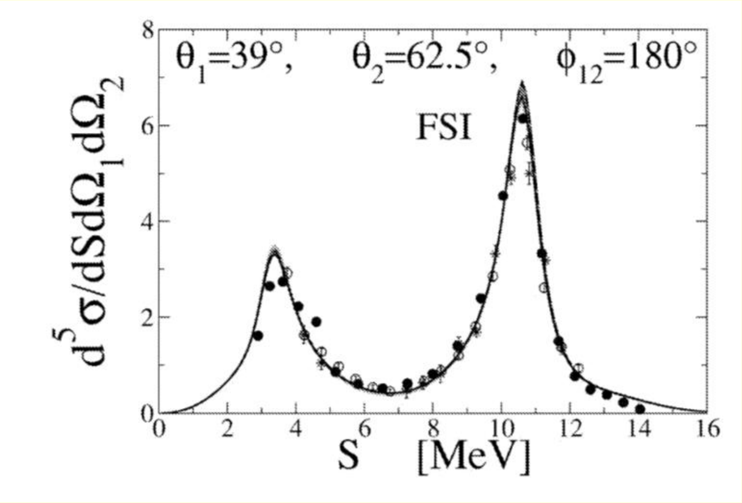


H. Witala et al. Eur. Phys. J. A41, 385 (2009)

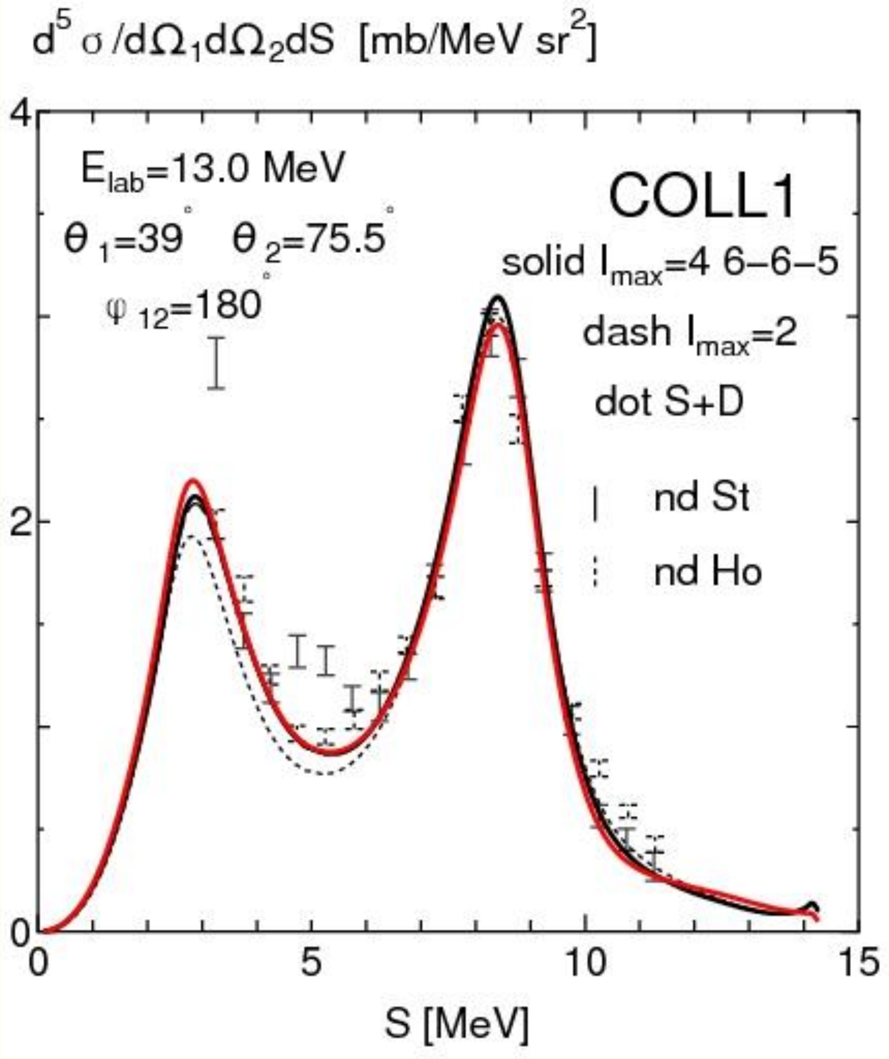


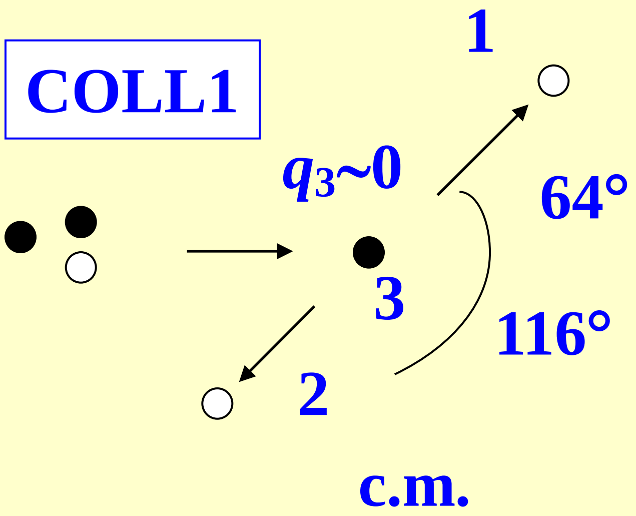
FSI

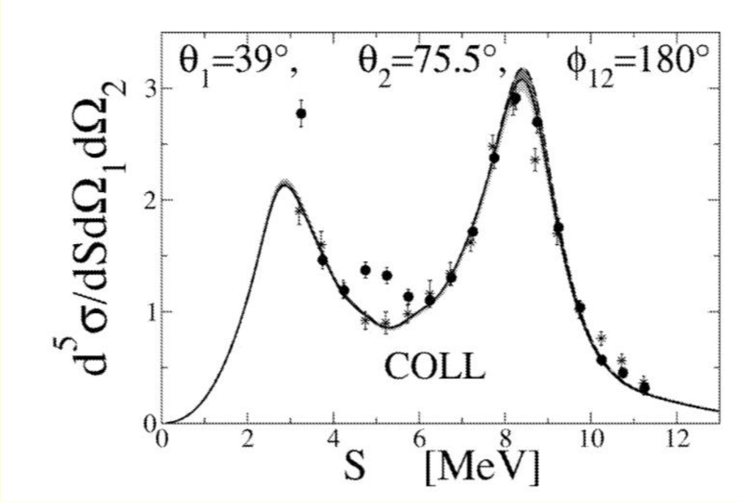




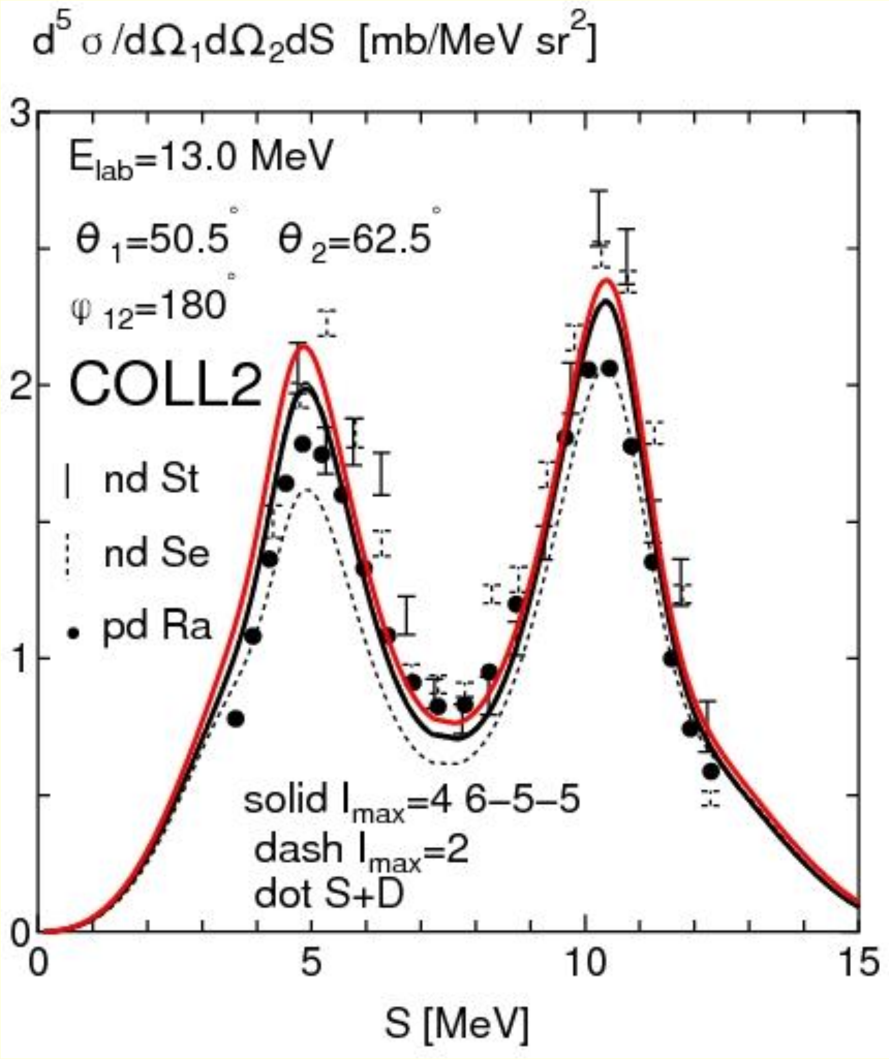
Kuroś-Żolnierczuk et al. (2002)

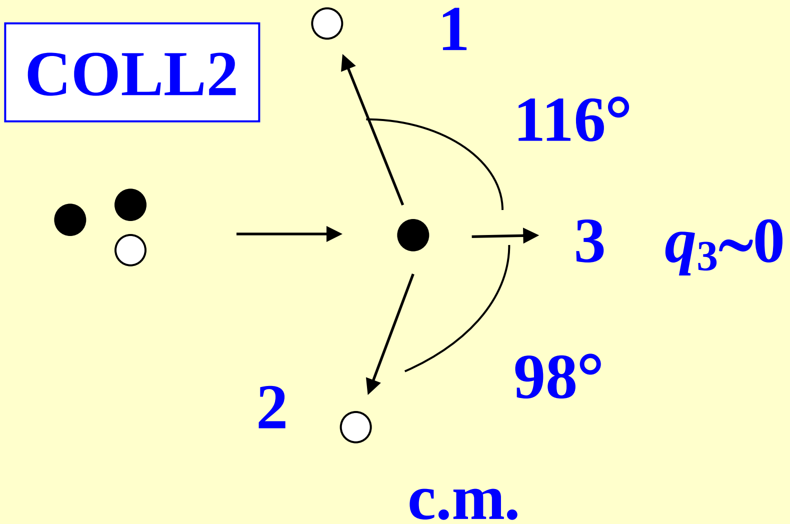


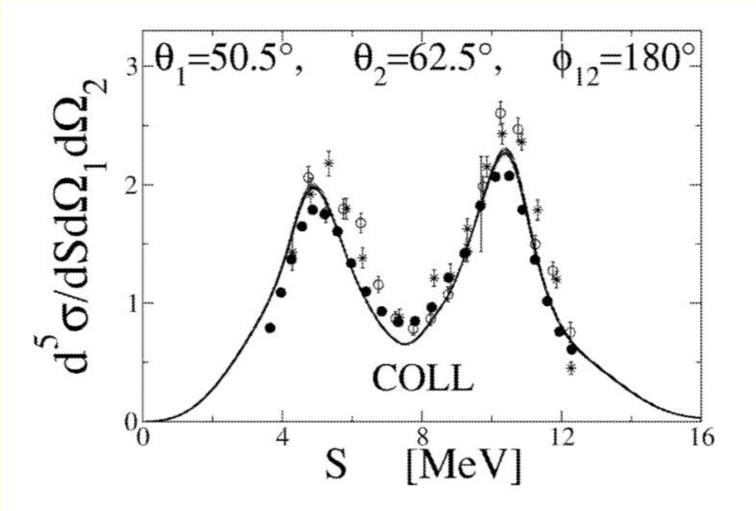




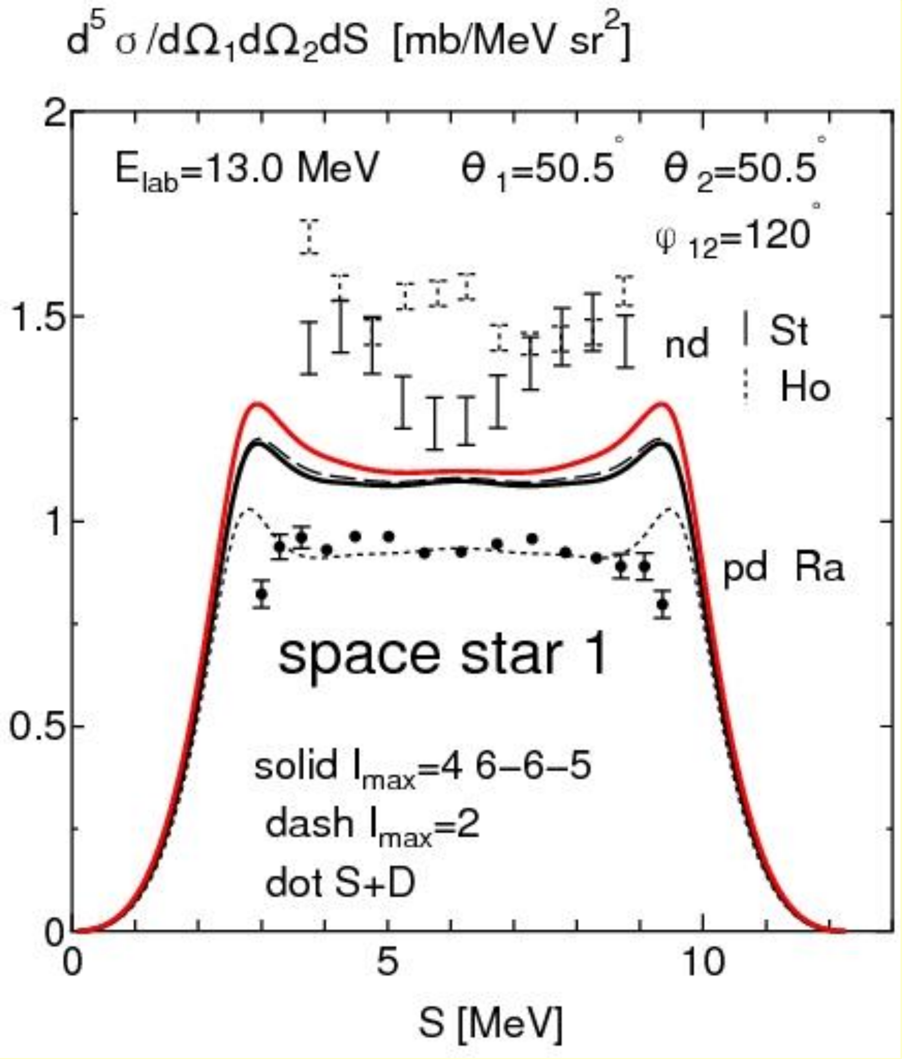
Kuroś-Żolnierczuk et al. (2002)



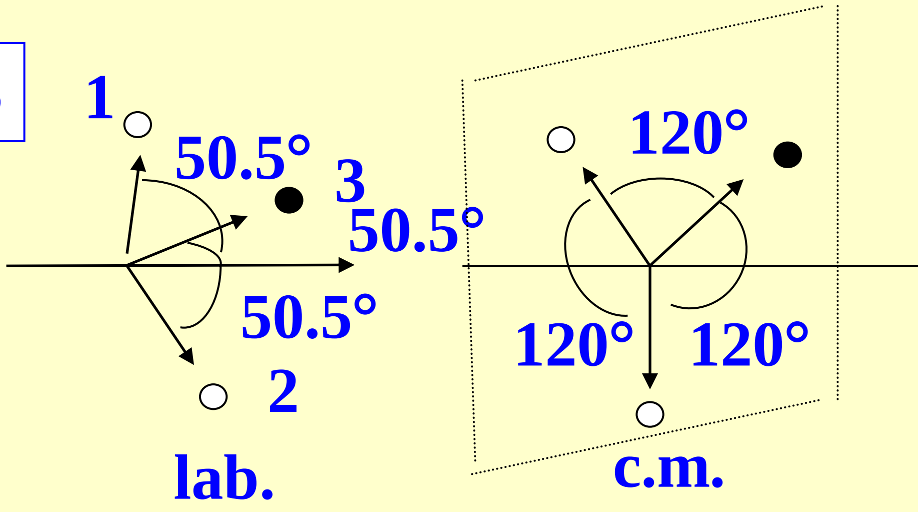


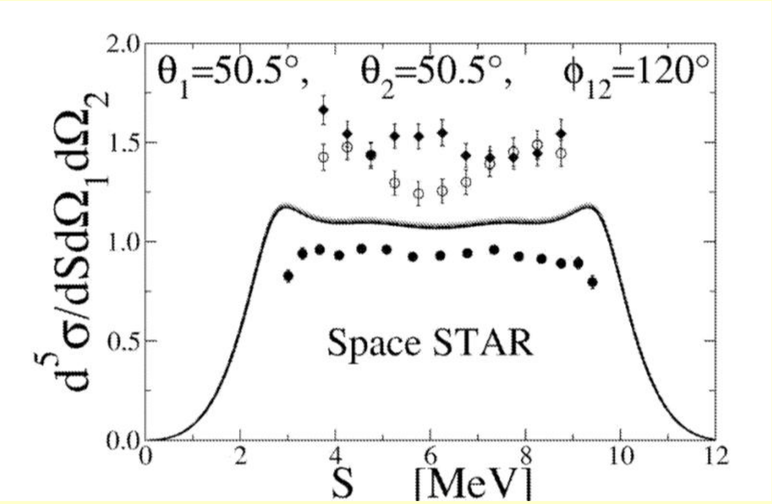


Kuroś-Żolnierczuk et al. (2002)



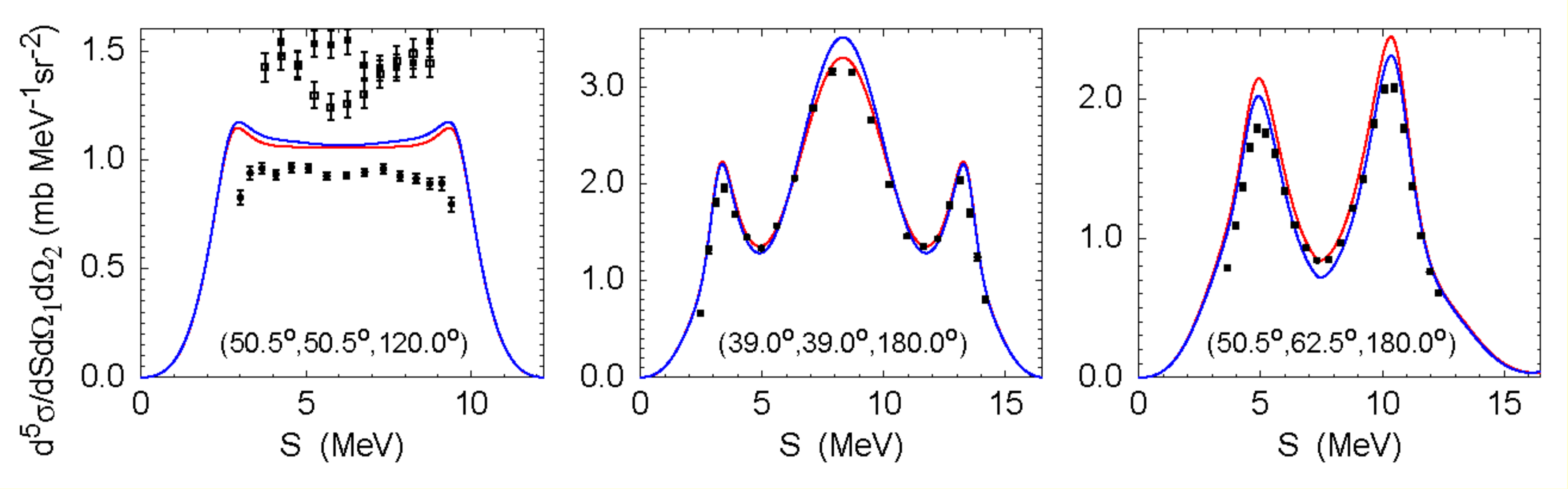
SS



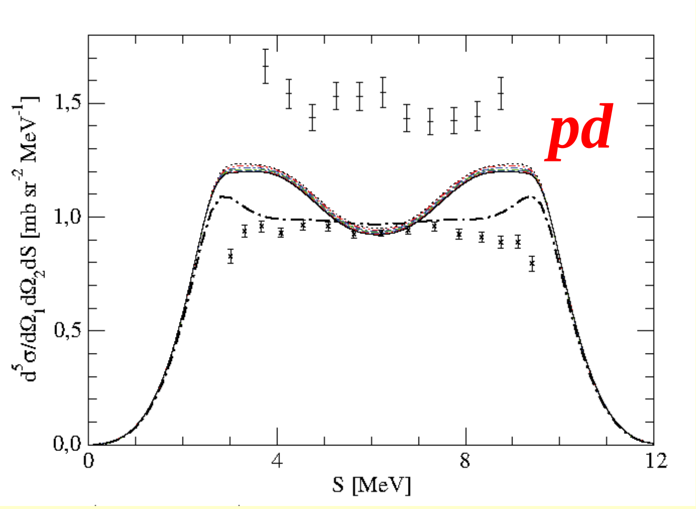


Kuroś-Żolnierczuk et al. (2002)

red *pd* **blue *nd***



A. Deltuva EPJ Web of Conference 3, 01003(2010) fb19: (2008)



H. Witala et al. Eur. Phys. J. A41, 385 (2009)

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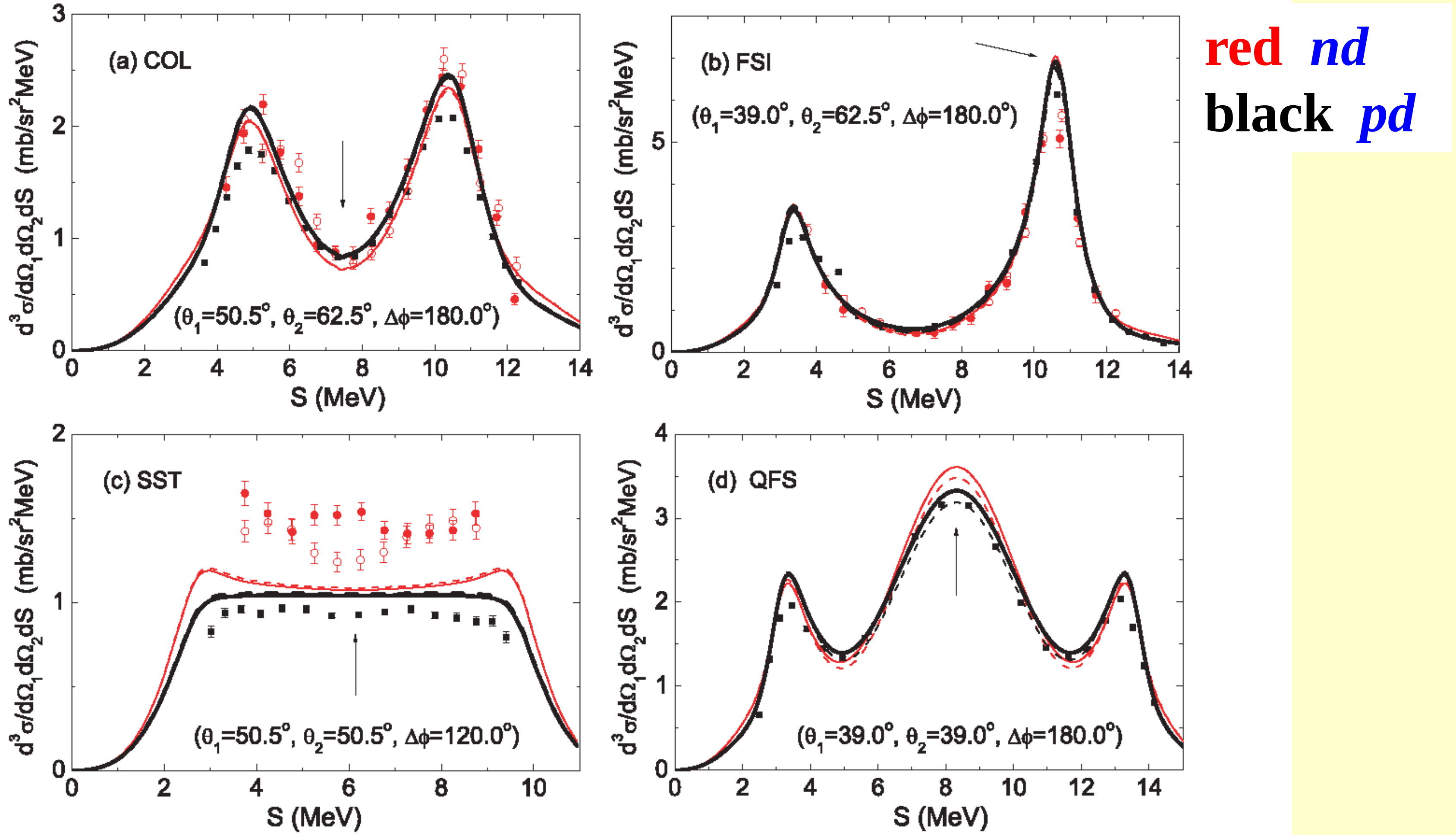
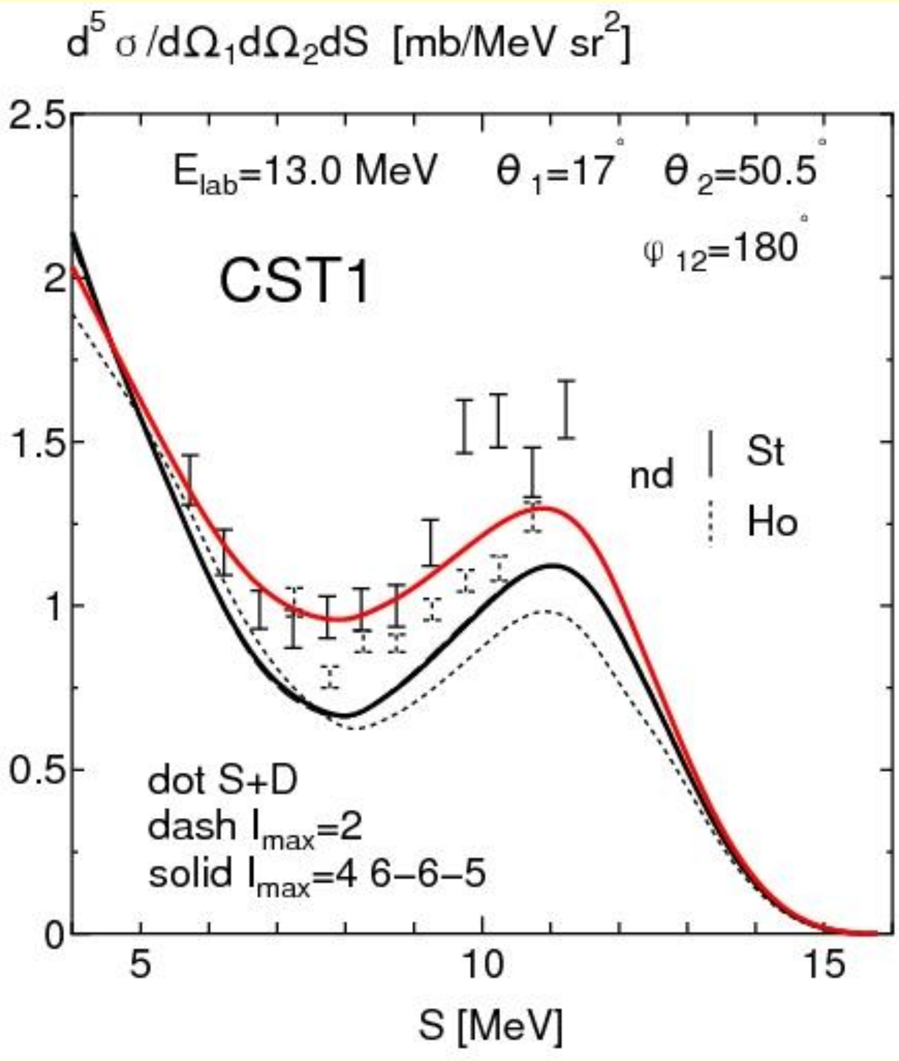
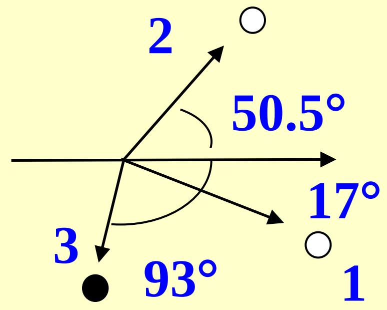


FIG. 14. (Color online) Differential cross sections of *pd*- and *nd*-breakup reactions for (a) the COL configuration, (b) the FSI configuration, (c) the SST configuration, and (d) the QFS configuration at $E_N = 13.0$ MeV. The bold curves are for *pd* scattering and the thin curves for *nd* scattering. The dashed curves denote the calculations with the AV18 potential, and the solid curves those with the AV18 + BR_{660} potential. Experimental data are from Ref. [41] (solid squares) for *pd* scattering and Ref. [42] (open circles) and Ref. [43] (solid circles) for *nd* scattering. The arrows indicate the kinematical points that match the typical configurations.

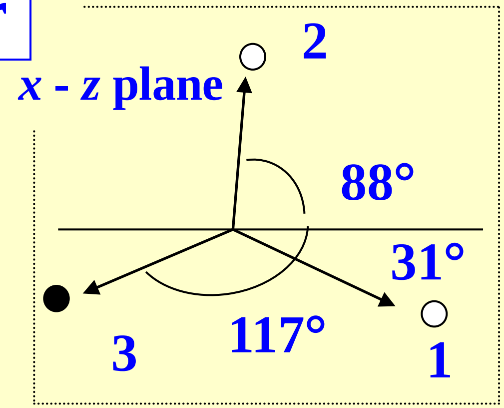
S. Ishikawa, Phys. Rev. C80, 054002 (2009)

Coplanar star

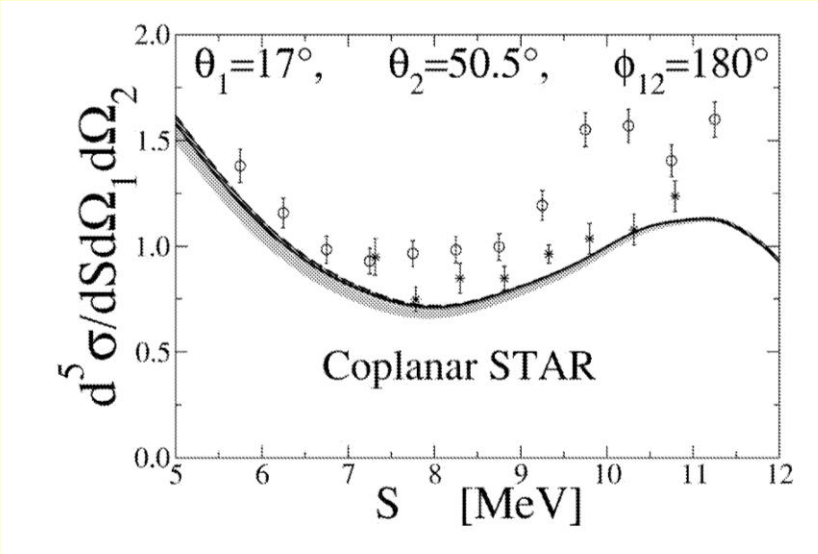




lab.

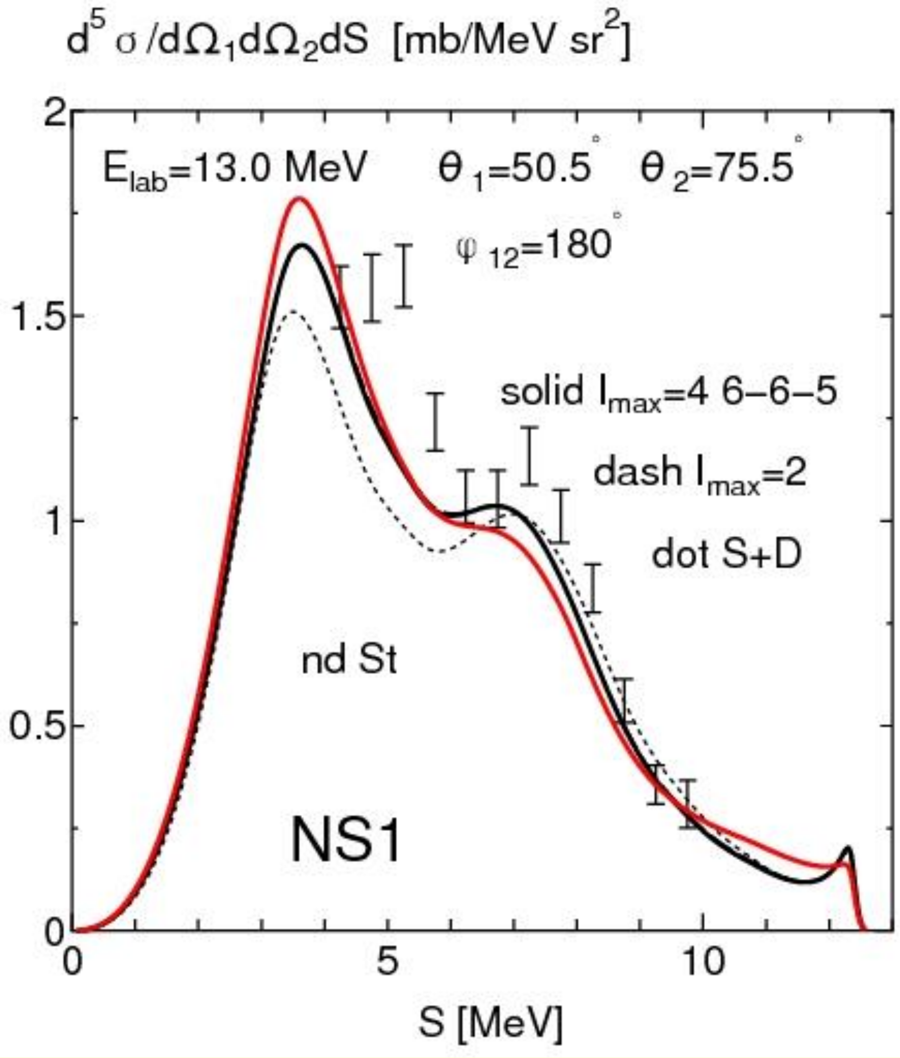


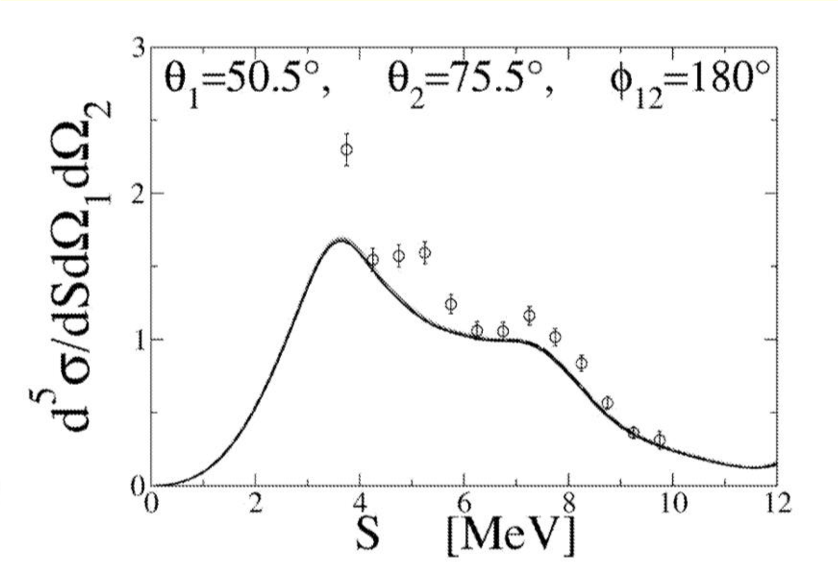
c.m.



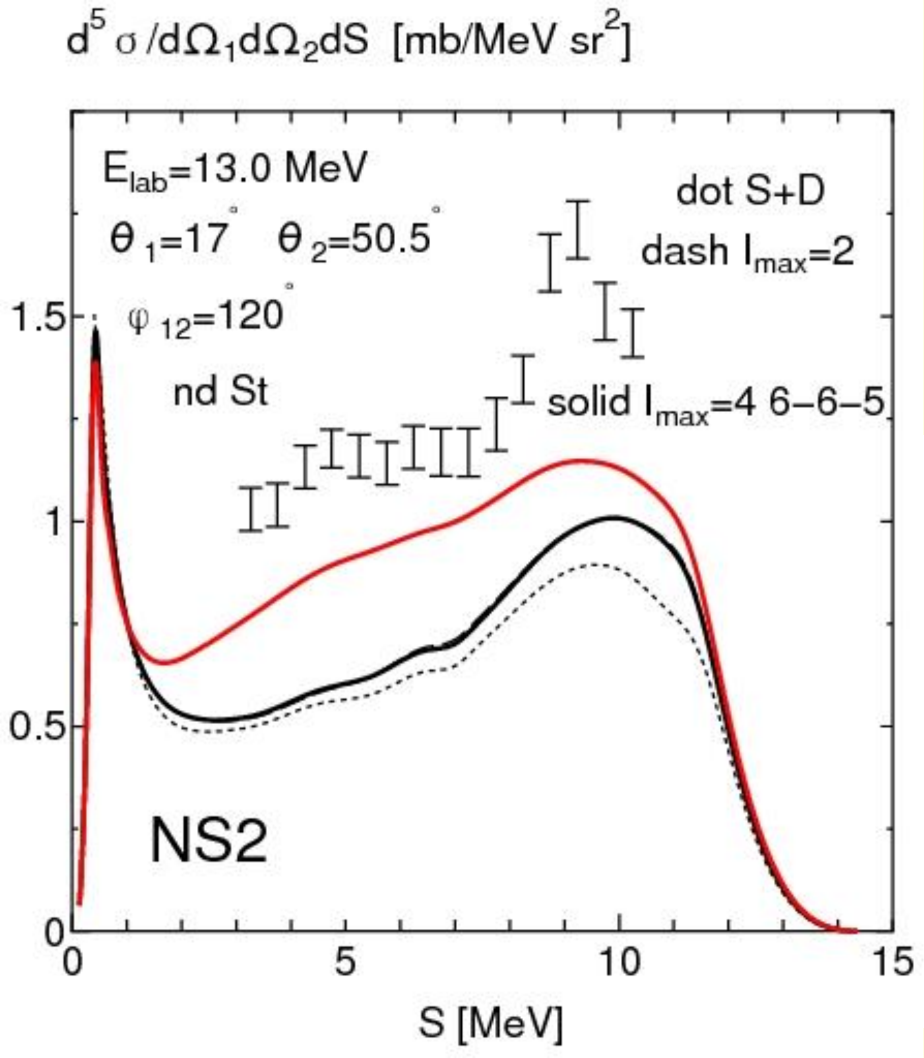
Kuroś-Żolnierczuk et al. (2002)

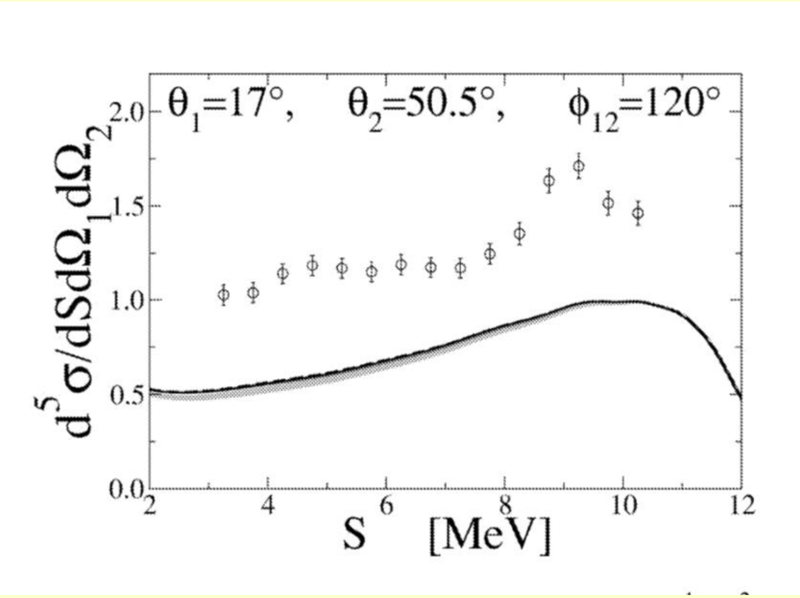
Non-standard configurations





Kuroś-Żolnierczuk et al. (2002)





Kuroś-Żolnierczuk et al. (2002)

結論

Coulomb 力の取り扱いを改良 (inc. breakup)

fss2 により *nd, pd* 散乱の大まかな特徴を E_N = 65 MeV 以下で再現 Coulomb 力は概ね実験との一致を改良

RGM kernel のエネルギー依存性を正しく扱うと

- 弾性散乱の微分断面積、スピン偏極量はほぼ再現される。

  しかし、A_y puzzle はまだ 3 MeV 以下で約 20% 程度の差がある。

  iT_{11} は低エネルギーサイド (~ 3 MeV) でクーロン力の影響が大きい。

  微分断面積の diffraction minimum は、高いエネルギーサイドで多少実験データより小さい。(論文 I の結果の訂正)

- breakup process の微分断面積に明らかに実験と合わないものが幾つか存在する。: symmetric space star (特に 13 MeV) および non-standard 配位の幾つか。22.7 MeV absolute value ? 19 MeV QFS?

  他 (QFS, FSI, CST,COLL) は、ほぼ合っている。Coulomb 力 : 要改良