RIA analysis of proton-elastic scattering from He isotopes

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Relativistic Impulse Approximation (RIA)

Tjon & Wallace, PRC32(1985)1667, PRC35(1987)280, PRC36(1987)1085

- optical potentials

Dirac equation for a projectile proton scattering from a target nucleus given by the optical model:

$$[\not{p} - m - \hat{U}(\mathbf{r})]\,\psi(\mathbf{r}) = 0,$$

$$\not{p} = \gamma_\mu p^\mu,\ p^\mu = (E, \mathbf{p}),\ \hbar = c = 1$$: natural unit

the momentum space Dirac equation

$$\left(\gamma^0 E - \boldsymbol{\gamma}\bullet\mathbf{p}' - m\right)\psi(\mathbf{p}') - \frac{1}{(2\pi)^3}\int d^3p\,\hat{U}(\mathbf{p}',\mathbf{p})\,\psi(\mathbf{p}) = 0$$

the generalized RIA optical potential

in the momentum space

$$\hat{U}(\mathbf{p}',\mathbf{p}) = -\frac{1}{4}\mathrm{Tr}\left\{\int \frac{d^3k}{(2\pi)^3}\,\hat{M}_{pp}(\mathbf{p},\mathbf{k}-\frac{\mathbf{q}}{2}\rightarrow \mathbf{p}',k+\frac{\mathbf{q}}{2})\,\hat{\rho}_p(\mathbf{k},\mathbf{q})\right\}$$

$$-\frac{1}{4}\mathrm{Tr}\left\{\int \frac{d^3k}{(2\pi)^3}\,\hat{M}_{pn}(\mathbf{p},\mathbf{k}-\frac{\mathbf{q}}{2}\rightarrow \mathbf{p}',k+\frac{\mathbf{q}}{2})\,\hat{\rho}_n(\mathbf{k},\mathbf{q})\right\}$$

optimal factorization : $(\mathbf{k}=0)$

simple

tρ-form

$$\hat{U}(\mathbf{p}',\mathbf{p}) = -\frac{1}{4}\mathrm{Tr}\left\{\hat{M}_{pp}(\mathbf{p},-\frac{\mathbf{q}}{2}\rightarrow \mathbf{p}',\frac{\mathbf{q}}{2})\,\hat{\rho}_p(\mathbf{q})\right\}$$

$$-\frac{1}{4}\mathrm{Tr}\left\{\hat{M}_{pn}(\mathbf{p},-\frac{\mathbf{q}}{2}\rightarrow \mathbf{p}',\frac{\mathbf{q}}{2})\,\hat{\rho}_n(\mathbf{q})\right\}$$

- amplitudes : NN-interactions
  - IA2 : full expansion

scalar Feynman amplitude

Covariants in terms of Dirac matrices

$$\hat{M}(p_1, p_2 \to p_1', p_2')$$

$$= \sum_{\rho_1\rho_2\rho_1'\rho_2'} \Lambda_1^{\rho_1'}(p_1')\,\Lambda_2^{\rho_2'}(p_2')\left[\sum_{n=1}^{13} M_n^{\rho_1'\rho_2'\rho_1\rho_2}\,\mathrm{K}_n\right]\Lambda_1^{\rho_1}(p_1)\,\Lambda_2^{\rho_2}(p_2)$$

$$M_n^{\rho_1'\rho_2'\rho_1\rho_2} \Longleftrightarrow \sum_{m=1}^{4}\frac{(g_{nm}^{ij})^2}{\mu_m^2 - t}\,\frac{\Lambda^2}{\Lambda^2 - t}$$

sum of four Yukawa terms

$$\Lambda^{\rho}(p) = \frac{\rho(\gamma^0 E - \boldsymbol{\gamma}\cdot\mathbf{p}) + m}{2m}, \quad \rho = \pm 1$$

covariant projection operator

- densities

Dirac wave function for target nucleons :

$$\psi_{\alpha}(r) = \psi_{njl\mu t}(r) = \begin{bmatrix} G_{njl\mu t}(r) \\ -i\sigma_{r} F_{njl\mu t}(r) \end{bmatrix} Y_{lj}^{\mu}(\hat{\mathbf{r}})$$

density matrix :

$$\hat{\rho}(r) = \rho_S(r) + \gamma^0 \rho_V(r) - \frac{i\hat{\boldsymbol{\alpha}} \cdot \hat{\mathbf{r}}}{2} \rho_T(r)$$

$$\hat{\rho}(q) = \int d^3r \, e^{i\mathbf{q}\cdot\mathbf{r}} \hat{\rho}(r)$$

$$= \rho_S(q) + \gamma^0 \rho_V(q) - \frac{i\hat{\boldsymbol{\alpha}} \cdot \mathbf{q}}{2m} \rho_T(q)$$

Fourier transformation of the coordinate-space density:

$$\begin{cases} \rho_S(q) = 4\pi \int_0^\infty dr\ r^2 j_0(qr)\ \rho_S(r) \\ \rho_V(q) = 4\pi \int_0^\infty dr\ r^2 j_0(qr)\ \rho_V(r) \\ \rho_T(q) = -4\pi \int_0^\infty dr\ r^2 \dfrac{j_1(qr)}{q} \rho_T(r) \end{cases}$$

coordinate-space density :

$$\begin{cases} \rho_S(r) = \sum_{nljt} \dfrac{2j+1}{4\pi} \left[G_{nljt}^2(r) - F_{nljt}^2(r) \right] \\ \rho_V(r) = \sum_{nljt} \dfrac{2j+1}{4\pi} \left[G_{nljt}^2(r) + F_{nljt}^2(r) \right] \\ \rho_T(r) = \sum_{nljt} \dfrac{2j+1}{4\pi} \left[4G_{nljt}(r) \times F_{nljt}(r) \right] \end{cases}$$

$j_m(qr)$: spherical Bessel function

Fourier transformation: $$\psi(\mathbf{p}') = \int d^3r \, e^{-i\mathbf{p}'\bullet\mathbf{r}} \, \psi(\mathbf{r})$$

Dirac equation:

$$\int d^3r \, e^{-i\mathbf{p}'\bullet\mathbf{r}} \left(\gamma^0 E + i\boldsymbol{\gamma}\bullet\nabla - m\right)\psi(\mathbf{r})$$

$$-\frac{1}{(2\pi)^3}\int d^3p \, \hat{U}(\mathbf{p}',\mathbf{p}) \int d^3r \, e^{-i\mathbf{p}'\bullet\mathbf{r}}\psi(\mathbf{r}) = 0$$

$$\left(\gamma^0 E + i\boldsymbol{\gamma}\bullet\nabla - m - \tilde{\hat{U}}(\mathbf{r})\right)\tilde{\psi}(\mathbf{r}) = 0,$$

$$\tilde{\hat{U}}(\mathbf{r}) = \tilde{S}(r) + \gamma^0\tilde{V}(r) - i\frac{\hat{\boldsymbol{\alpha}}\cdot\hat{\mathbf{r}}}{m}\tilde{T}(r) - \left[\tilde{S}_{LS}(r) + \gamma^0\tilde{V}_{LS}(r)\right]\boldsymbol{\sigma}\cdot\mathbf{L}$$

Schrödinger equivalent potential (IA1)

$$U_c = \frac{1}{2E}\left\{ \quad 2EV + 2mS - V^2 + S^2 - 2VV_C \right.$$

$$+\left(T^2 - \frac{T}{A}\frac{\partial A}{\partial r} + 2\frac{T}{r} + \frac{\partial T}{\partial r} \right)$$

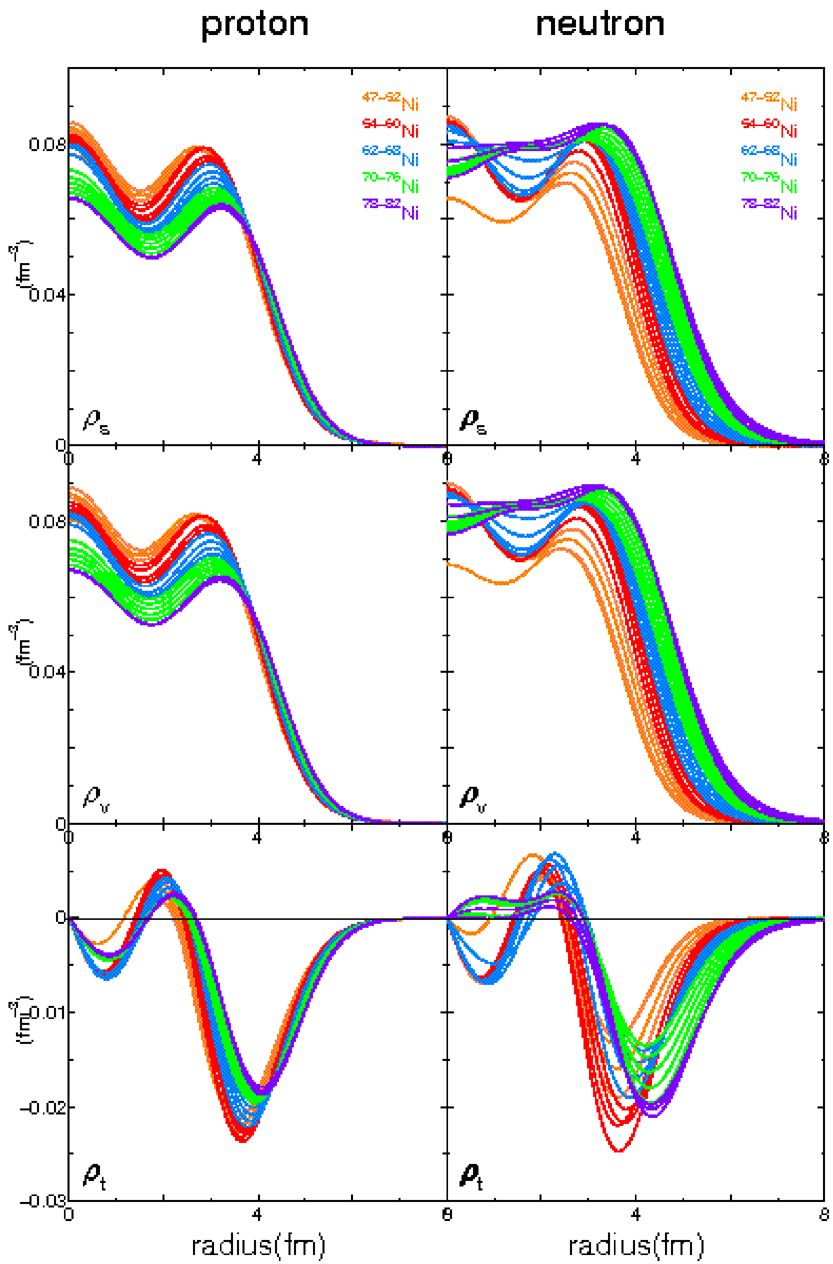
$$\left. +\left(-\frac{1}{2r^2 A}\frac{\partial}{\partial r}\left(r^2 \frac{\partial A}{\partial r} \right) + \frac{3}{4A^2}\left(\frac{\partial A}{\partial r} \right)^2 \right) \quad \right\}$$

$$U_{ls} = \frac{1}{2E}\left\{ \quad -\frac{1}{rA}\left(\frac{\partial A}{\partial r} \right) + 2\frac{T}{r} \quad \right\}$$

$$A = \frac{1}{E+m}\left\{ E - V + m + S - V_C \right\}$$

density distributions for Ni isotopes

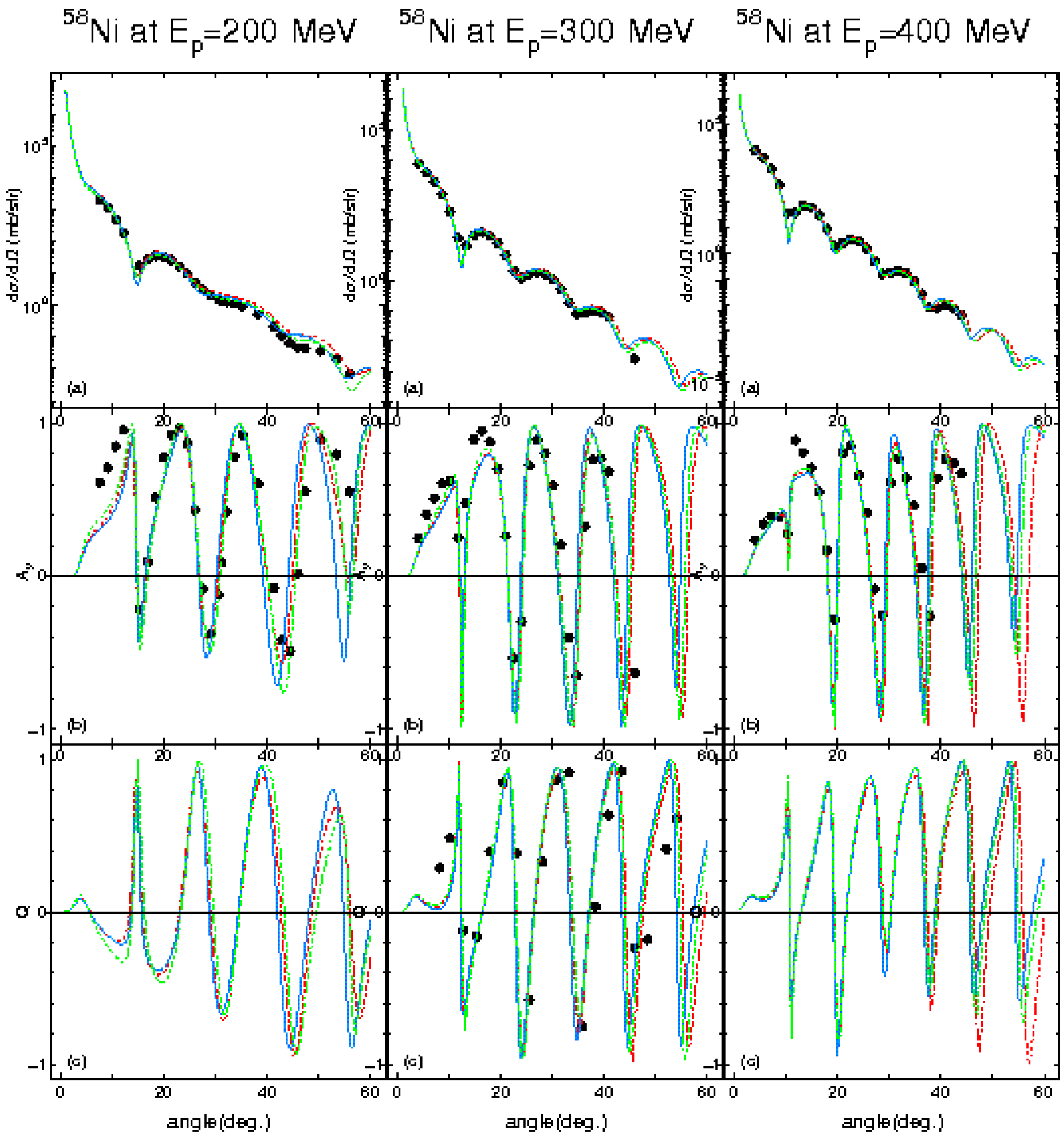
relativistic mean field theory (rmft)



TMA code :Y.Sugahara & H.Toki
NPA579 (1994) 557

Relativistic
Impulse
Approximation

^{58}Ni

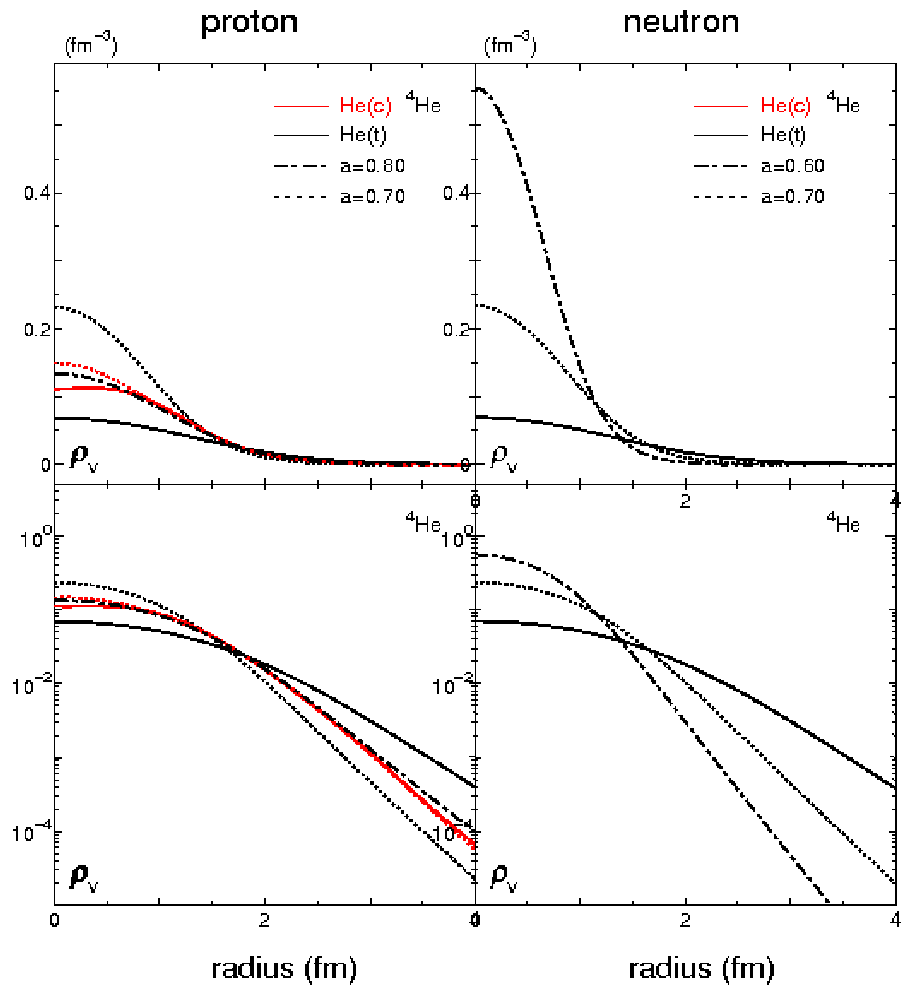


- - - 2nd
— 1st
-·- med.

exp. data
H.Sakaguchi et al.
PRC57(1998)1749

density distributions of ^{4}He

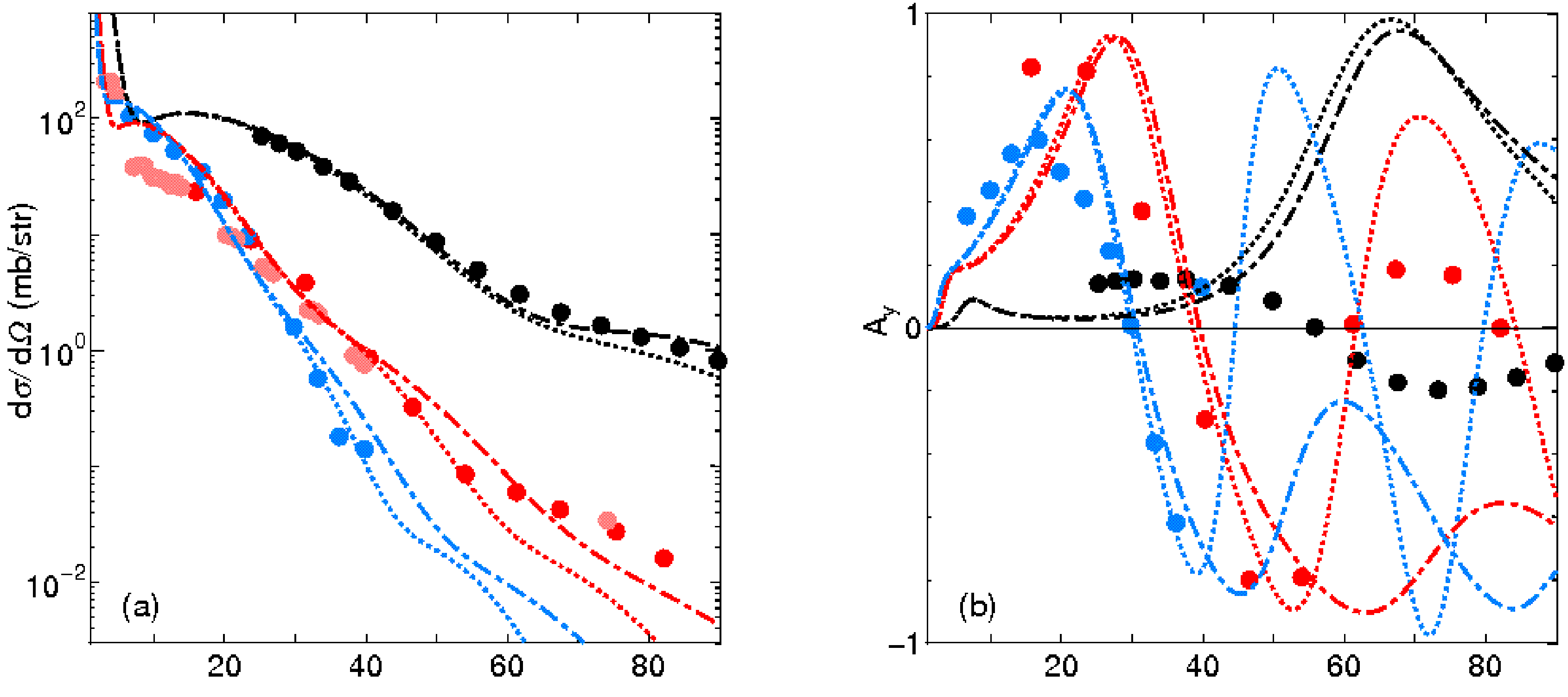
root-mean-square radius (fm)	proton	neutron
tma	2.150	2.137
tmav1	1.433	1.425
tmav2	1.720	1.068
charge	1.496	-

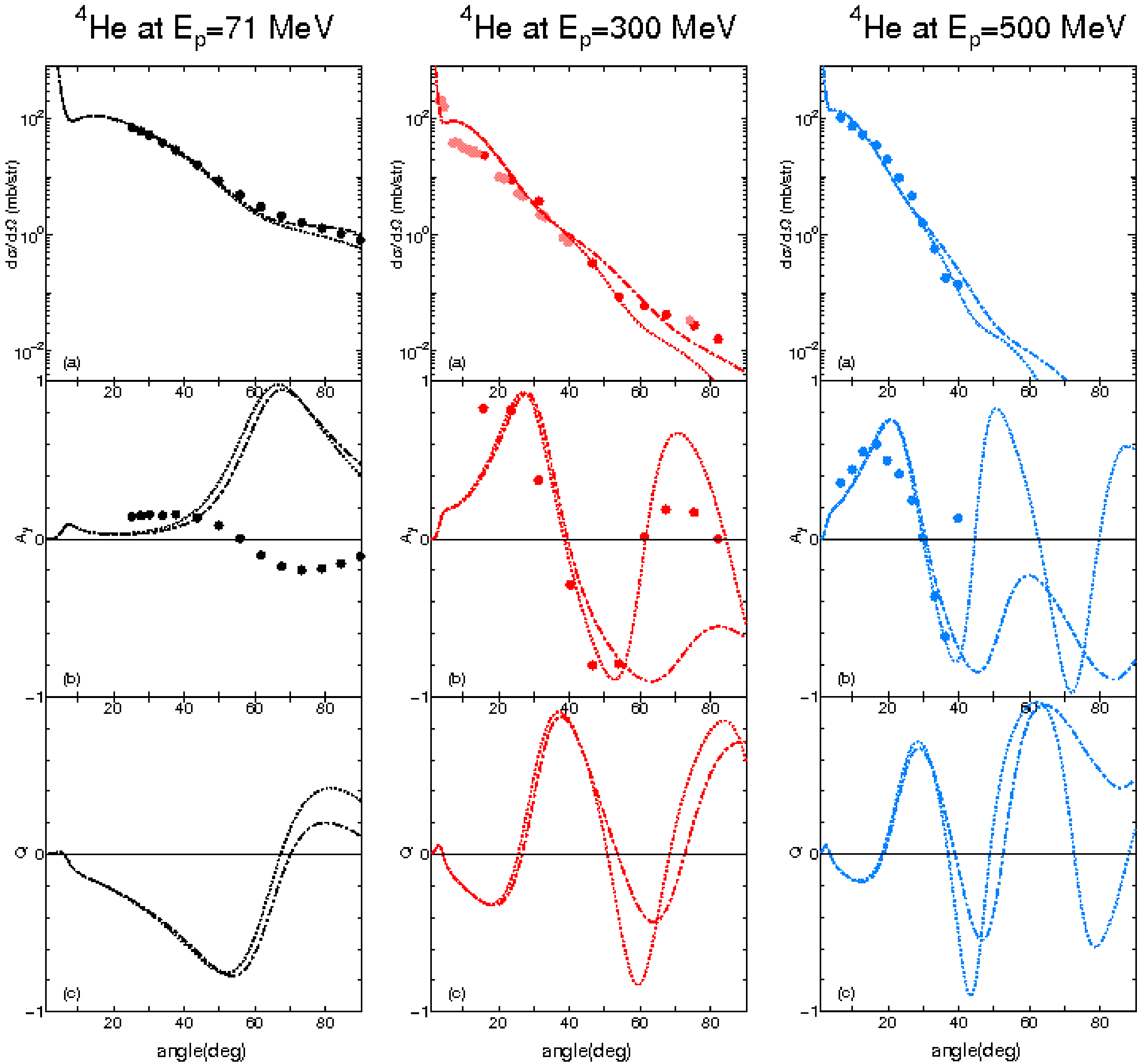


71MeV: S.Burzynski et al., PRC39 (1989) 56

300MeV:T.Yamagata et al., PRC74(2006)014309
: M.Yoshimura et al., PRC63(2001)034618

500MeV:S.M.Sterbenz et al., PRC45(1992)2578



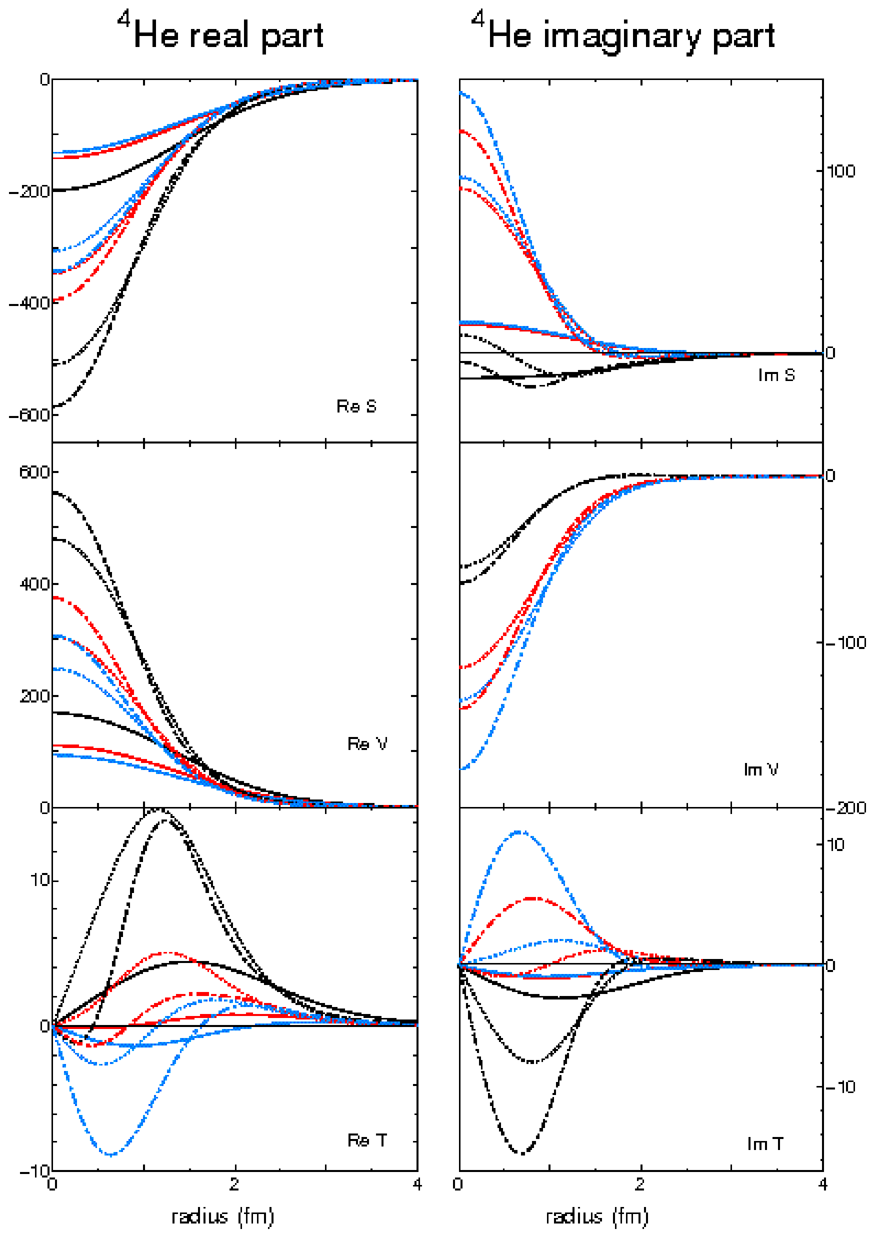


4He at Ep=71 MeV
4He at Ep=300 MeV
4He at Ep=500 MeV
dσ/dΩ (mb/str)
Ay
Q
(a)
(b)
(c)
angle(deg)

Dirac optical potential

tma
tmav1
tmav2

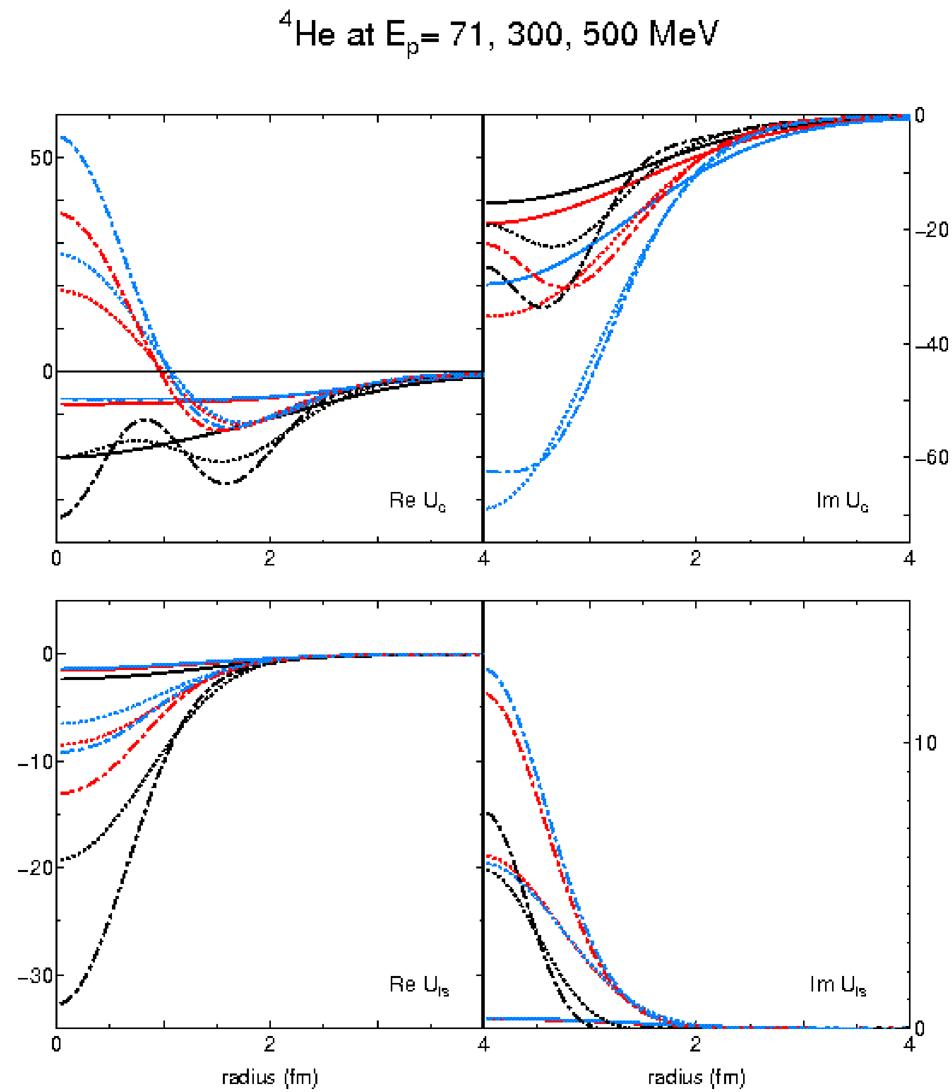
71 MeV
300 MeV
500 MeV



Schrödinger equivalent potential

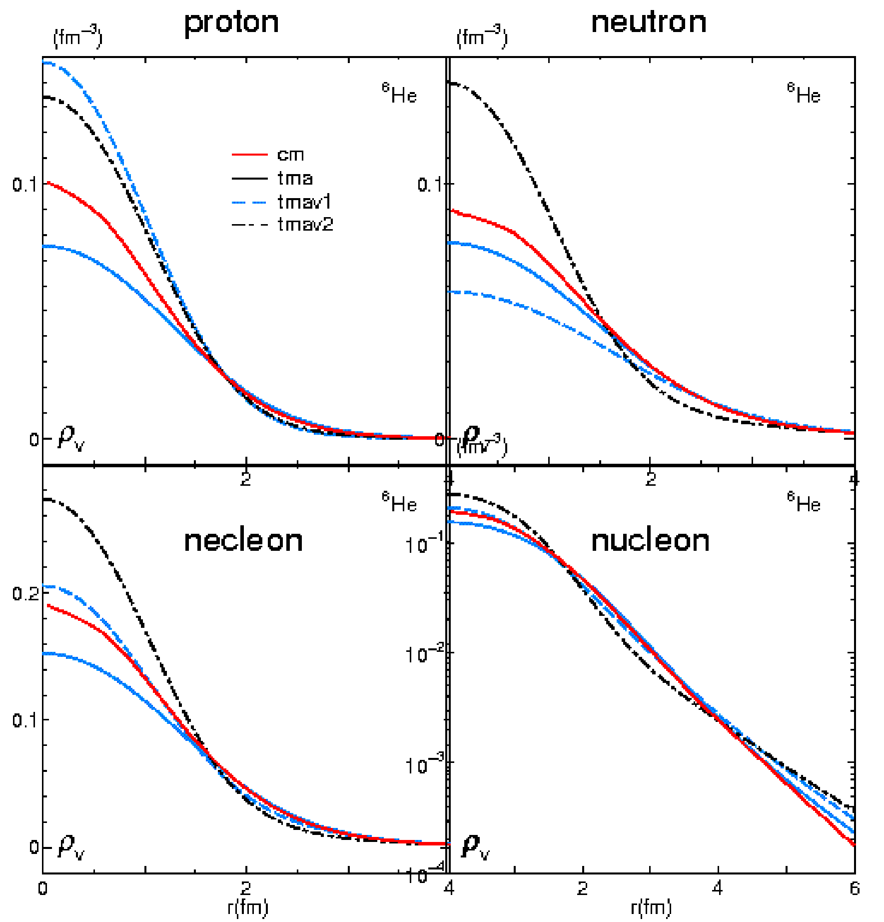
tma
tmav1
tmav2

71 MeV
300 MeV
500 MeV

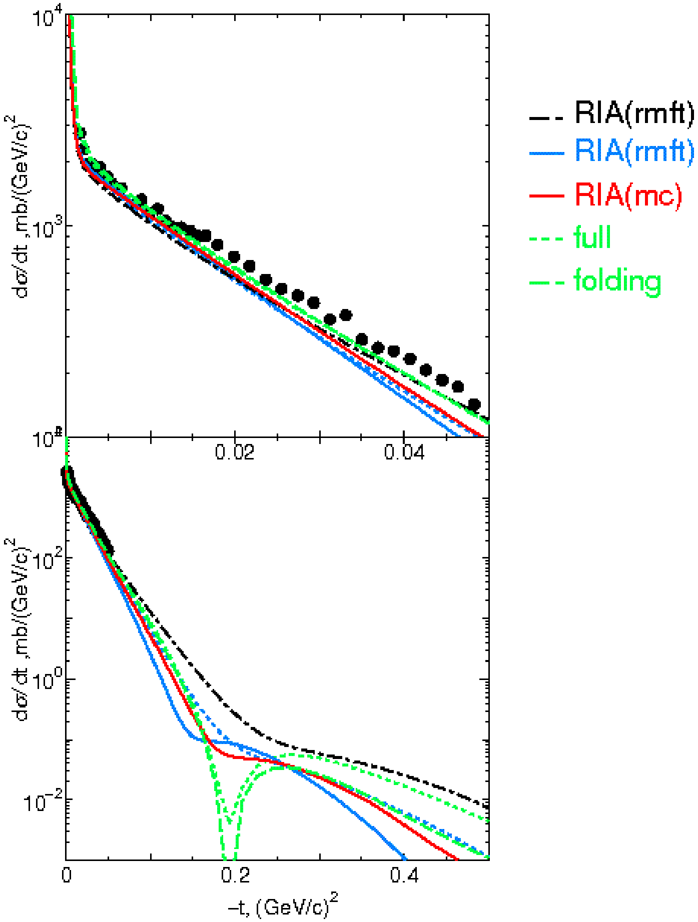


density distributions of ^{6}He

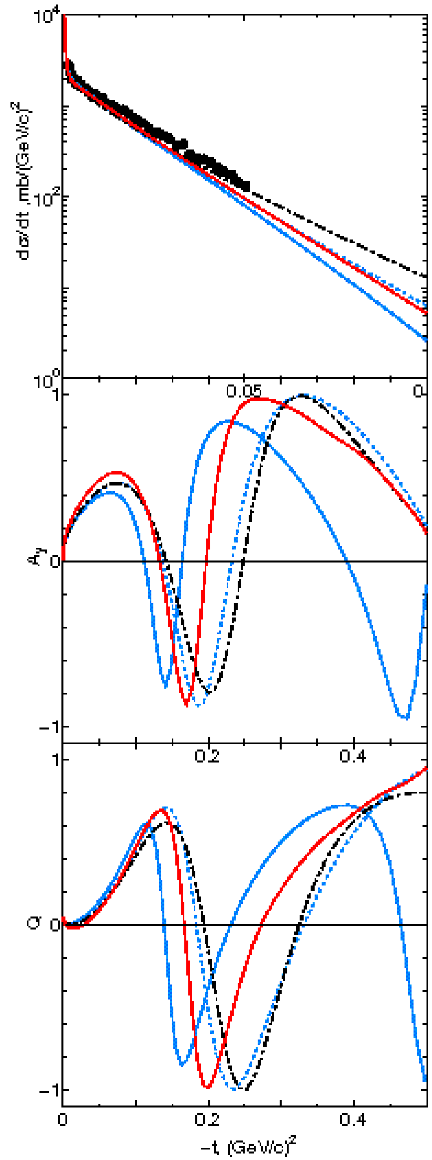
root-mean-square radius (fm)	proton	neutron
tma	2.044	3.054
mc	1.928	2.871
tmav1	1.635	3.352
tmav2	1.720	3.385
charge	1.955	-



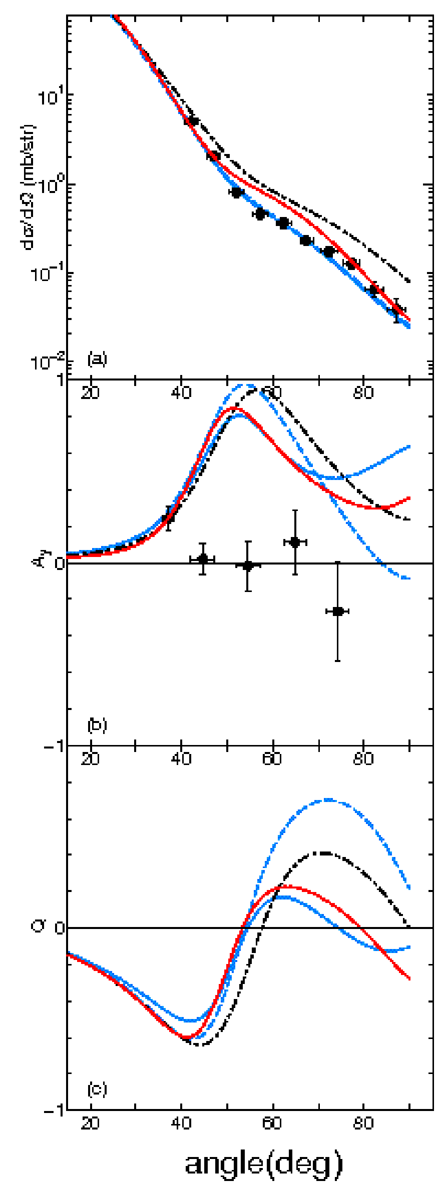
tmav1: p080a110
tmav2: pa080an12
mc: K.Varga et al., PRC66(2002)034611



^{6}He at E_p=0.7 GeV



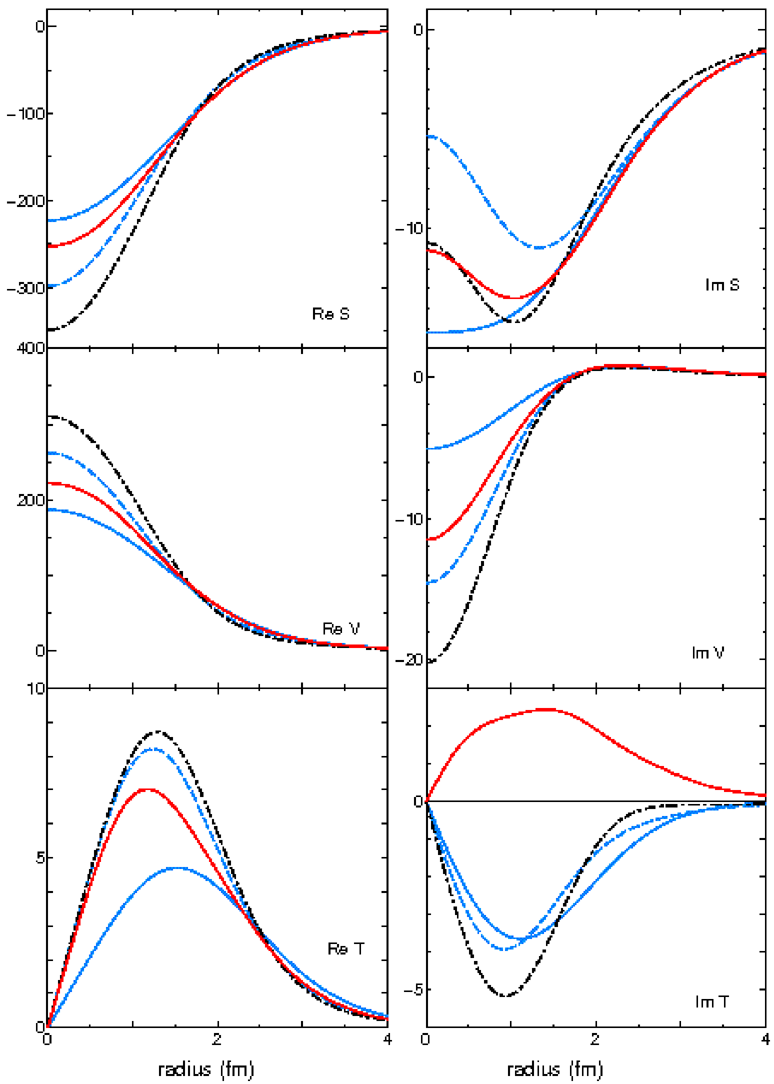
^{6}He at E_p=71 MeV



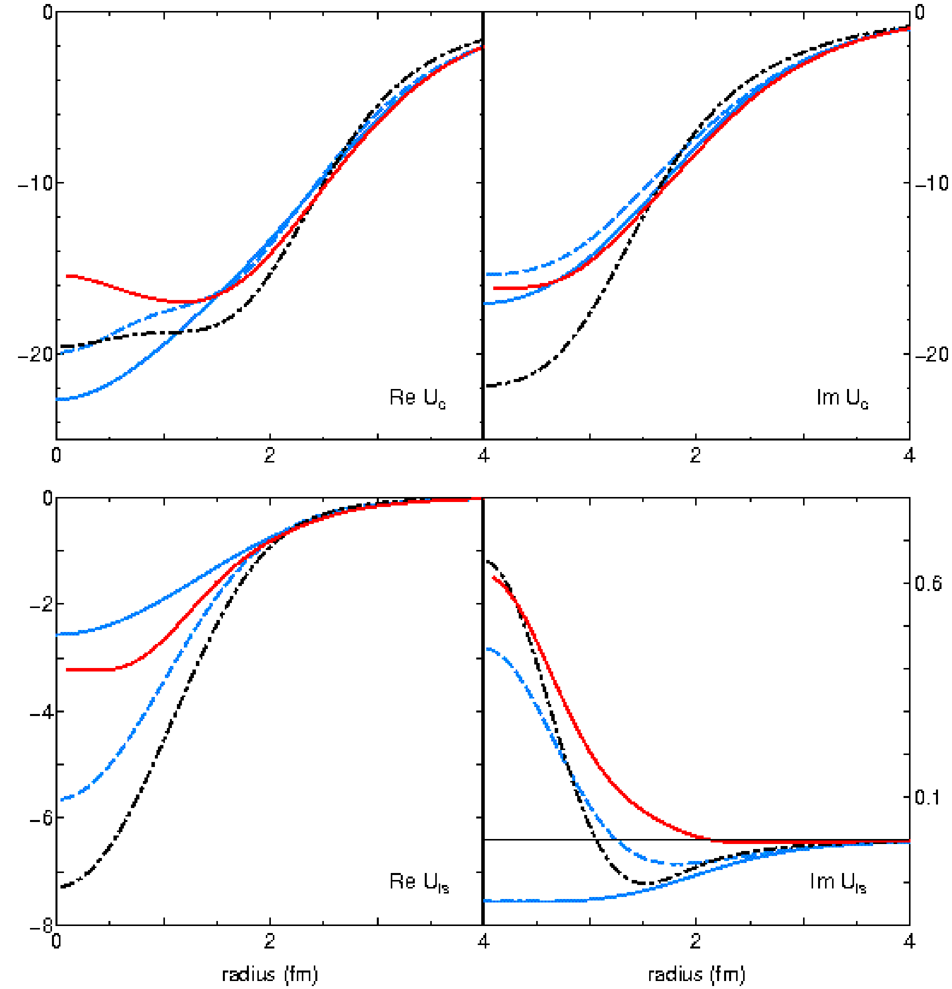
71MeV: T.Uesaka et al., PRC82(2010)021602R
0.7GeV: S.R.Neumaier et al.,NPA712(2002) 247

E_p= 71 MeV

^{6}He real part ^{6}He imaginary part

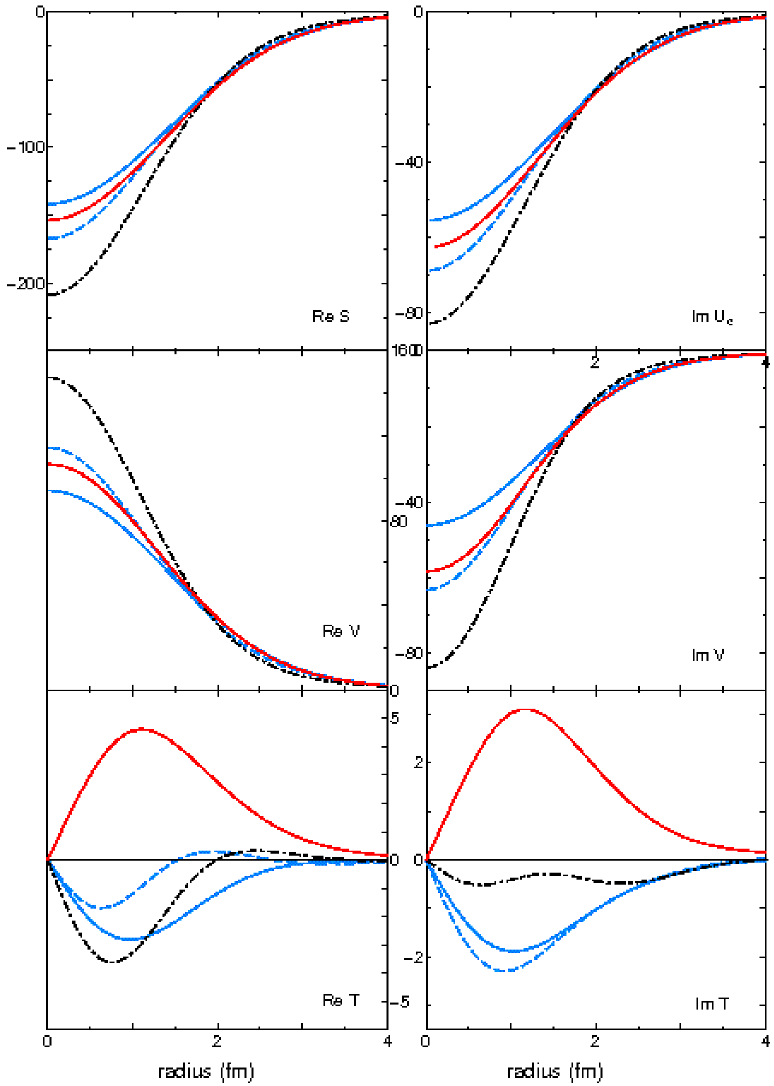


^{6}He at E_p= 71 MeV

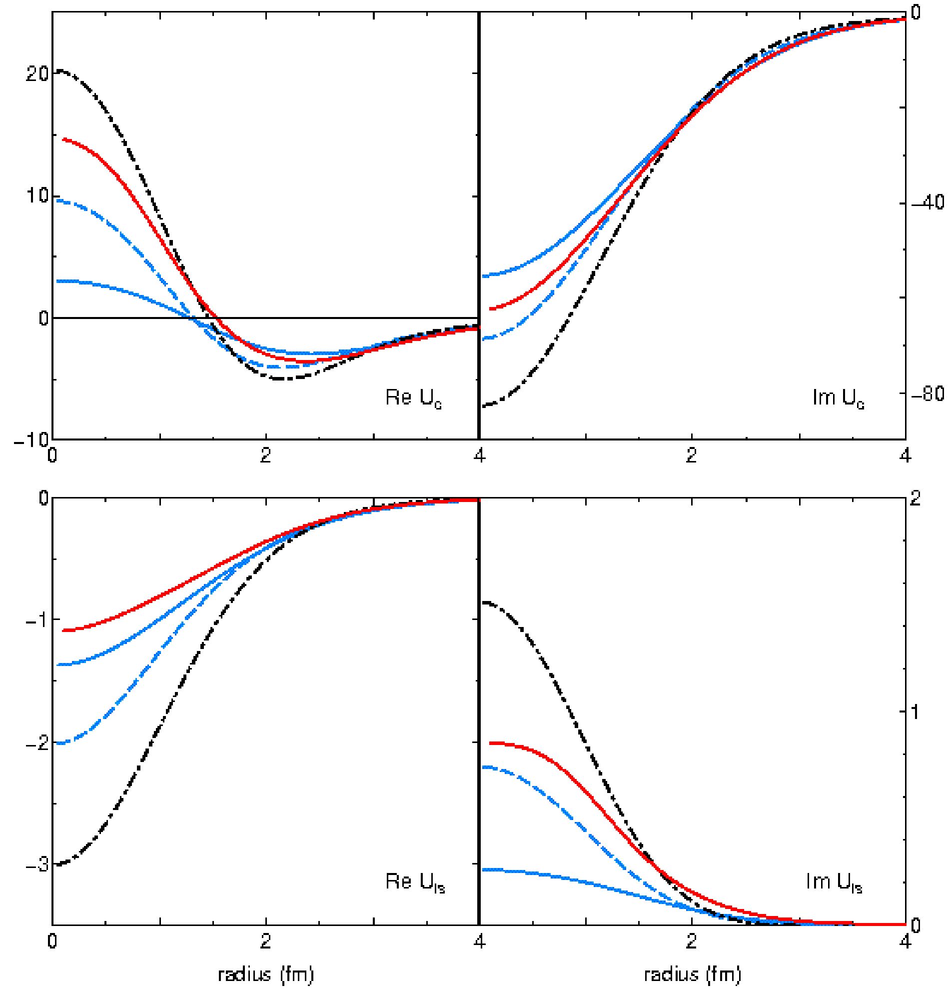


$E_p = 0.7$ GeV

^{6}He real part ^{6}He imaginary part

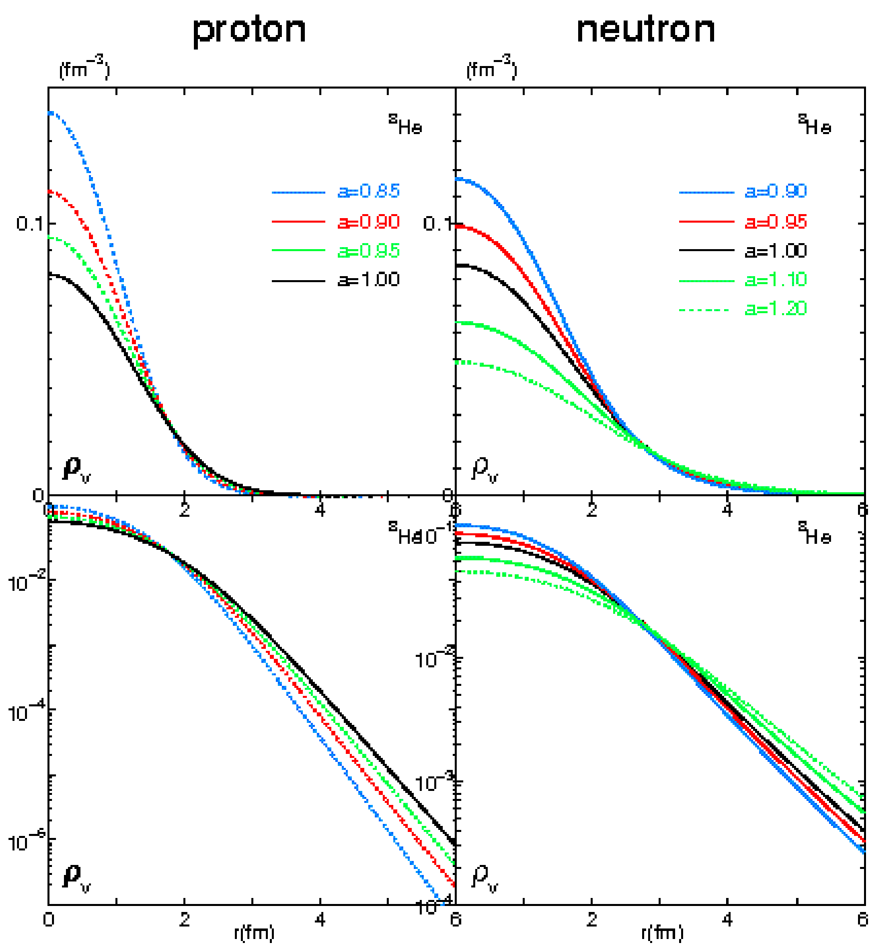


^{6}He at $E_p = 0.7$ GeV

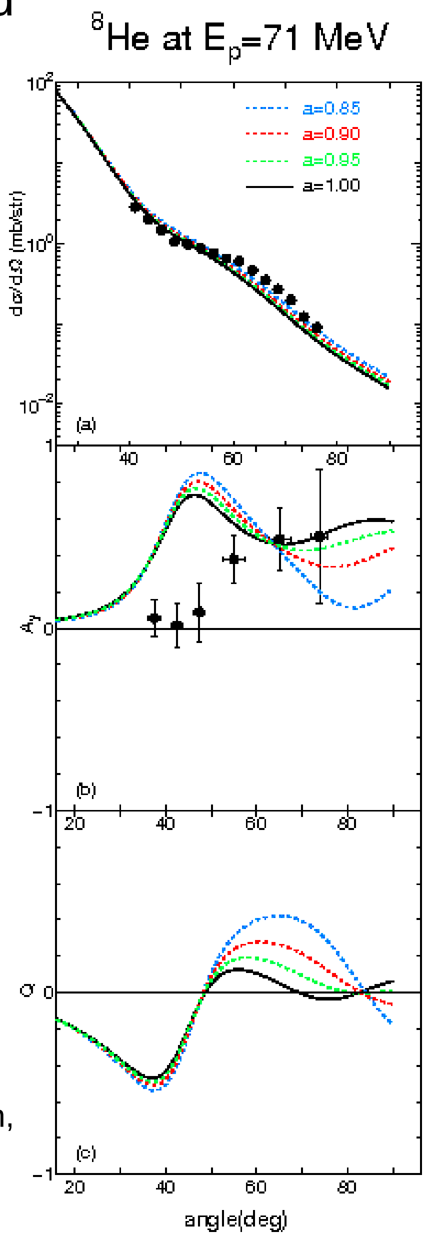


density distributions of ^{8}He

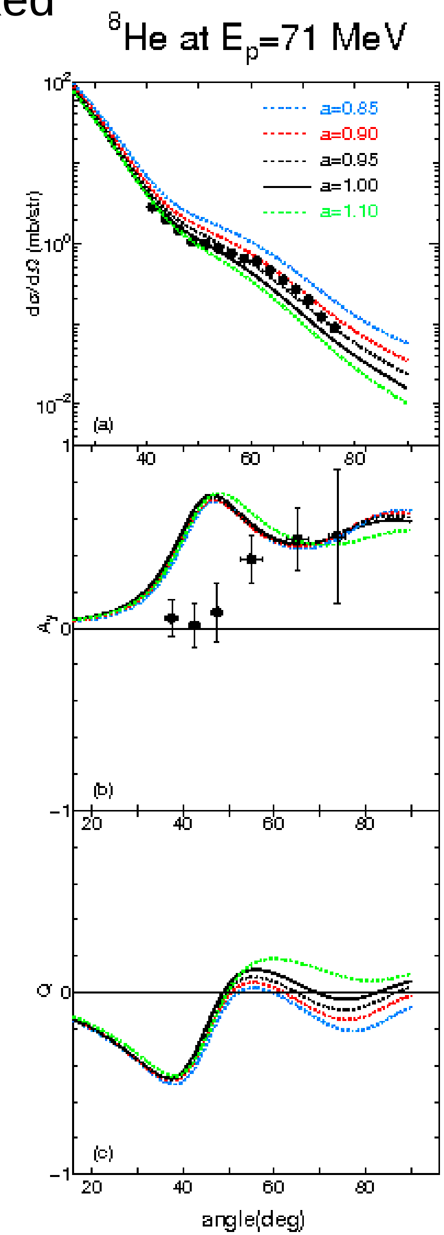
root-mean-square radius (fm)	proton	neutron
tma	1.975	3.193
a=0.85	1.646	-
a=0.90	1.778	2.873
a=0.95	1.876	3.033
a=1.10	-	3.510
a=1.20	-	3.819
charge	1.775	-



neutron: fixed



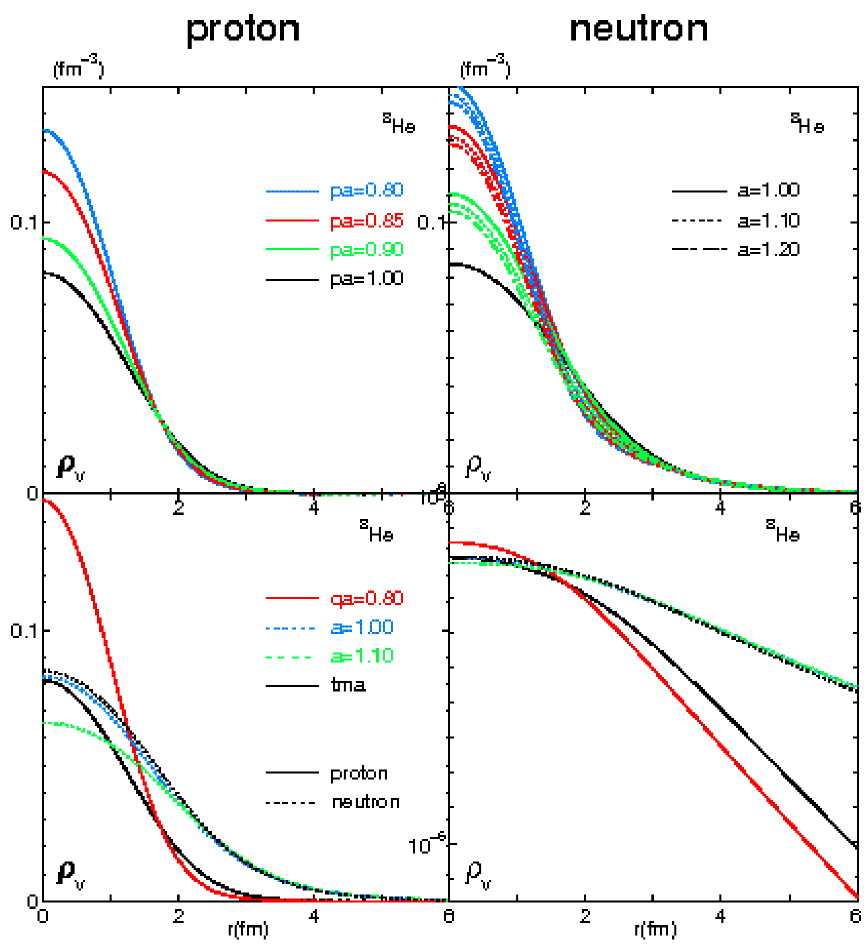
proton: fixed



S.Sakaguchi ,
Doctoral Dissertation,
Univ. of Tokyo, 2008

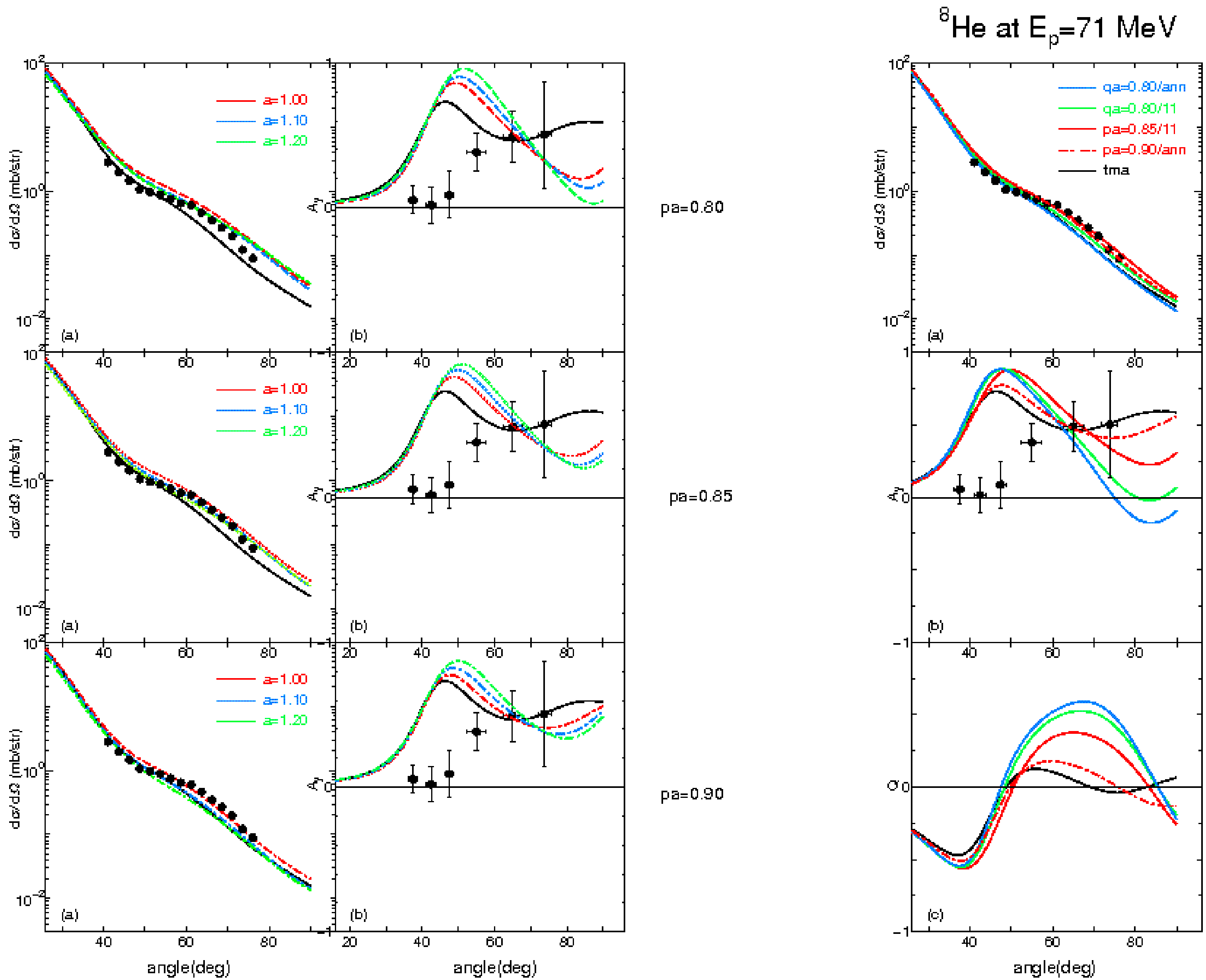
density distributions of ^{8}He

root-mean-square radius (fm)	proton	neutron
tma	1.975	3.193
pa=0.80/12	1.720	3.657
pa=0.85/11	1.792	3.396
pa=0.90/an	1.936	3.422
qa=0.80/11	1.635	3.321
qa=0.80/an	1.635	3.389
charge	1.775	-



pa: ^{4}He core + 4 neutrons

qa: ^{6}He core + 2 neutrons



8He at Ep=71 MeV
a=1.00
a=1.10
a=1.20
pa=0.80
pa=0.85
pa=0.90
qa=0.80/ann
qa=0.80/11
pa=0.85/11
pa=0.90/ann
tma
dσ/dΩ (mb/str)
Ay
Q
(a)
(b)
(c)
angle(deg)

Summary & conclusion

1. RIA analysis of proton-elastic scattering from ^{4}He at 71, 300 and 500 MeV
   - two density distributions

     differential cross section ~ similar accuracy

     analyzing power ~ large difference in large angles
   - experimental data favor the one of them (300 and 500 MeV)
2. from ^{6}He at 71 MeV (and 0.7GeV)
   - two different density distributions

     differential cross section ~ similar accuracy

     analyzing power ~ large difference in large angles
   - to fail in reproducing analyzing power data at 71 MeV
3. from ^{8}He at 71 MeV
   - ^{4}He+4neutrons ~ applicable differential cross section

     → not simple distributions ?
   - contribution of protons to Ay ~ much larger than that of neutrons
   - again to fail in reproducing analyzing power data at 71 MeV

     → scattering mechanism other than the impulse approximation ?

To determine the density distributions, both data of cross section and Ay are need in the region of several handed MeV.