クォーク模型バリオン間相互作用 fss2 の核子−重陽子弾性散乱への適用

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1. Introduction
• Motivation

Y. Fujiwara, Y. Suzuki and C. Nakamoto: Prog. Part. Nucl. Phys. 58 439(2007)

QMPACK homepage http://qmpack.homelinux.com/~qmpack/index.php

- **Few-nucleon systems are most appropriate to study the underlying nuclear interaction**
- **Ambiguity of the *NN* and 3*N* force in the 3*N* system**
- Different way of describing the short range repulsion

Meson-exchange **phenomenological repulsive core**

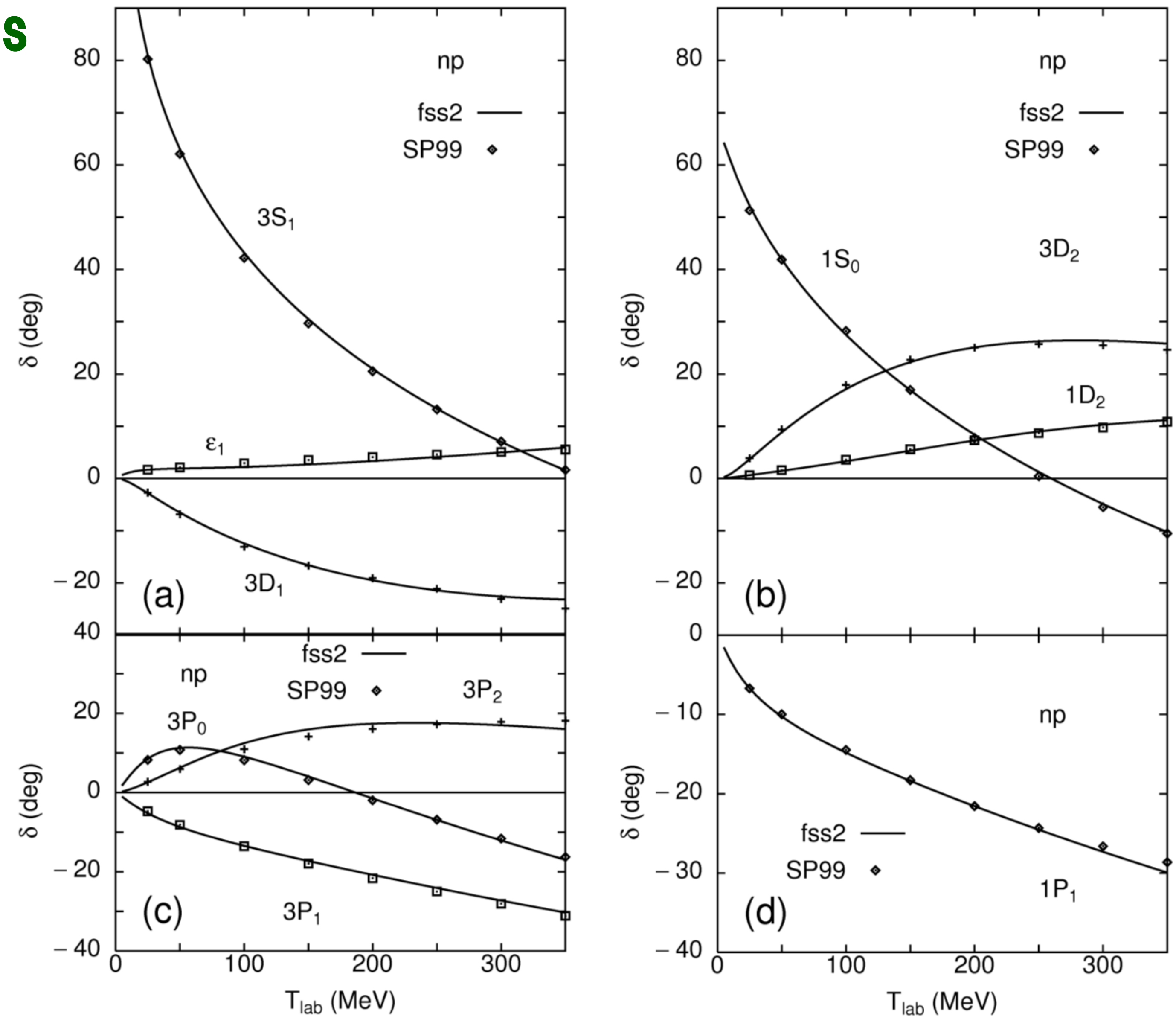
Quark model **nonlocal exchange kernel (from the 3-quark structure)**

The (*3q*)-(*3q*) RGM (Resonating-group Method)

$$\left\langle \phi(3q)\phi(3q) \,|\, E-H \,|\, \mathcal{A}\left\{\phi(3q)\phi(3q)\chi(\mathbf{r})\right\}\right\rangle = 0$$

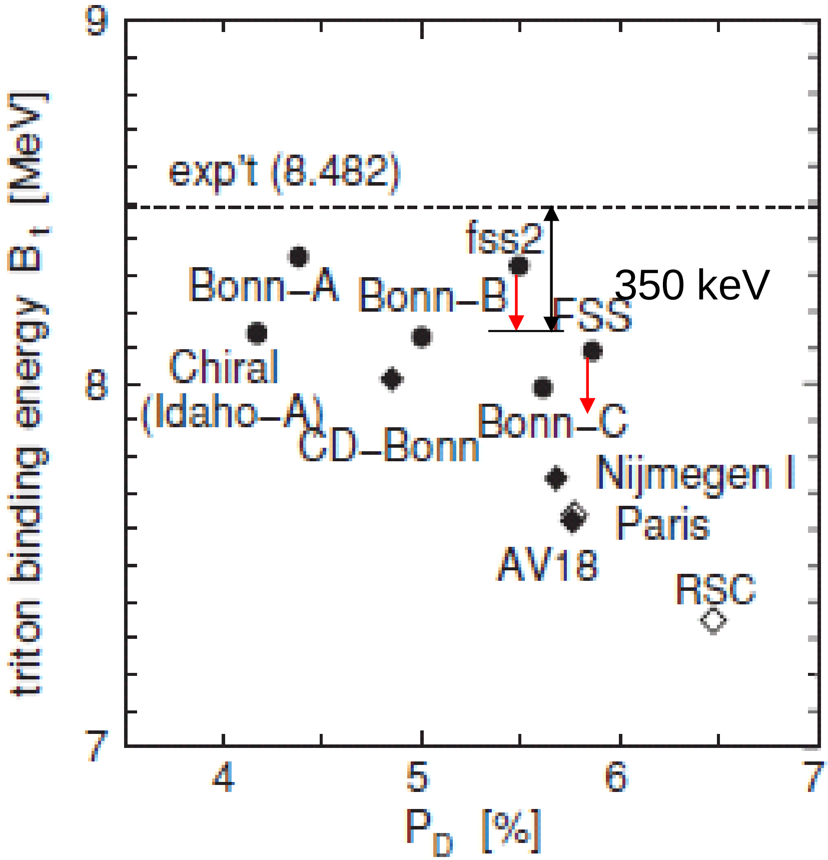
- We therefore construct the nonlocal Gaussian potential based on the fss2 and study **the effect of non-locality** and **the off-shell effect** of the short-range *NN* interaction by the QM BB interaction.

NN phase shifts by fss2



Y. Fujiwara, M. Kohno, C. Nakamoto and Y. Suzuki, Phys. Rev. C64, 054001 (2001).

^{3}H binding energy predicted by fss2



Deuteron D−state probability

Lack of energy

Meson exchange **0.5 MeV~1 MeV**

fss2 (Quark model) ~ **0.35 MeV**

The 3*N* force certainly exist, **but its effect on the ^{3}H binding energy might not be as large as generally believed from the meson−exchange potential.**

How well QM NN interaction describes the 3−body scattering system ?

($J^\pi = 1/2^+, 1/2^-, 3/2^+ \cdots\cdots$)

Triton channel

Y. Fujiwara et al. PRC77 (2008) 027001

◆ : take into account the charge dependence

● : do not take into account charge dependence of the order of 190 keV

R. Machleidt, Adv. Nucl. Phys. 19 (1989) 189

2. Formulation

- **Alt-Grassbarger-Sandahs (AGS) equarion**

$$U|\phi\rangle = G_0^{-1}P|\phi\rangle + PtG_0U|\phi\rangle \quad \text{with } |\phi\rangle = |\mathbf{q_0}, \psi_d\rangle$$

$$T|\phi\rangle = tP|\phi\rangle + tPG_0T|\phi\rangle, \; T = tG_0U$$

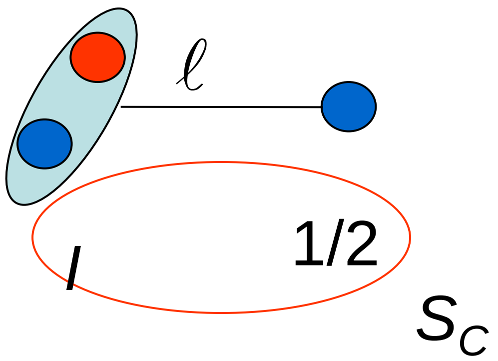
U : three-body scattering matrix $\quad P = P_{(123)} + P_{(123)}^2$

t : NN t-matrix $\quad G_0$: three-body Green function for free motion

- **Channel-spin formalism**

Channel coupling, LS coupling, jj coupling

$$\ell\left[(\lambda s)I\frac{1}{2}\right]S_c;\; JJ_z;\left(0\frac{1}{2}\right)\frac{1}{2}T_z$$



S_C=3/2 (quartet channel) **Pauli principle**

S_C=1/2 (doublet channel) **Deuteron distortion effect**

- ***S* - matrix** (3×3 or 2×2 matrix)

S-matrix can be parameterized in terms of the eigenphase shift δ_{ℓ,S_c}^J and mixing parameters.

$$S_{(\ell S_c),(\ell' S'_c)}^J = \delta_{\ell,\ell'}\delta_{S_c,S'_c} - 4\pi i\frac{q_0 M}{3\hbar^2}U_{(\ell S_c),(\ell' S'_c)}^J$$

3. Results

3.1 The *J*-averaged *nd* central phase shift

$$\delta^C\left({}^{2S_c+1}\ell\right)=\frac{1}{(2\ell+1)(2S_c+1)}\sum_J(2J+1)\delta^J_{\ell,S_c}$$

The *J*-splitting between the eigenphase shifts with the same (ℓS_c) is very small.

Model	fss2	AV18	fss2	AV18	fss2	AV18
E_n(MeV)	1.0	1.0	2.0	2.0	3.0	3.0
^2S	−14.9	−18.1 (−14.3)	−24.3	−28.3 (−24.0)	−30.8	−35.3 (−30.8)
^2P	−4.09	−4.10	−6.45	−6.46	−7.21	−7.28
^2D	0.560	0.562	1.49	1.50	2.32	2.34
^2F	−0.063	−0.063	−0.250	−0.249	−0.461	−0.463
^4S	−46.7	−46.7	−60.6	−60.8	−69.6	−69.9
^4P	13.0	13.2	21.2	21.6	25.3	25.7
^4D	−1.05	−1.05	−2.73	−2.74	−4.15	−4.18
^4F	0.127	0.127	0.509	0.508	0.948	0.950

^2S phase shifts in parentheses predictions by AV18 + Urbana 3N potential

AV18 (+UR 3N). A. Kievsky et al. NPA 607 ,402 (1996).

^2S : 1. very similar to the AV18+UR 3*N* potential

2. more attractive than AV18

^4S and higher partial wave: The difference from the AV18 or AV18+UR3N is small.

Good correspondence of phase shifts between the fss2 and the AV18+UR3N

3.2 Effective range expansion

Larger triton binding energy —— smaller doublet scattering length

Correlation

A. C. Phillips, Rep. Prog. Phys. 40, 905(1977)

If this correlation holds for the QM NN interaction, we can expect that the good reproduction of the *nd* doublet scattering length.

L. Schlessinger, Phys, Rev. 167 (1968), 1411

$$K(q) = \frac{K(q_1)}{1+}\,\frac{a_1(q^2-q_1^2)}{1+}\,\frac{a_2(q^2-q_2^2)}{1+}\cdots\frac{a_N(q^2-q_N^2)}{1} \qquad K(q) = q_0 \cot\delta$$

$$K(q_1),\cdot\ \cdot\ \cdot\ \cdot, K(q_{N+1}) \rightarrow a_1, a_2, \cdot\ \cdot\ \cdot\ \cdot, a_{N+1}$$

Merit

1. convenient for approximating a function with a pole

(Pole structure of the doublet effective range function)

L.M. Delves, PR 118, 1318 (1960)

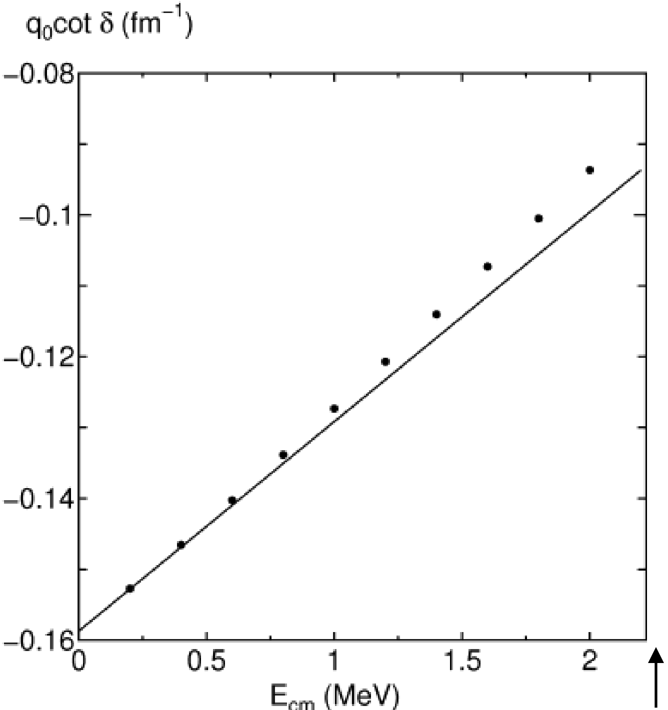
$$K(q_0) = \frac{-\frac{1}{a} + \frac{1}{2}\tilde{r}_e q_0^2 + \mathcal{O}(q_0^4)}{1+(q_0/q_Q)^2}$$

2. taking into account the contributions of higher terms

nd effective-range parameters

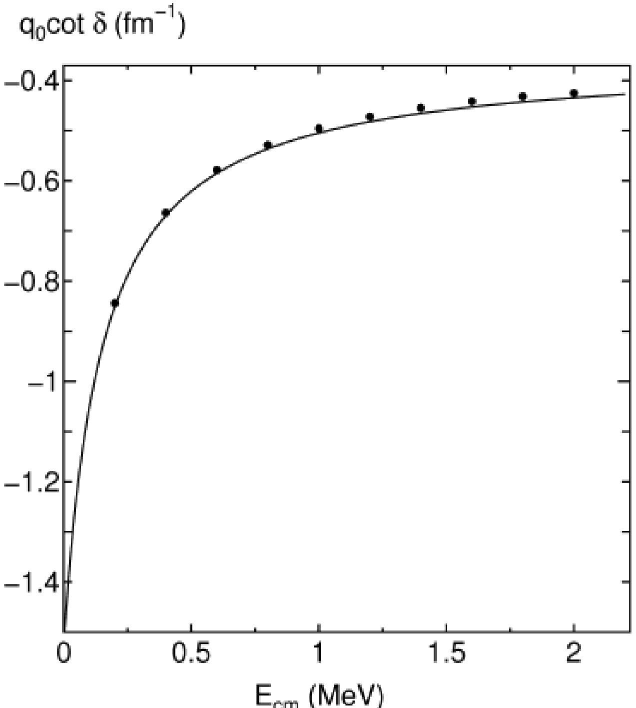
Quartet S-channel

$$K(q_0) = -\frac{1}{a} + \frac{1}{2} r_e q_0^2 + \mathcal{O}(q_0^4)$$



Doublet S-channel

$$K(q_0) = \frac{-\frac{1}{a} + \frac{1}{2}\tilde{r}_e q_0^2 + \mathcal{O}(q_0^4)}{1 + (q_0/q_Q)^2}$$



Deuteron breakup threshold

${}^{4}a_{nd}$ =**6.30** fm (exp. **6.35±0.02** fm)

$\left({}^{4}r_e\right)_{nd}$=**1.84** fm

W. Dilg et al. PL 36B, 208 (1971)

Pole position: $-\frac{3q_0^2}{4M} \approx -0.13$ MeV.

${}^{2}a_{nd}$ =**0.66** fm (exp. **0.65±0.04** fm)

$\left({}^{2}\tilde{r}_e\right)_{nd}$ =**−148 fm**

nd scattering lengths: Comparison with other calculations

model	$E_B(^3H)$	(MeV)	$^2a_{nd}$ (fm)		$^4a_{nd}$ (fm)
	NN	NN+TM99	NN	NN+TM99	NN(+TM99)
fss2	8.307	—	0.66	—	6.30
CD-Bonn 2000	8.005	8.482	0.925	0.569	6.347
AV18	7.628	8.482	1.248	0.587	6.346
Nijm I	7.742	8.485	1.158	0.594	6.342
exp	8.482		0.65 ± 0.04		6.35 ± 0.02

Doublet scattering length

Ref. H. Witala et al. PRC68 ,034002 (2003).

- Quark-model baryon-baryon interaction (fss2) ~ **0.6 fm** (without CIB)
- meson exchange potential **1.0 ~1.3 fm**
- meson-exchange potentials + TM 3N force ~ **0.6 fm**

The effect of charge independence breaking : 0.13 fm
(estimated from the Phillips line for fss2)

$^2a_{nd}$ predicted by fss2 : 0.76fm ~ 0.80 fm

The model fss2 almost reproduces ^{3}H binding energy and $^2a_{nd}$
(consistent with the results for ^{2}S phase shifts)

The description of NN short-range part influences deuteron distortion effect

3.3 Proton-Deuteron Scattering

Because of its 1/r nature, the 3-body problem including the Coulomb force is challenging.

"Richness and precision of the pd experimental data lead to the stringent test of the nuclear force" **E.O. Alt et al. PRC 65, 064613 (2002).**

Treatment of the Coulomb force

- **Coulomb externally corrected approximation**

$$f(\theta) = \left| f^C(\theta) + \sum_\ell (2\ell+1) e^{2i\sigma_\ell} f_\ell^N P_\ell(\cos\theta) \right|$$

P. Doleschall et al. NPA 380, 72 (1982).

($f^C(\theta)$: Rutherford scattering amplitude between proton and deuteron)

Advanced treatment in recent years

- **Variational method** **A. Kievsky, M. Viviani and S. Rosati, PRC 64, 024002 (2001).**
- **Screening Coulomb and Renormalization method**

(Faddeev, momentum representation)

$$w_R(r) = \frac{\alpha}{r} e^{-(r/R)^n} \quad (R \to \infty)$$ **A. Deltuva, A.C. Fonseca and P.U. Sauer, PRC 71, 054005 (2005)**

- Sasakawa-Sawada's method (Faddeev, coordinate representation)

S. Ishikawa, Few-Body Syst. 32, 229 (2003). T. Sasakawa and T. Sawada, PRC 20, 1954 (1979).

•Our method

Vincent-Phatak approach to the AGS Equation

C. M. Vincent and S. C. Phatak, PRC 10, 391 (1974).

Merit

1. **The method to obtain the accurate nuclear phase shifts for short-range nuclear force**
2. **We can choose the relatively small cut-off radius R.**

Problem

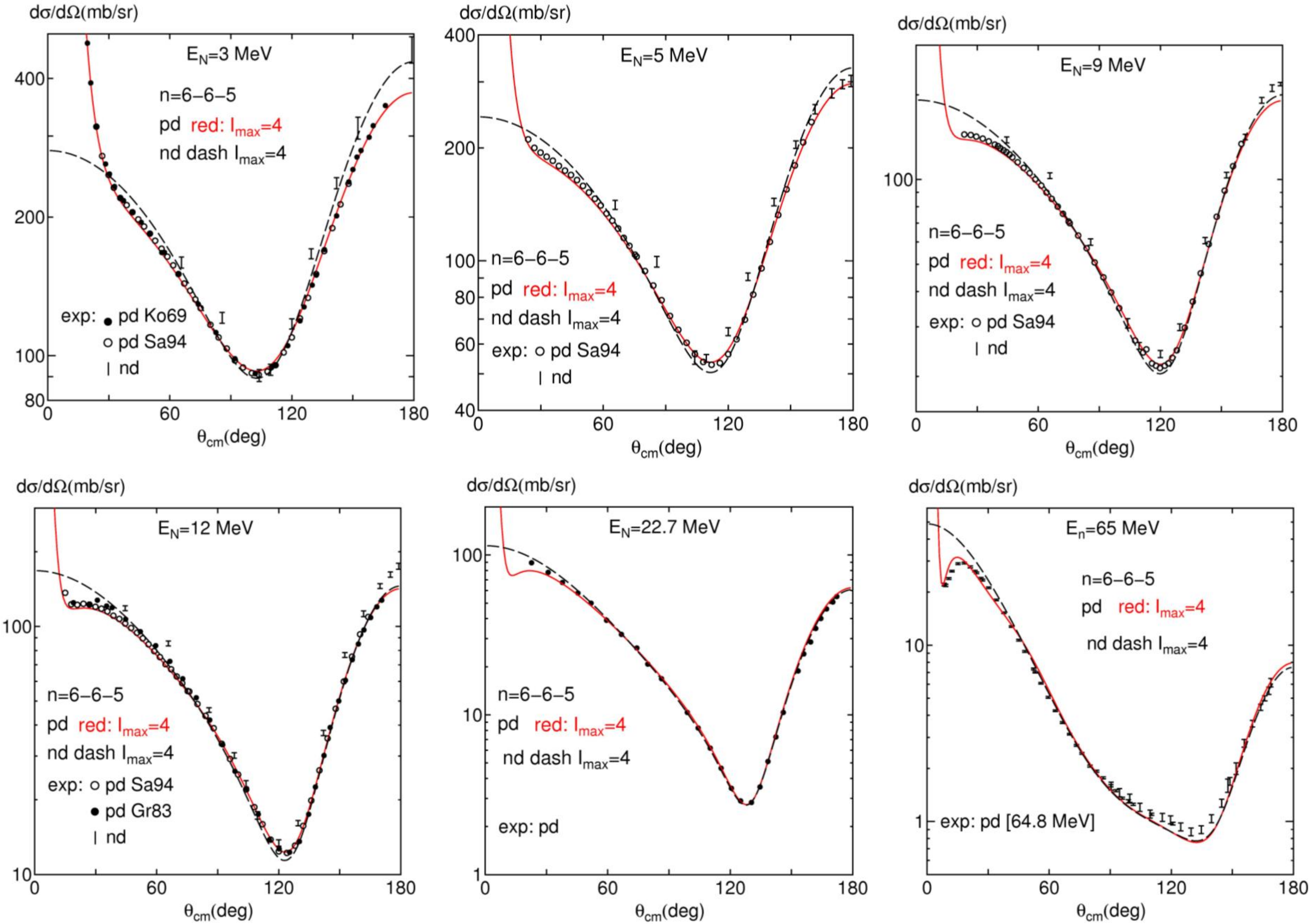
How we choose the most appropriate cut off-scheme ?

(Cut-off radius R, the convergence with respect to the partial waves)

Coulomb force

- Sharp cut-off Coulomb at the quark level $\frac{1}{r}\theta(R-r)$
- Screened error function at the nucleon level
- Folded Coulomb potential by realistic deuteron wave function

Differential cross sections



Nucleon vector analyzing Power $A_y(\theta)$

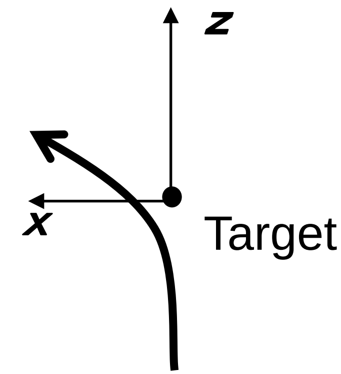
A_y puzzle (Vector analyzing power puzzle)

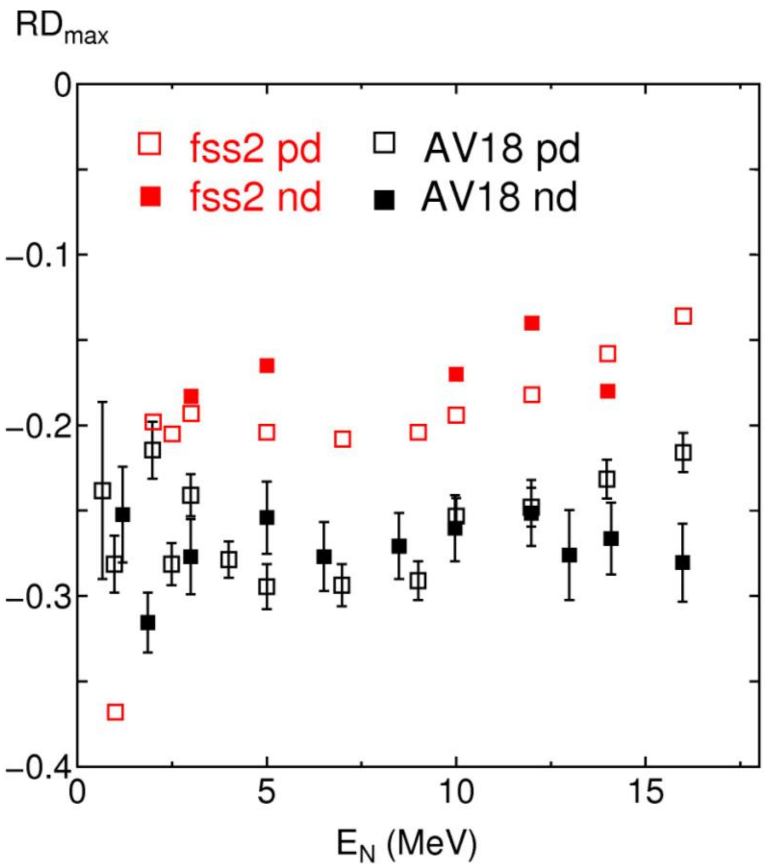
- Nucleon analyzing power $A_y(\theta)$ at the peak is **exceptional too small by more than 20%** in the meson exchange potentials

$$RD_{\max} = \frac{A^{th}_{y\,\max} - A^{ex}_{y\,\max}}{A^{ex}_{y\,\max}}$$

$$p_y A_y = \frac{N_L - N_R}{N_L + N_R}$$

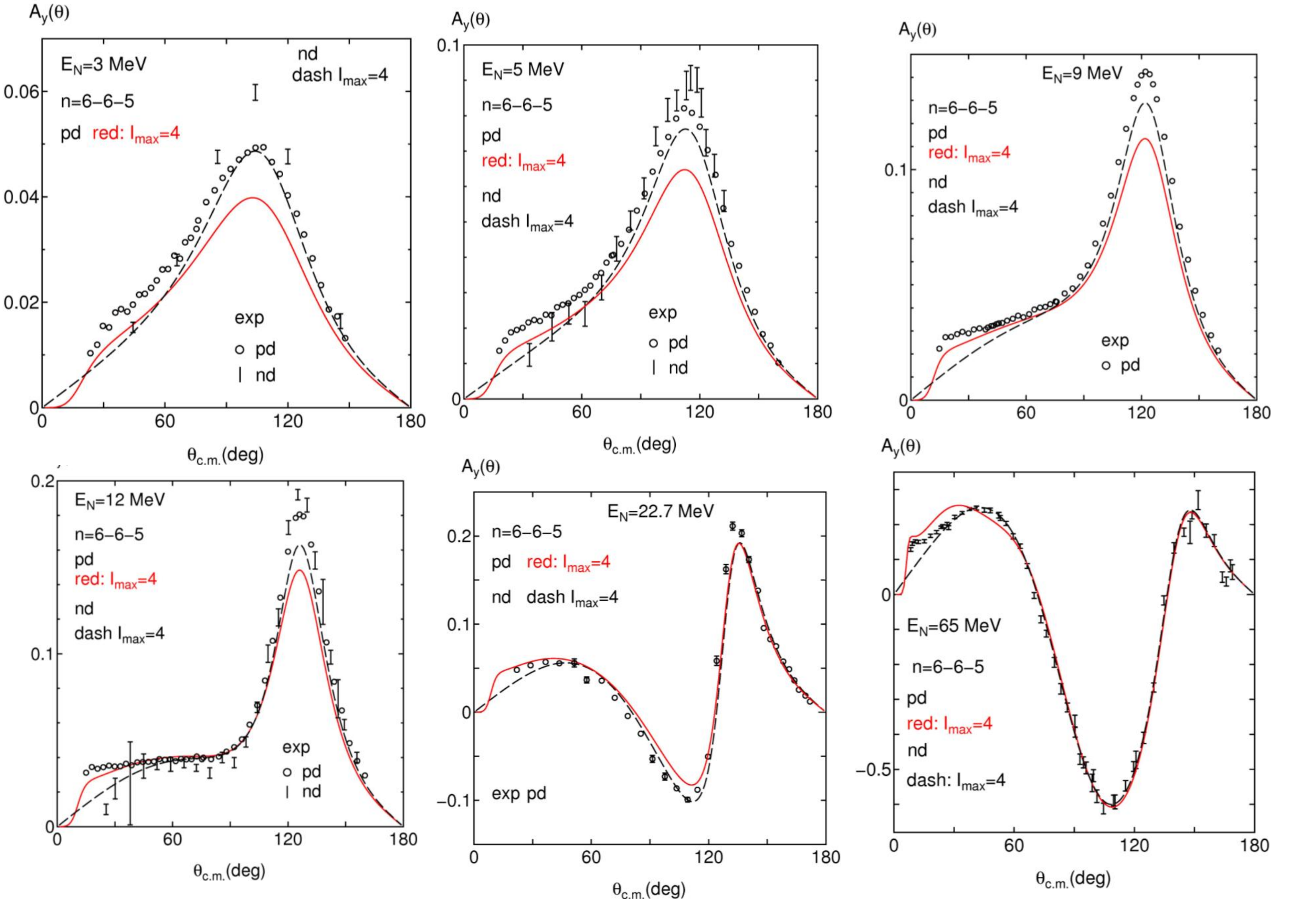
$$A_y(\theta)\boldsymbol{e}_y \equiv \frac{Tr\{f\boldsymbol{\sigma}f^\dagger\}}{Tr\{ff^\dagger\}}$$



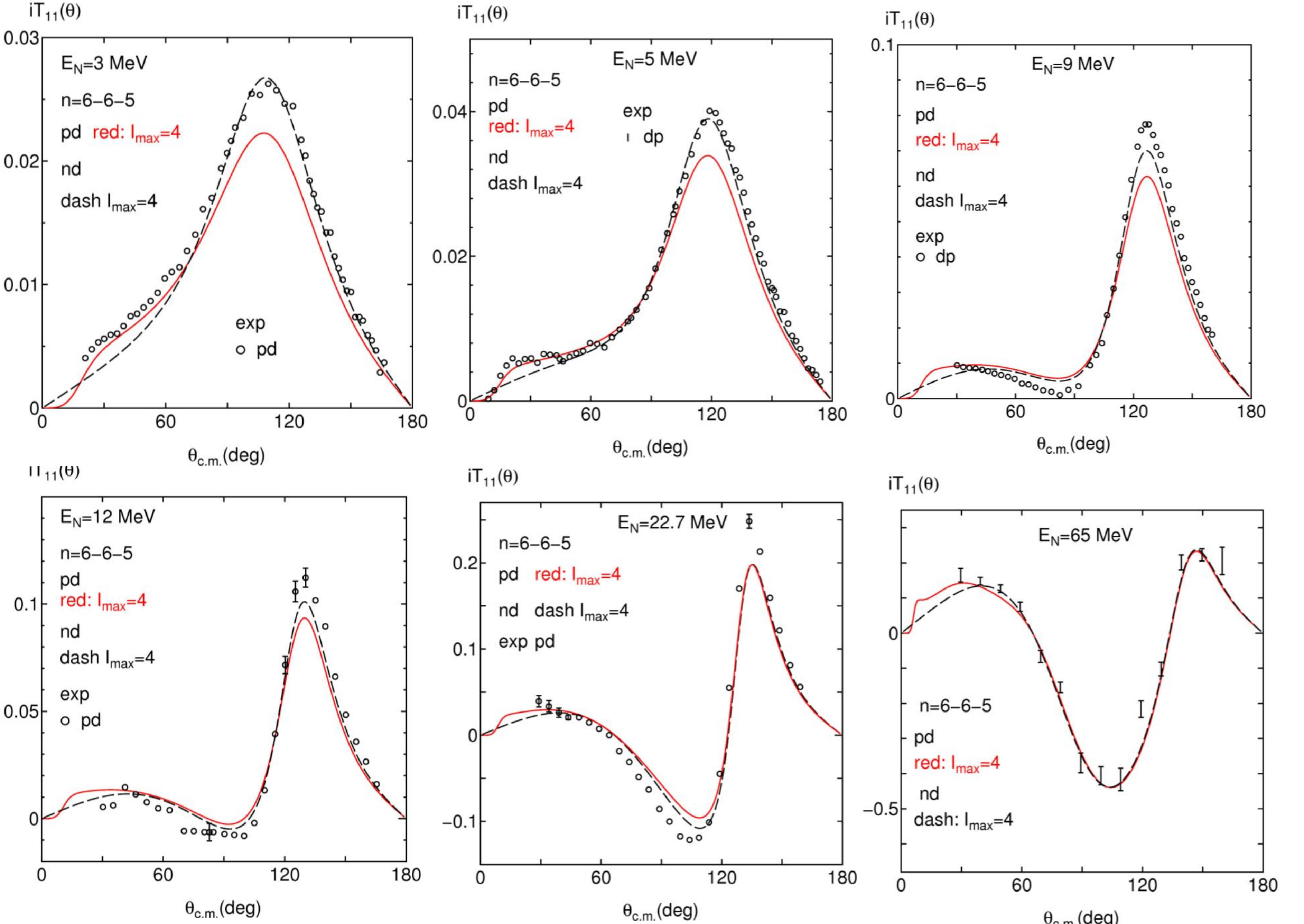


W. Tornow, J. Phys. G: Nucl. Part. Phys. 35 (2008), 125104

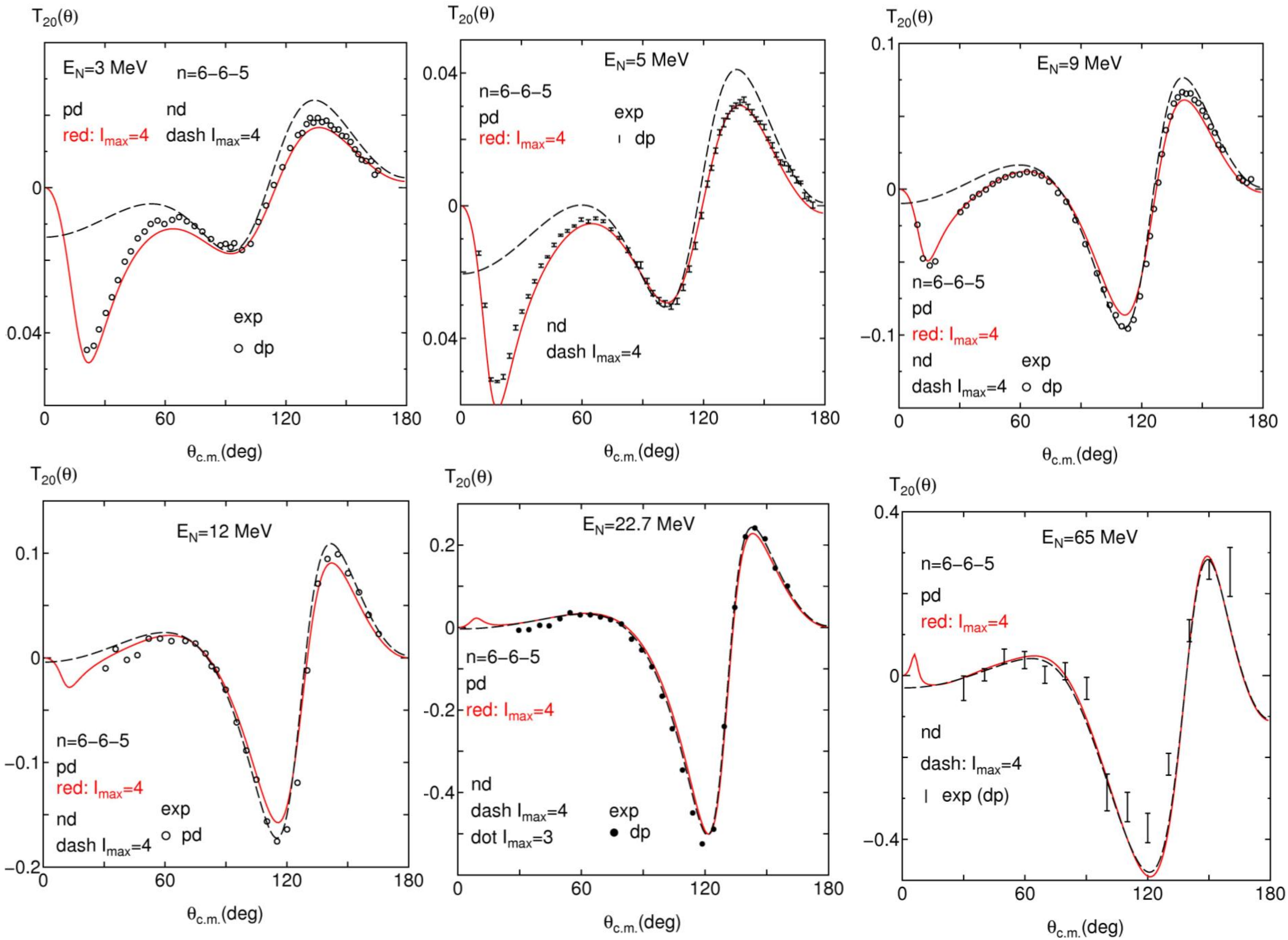
Nucleon vector analyzing Power $A_y(\theta)$



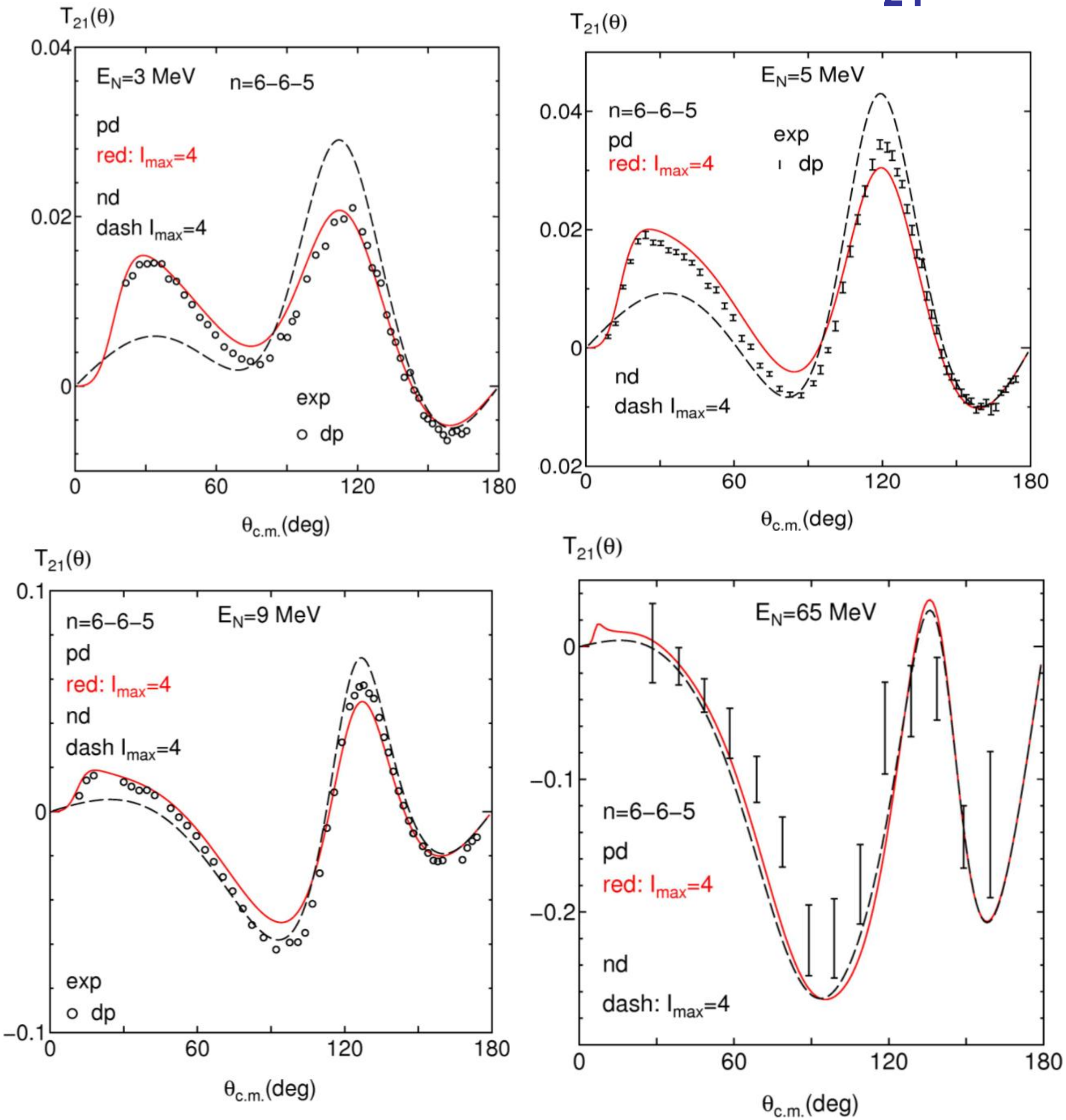
Deuteron vector analyzing Power $iT_{11}(\theta)$



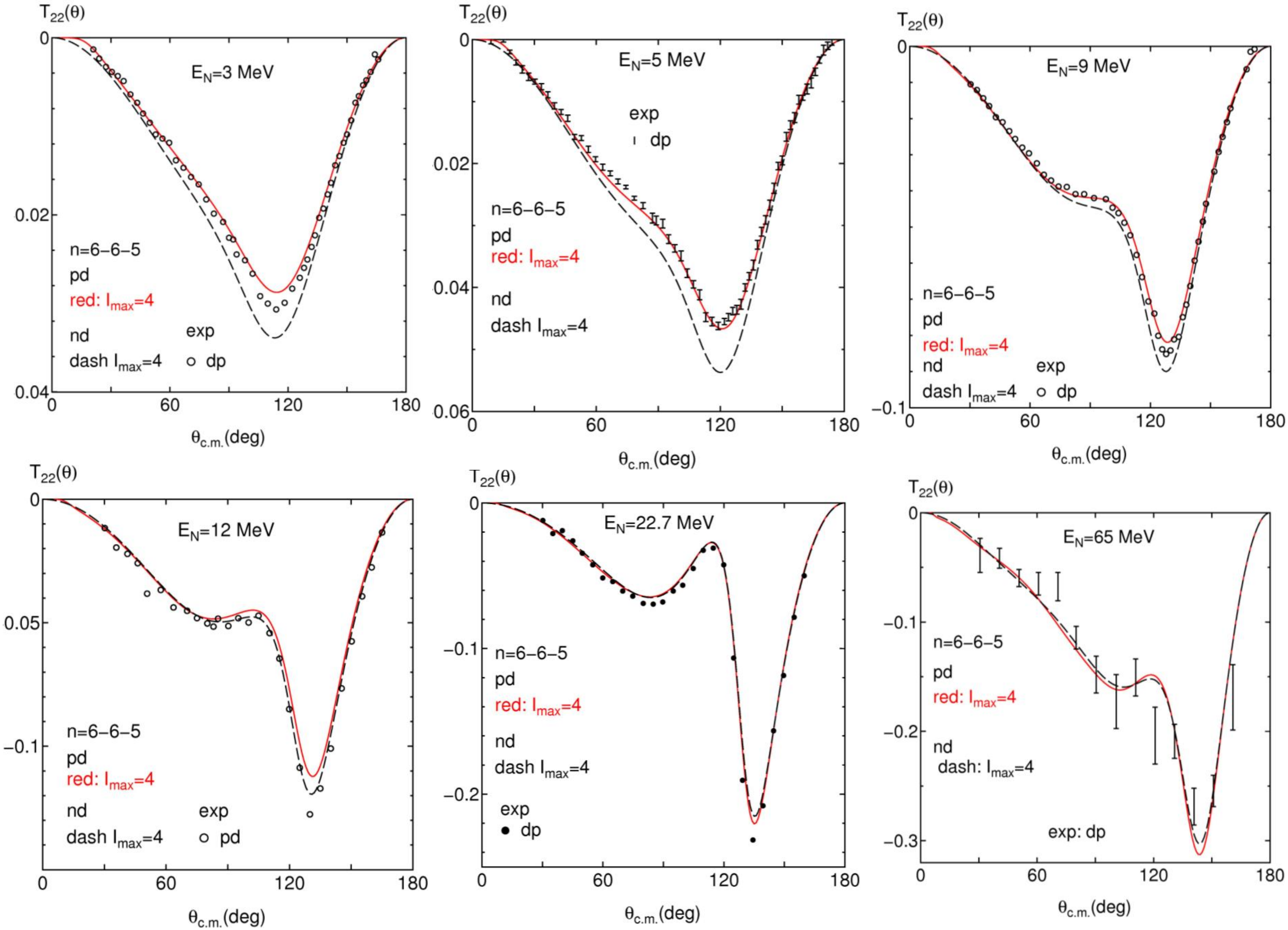
Deuteron tensor analyzing Power $T_{20}(\theta)$



Deuteron tensor analyzing Power $T_{21}(\theta)$



Deuteron tensor analyzing Power $T_{22}(\theta)$



4. Summary & Future work

Summary

1. We have applied the QM BB interaction fss2 to the *nd* and *pd* scattering, and calculated the *nd* scattering lengths, differential cross sections and many polarization observables in the low energy region $E_N \leq 65\,\mathrm{MeV}$.

2. **The model fss2 is attractive enough in the $^2S_1$ channel below the deuteron breakup threshold.**

(The triton binding energy, the eigenphase shift and $^2a_{nd}$)

This is the result of the nonlocal effect of the QM BB interaction.

3. The deficiency of $A_y(\theta)$ at the maximum point is about 15-20% for $E_N \geq 2\,\mathrm{MeV}$.

Future work

Intermediate energy region ($E_N = 100 \sim 250\ \mathrm{MeV}$)

(The parallelization of the computer program has to be made because we need to take more partial waves into account)