

Turning points in the development of the study of nuclear reactions

M. Kawai

A talk given at the symposium on Microscopic Theory of Nuclear Reactions,
at The Institute of Fundamental Physics, Kyoto, August 2, 2012

[Foreword]

“Microscopic theory of nuclear reactions”, the theme of
this meeting, is based on

studies of $\left\{ \begin{array}{l} \text{reaction mechanisms} \\ \text{NN effective interaction} \end{array} \right.$

This talk is a review of some turning points in the study
of nuclear reaction *mechanisms*, in particular basic ideas and
developments of phenomenological models of multi-step direct
reactions of light ions.

[Experimental discoveries and theoretical models]

symbols : \Rightarrow original Model, \rightarrow development, * Basic pictures

- Atomic nucleus (1901~1911)
- Neutron (n) (1932)
- Resonance and large gamma emission probability in
low energy neutron induced reactions (1934~5)
 \Rightarrow Compound Nucleus (CN) model (Bohr, Breit-Wigner, 1936)
* Strong correlation \rightarrow Liquid drop model

- Non-CN process at $\sim 100\text{MeV}/N$ (1940s)
 - \Rightarrow Serber's Direct Reaction (DR) model (Serber, 1947)
 - * Weak NN correlation in short time \rightarrow Impulse App.

- Low energy DR leading to discrete final nuclear states.
 - \Rightarrow Deuteron stripping model (Butler, 1951)
 - *Shell model \rightarrow Spectroscopic applications

- Gross structure in neutron total cross sections vs A and E at $0\sim 3\text{MeV}$ (Barshall, 1952)
 - \Rightarrow Optical Model (Feshbach-Porter - Weisskopf, 1954)
 - * Energy average = observation by wave packet

- Intermediate resonance in energy averaged cross section
 - (e.g. Giant Resonance, Isobaric analog resonance)
 - \Rightarrow Theory of pre-equilibrium process through
Doorway, Hallway etc. states (Block-Feshbach, 1963)
 - * Size of energy average interval = time interval of observation

- Heavy Ion reactions at low energy
 - \Rightarrow Semi-classical models
 - \Rightarrow QMD & AMD simulations

- Heavy Ion reactions at high and ultra-high energies
 - \Rightarrow VVU eqs. Quark-Gluon plasma

[Developments of reaction theories]

\$ Compound Nucleus (CN) model

- Breit-Wigner formula,
- Dispersion formula of S-matrix : Decaying state, R –matrix,
S- matrix pole
 - * Resonance state as a black-box with only a few strings,
i.e. resonance energy and total and partial widths.
- Statistical CN models :
 - ⇒ Hauser-Feshbach formula for transition to discrete states
 - ⇒ Evaporation model (concept of ‘temperature’) for transition to continuous states, in particular multi-particle emission
- Models for pre-equilibrium process :
 - ⇒ Semi-classical models , Exciton model etc.
 - ⇒ Quantum theories of multi-step processes (FKK, TU, NWY, SCDW)

\$ Models of DR at intermediate energies

- Optical model for A-dependence of neutron total cross sections
- Strippig and pick up model of (d, n) and (n, d) reactions
- Intra-Nuclear Cascade model (INC)
 - ⇒ Monte Carlo calculations
- Impulse approximation
 - ⇒ Multiple scattering theory → Theories of Effective NN interaction in nuclear medium → in progress.

\$ Models of DR at low energies

- Optical model for energy averaged elastic scattering, total and total reaction cross sections and polarizations
 - ⇒ Optical potential (OP) search by analysis of exp. data
 - ⇒ Theoretical derivation of optical model
 - ⇒ Microscopic calculation of OP → in progress

\$\$ One-step DR

- DWBA for Inelastic scattering & re-arrangement reactions
 - Spectroscopic applications → in progress
- Diffraction model for scattering of strongly absorbed projectile
- DWIA for inelastic & charge exchange scattering
 - ⇒ Theories of effective NN interaction → in progress

\$\$ Multi-step DR (MSDR)

- Method of Coupled Channels (CC)
 - ⇒ CC for collective excitations
 - ⇒ CRC (Coupled Rearrangement Channels)
for rearrangement reactions
 - ⇒ CDCC (Continuum Discretised CC)
for real or virtual break up process.
 - ⇒ 2nd order DWBA for rearrangement reactions
 - ⇒ Resonating Group Method (RGM)
 - ⇒ Glauber model

[Method of Coupled Channels (CC)]

The most general phenomenological DW theory of DR, first introduced to nuclear physics by S. Yoshida (1956)

\$ Early references:

(1) CC for collective excitations

S. Yoshida, Proc. Phys. Soc. 69A, 668 (1956)

Applications to collective excitations by (p,p') , (α,α') etc.

T. Tamura, Rev. Mod. Phys, 37, 679, (1965)

(2) CRC : Coupled Rearranged Channels

J.A. Wheeler, Phys.Rev., 52, 1083, 1107 (1937) (RGM)

G.H.Rawitscher, Phys. Lett, 21, 444 (1966), Phys. Rev. 163, 1223 (1967)

B.Buck and A.D.Hill, Nucl. Phys. A95,271, (1967)

A.P.Stamp, Nucl. Phys, 83, 232, ('69)

T. Ohmura, B.Imanishi, M.Ichimura,M.Kawai,
Prog. Theor. Phys. 41, 391(1969) , ibid,43, 347('70),
ibid 44, 1212 ('70)

R.J.Ascuitto, N.K.Glendenning,B.Sorensen,
Nucl. Phys.A, 170, 65(1971)

T. Udagawa, H.H.Wolter, W.W. Coker, PRL, 31, 1507(1973)

(3) CDCC : Continuum Discretised Coupled Channels

M.Kamimura, M. Yahiro, Y. Iseri, Y. sakuragi, M. Kawai,
Prog.Theor. Phys. Suppl. 89, (1986)

N. Austern, Y. Iseri, M. Kamimura, M. Kawai, G.Rasitzcher,
M.Yahiro, Phys. Rep. 154, 126 (1987)

N. Austern, M. Kawai, M. Yahiro, Phys. Rev, Lett. 61, 2694
(1996), Phys. Rev. C53, 314 (1996)

[Formalism of CC]

§ Model space, \mathcal{D}

- spanned by the internal w.fs. of strongly coupled channels :

$$\{\Phi_c, c = \alpha \text{ (initial), } \beta \text{ (final), } \gamma_1, \gamma_2, \dots, \gamma_N \text{ (intermediate)}\}$$

- Effective Hamiltonian in \mathcal{D} : $H(P)$

its energy average over interval I : $\langle H(P) \rangle_I$

§ Phenomenological substitute : $\langle H(P) \rangle_I \Rightarrow \bar{H}$

- Form of \bar{H} in channel c : $\bar{H}^{(c)} = h_c + T_c + \bar{V}_c$

where $h_c \Phi_c = \varepsilon_c \Phi_c$, T_c = K.E. of relative motion,

\bar{V}_c = interaction. With channel diagonal 'bare potential', U_c ,

and non-diagonal 'residual interaction', \hat{V}_c ,

$$\bar{H}^{(c)} = h_c + T_c + U_c + \hat{V}_c$$

§ CC w.f.: $\Psi^{CC} = \chi_\alpha \Phi_\alpha + \sum_\gamma \chi_\gamma \Phi_\gamma + \chi_\beta \Phi_\beta$

- Ψ^{CC} satisfies $\langle \Phi_c | E - \bar{H} | \Psi^{CC} \rangle = 0$ which leads to

$$(E_c - T_c - U_c) \chi_c = \sum_{m \neq c} U_{cm} \chi_m$$

with 'channel coupling potential',

$$U_{cm} \equiv \langle \Phi_c | (\bar{H} - E) | \Phi_m \rangle$$

- Coupled integral equations under scattering B.C.:

$$\chi_c^{(+)} = \delta_{c\alpha} (1 + G_c^{(+)} U_c) \varphi_\alpha \Phi_\alpha + G_c^{(+)} \sum_{m \neq c} U_{cm} \chi_m^{(+)}$$

with $G_c^{(+)} = (E_c + i\varepsilon - T_c - U_c)^{-1}$.

Solution for Ψ^{CC} :

$$\Psi^{CC} = \delta_{c\alpha} (1 + G_c^{(+)} U_c) \varphi_\alpha \Phi_\alpha + \sum_c G_c^{(+)} \sum_{m \neq c} U_{cm} \chi_m^{(+)}$$

\$ Iterative solution and T - matrix

$$T_{\beta\alpha} = T_{\beta\alpha}^{(0)} + T_{\beta\alpha}^{(1)} + T_{\beta\alpha}^{(2)} + \dots$$

\$\$ 0-th order: $\chi_c^{(+)} \approx \chi_c^{(0)(+)} = \delta_{c\alpha} (1 + G_c^{(+)} U_c) \varphi_\alpha \Phi_\alpha$

$$T_{\beta\alpha}^{(0)} = \delta_{\beta\alpha} \langle \varphi_\beta \Phi_\beta | U_\beta | \chi_\alpha^{(0)(+)} \rangle$$

\$\$ 1st order: for $\beta \neq \alpha$

$$\chi_\beta^{(1)(+)} = G_\beta^{(+)} U_{\beta\alpha} \chi_\alpha^{(0)(+)}$$

$$T_{\beta\alpha}^{(1)} = \langle \chi_\beta^{(0)(-)} | U_{\beta\alpha} | \chi_\alpha^{(0)(+)} \rangle = T_{\beta\alpha}^{DWBA} \leftarrow \text{DWBA}$$

with the distorting potentials

$$\chi_\alpha^{(0)(+)} \text{ and } \chi_\beta^{(0)(-)} = (1 + G_\beta^{(-)}) \varphi_\beta \Phi_\beta \quad \text{where } G_\beta^{(-)} = G_\beta^{(+)*}$$

\$\$ 2nd order for $\beta \neq \alpha$:

$$\chi_\beta^{(2)(+)} = G_\beta^{(+)} \sum_{m \neq \alpha, \beta} U_{\beta m} G_m^{(+)} U_{m\alpha} \chi_\alpha^{(0)(+)}$$

$$T_{\beta\alpha}^{(2)} = \sum_{m \neq \alpha, \beta} \langle \chi_\beta^{(0)(-)} | U_{\beta m} G_m^{(+)} U_{m\alpha} | \chi_\alpha^{(0)(+)} \rangle \equiv \sum_{m \neq \alpha, \beta} T_{\beta m \alpha}^{(2)}$$

with $T_{\beta m \alpha}^{(2)} = \langle \chi_{\beta}^{(0)(-)} | U_{\beta m} G_m^{(+)} U_{m \alpha} | \chi_{\alpha}^{(0)(+)} \rangle \leftarrow$ 2nd order DWBA

\$\$ Coupling potential $U_{cm} \equiv \langle \Phi_c | (\bar{H} - E) | \Phi_m \rangle$

- Two choices of U_{cm} , with $\bar{H}^{(c)}$ or $\bar{H}^{(m)}$

For $d = c$ or m , $U_{cm}^{(d)} = \langle \Phi_c | (\bar{H}^{(d)} - E) | \Phi_m \rangle$

$$= \langle \Phi_c | (T_d + U_d + \hat{V}_d + h_d - E) | \Phi_m \rangle$$

$$= \underbrace{\langle \Phi_c | \hat{V}_d | \Phi_m \rangle}_{\text{Interaction term}} + \underbrace{\langle \Phi_c | \Phi_m \rangle (E_d - T_d - U_d)}_{\text{Non-orthogonality (NO) term}}$$

Interaction term Non-orthogonality (NO) term

NO term $\neq 0$ in rearrangement reaction $\because \langle \Phi_c | \Phi_m \rangle \neq 0$

\$\$ Two different forms of DWBA:

- $T_{\beta \alpha}^{DWBA}(\text{post}) = \langle \chi_{\beta}^{(0)(-)} | U_{\beta \alpha}^{(\beta)} | \chi_{\alpha}^{(0)(+)} \rangle = \langle \chi_{\beta}^{(0)(-)} | \hat{V}_{\beta} | \chi_{\alpha}^{(0)(+)} \rangle$

$$T_{\beta \alpha}^{(1)}(\text{prior}) = \langle \chi_{\beta}^{(0)(-)} | U_{\beta \alpha}^{(\alpha)} | \chi_{\alpha}^{(0)(+)} \rangle = \langle \chi_{\beta}^{(0)(-)} | \hat{V}_{\alpha} | \chi_{\alpha}^{(0)(+)} \rangle.$$

NO terms do not contribute since.

$$(E_{\sigma} - T_{\alpha} - U_a) \chi_{\alpha}^{(0)(+)} = (E_{\sigma} - T_{\alpha} - U_a) * \chi_{\beta}^{(0)(-)} = 0$$

- $T_{\beta \alpha}^{(1)}(\text{post})$ is used for stripping, $T_{\beta \alpha}^{(1)}(\text{prior})$ for pick up

- If $U_{cm}^{(c)} = U_{cm}^{(m)} \rightarrow$

$$T_{\beta \alpha}^{(1)}(\text{post}) = T_{\beta \alpha}^{(1)}(\text{prior}), \quad \text{“prior-post identity”}$$

\$\$ Four different forms of 2nd order DWBA, $T_{\beta \alpha}^{(2)} = \sum_{m \neq \alpha \beta} T_{\beta m \alpha}^{(2)}$

For each of the potentials $U_{m\alpha}$ and $U_{\beta m}$ in $T_{\beta m \alpha}^{(2)}$, there are prior and post forms, hence altogether four forms. For each step of the transition the one that matches the corresponding DWBA form factor is: *prior form* for *pick up*, and *post form* for *stripping*.

\$\$ Examples of application of 2nd order DWBA

- **Example (1)** : $(^3He, \alpha)(\alpha, t)$ in $A(^3He, t)B^*$

$$(\alpha) A(C+n)+h \rightarrow (\gamma) C+\alpha(h+n) \rightarrow (\beta) B(C+p)+t$$

$$1^{st} \text{ step } (\alpha) \rightarrow (\gamma) \text{ pickup } \rightarrow \text{ prior form } U_{\gamma\alpha}^{(\alpha)}$$

$$2^{nd} \text{ step } (\gamma) \rightarrow (\beta) \text{ stripping } \rightarrow \text{ post form } U_{\gamma\alpha}^{(\beta)}$$

$$T_{th\alpha}^{(2)} = \langle \chi_{\beta}^{(0)(-)} | \langle \Phi_t | \hat{V}_{\beta} | \Phi_h \rangle G_h^{(+)} \langle \Phi_h | \hat{V}_{\alpha} | \Phi_{\alpha} \rangle | \chi_{\alpha}^{(0)(+)} \rangle$$

$$\text{No NO terms } \because (T_{\alpha} + U_{\alpha} - E_{\alpha})\chi_{\alpha}^{(0)(+)} = 0, (T_t + U_t - E_t)\chi_t^{(0)(-)} = 0$$

- * M. Toyama, Phys. Lett. **38B**, 147 (1972),
R.Schaeffer & Bertsch *ibid*, 159,(1972)

\$\$ Example (2) : Successive transfer, $(p, d^*)(d^*, t)$ in $A(p, t)B$

d^* = the ground and excited states of the p-n system

$$(\alpha) A(B+2n)+p \rightarrow (\gamma) C(B+n)+d^*(p+n) \rightarrow (\beta) B+t(p+2n)$$

- **Choice of the coupling potentials:**

$$1^{st} \text{ step } (\alpha) \rightarrow (\gamma) \text{ pickup } \text{ prior form } U_{\gamma\alpha}^{(\alpha)}$$

$$2^{nd} \text{ step } (\gamma) \rightarrow (\beta) \text{ pickup } \text{ prior form } U_{\gamma\alpha}^{(\beta)}$$

$$\bullet T_{td^*p} = T_{td^*p}^{vv} (\text{prior} - \text{prior}) + T_{td^*p}^{NO} (\text{prior} - \text{prior})$$

$$\bullet T_{td^*p}^{vv} (\text{prior} - \text{prior}) = \langle \Phi_t | \hat{V}_{d^*} | \Phi_{d^*} \rangle G_{d^*}^{(+)} \langle \Phi_{d^*} | \hat{V}_p | \Phi_p \chi_p^{(+)} \rangle,$$

$$T_{td^*p}^{NO}(\text{prior-prior}) = - \langle \chi_t^{(0)(-)} | \langle \Phi_t | \Phi_{d^*} \rangle \langle \Phi_{d^*} | \hat{V}_p | \Phi_p \rangle | \chi_p^{(0)(+)} \rangle$$

- If $\{\Phi_{d^*}\}$ were a complete set ,

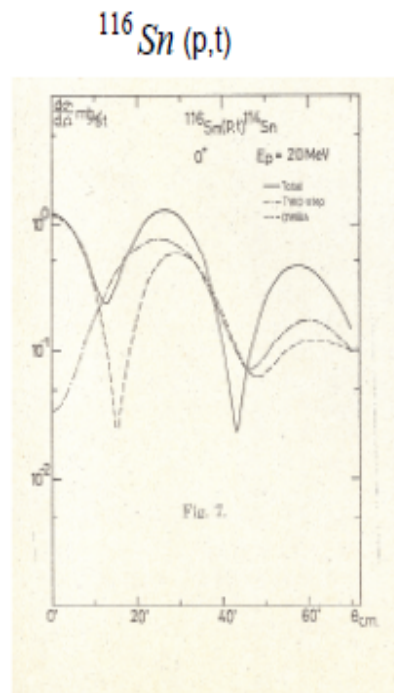
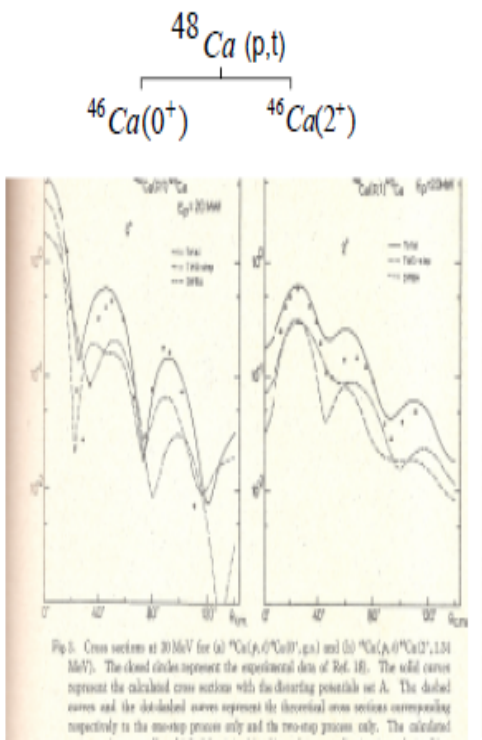
$$\begin{aligned} \sum_{d^*} T_{td^*p}^{NO}(\text{prior-prior}) &= - \langle \chi_\beta^{DW(-)} | \langle \Phi_\beta | \hat{V}_\alpha | \Phi_\alpha \rangle | \chi_\alpha^{DW(+)} \rangle \\ &= -T_{\beta\alpha}^{DWBA}(\text{prior}) \quad \because \quad \sum_\gamma |\Phi_\gamma\rangle \langle \Phi_\gamma| = 1. \end{aligned}$$

Reduction of 1-step contribution is anticipated even though E

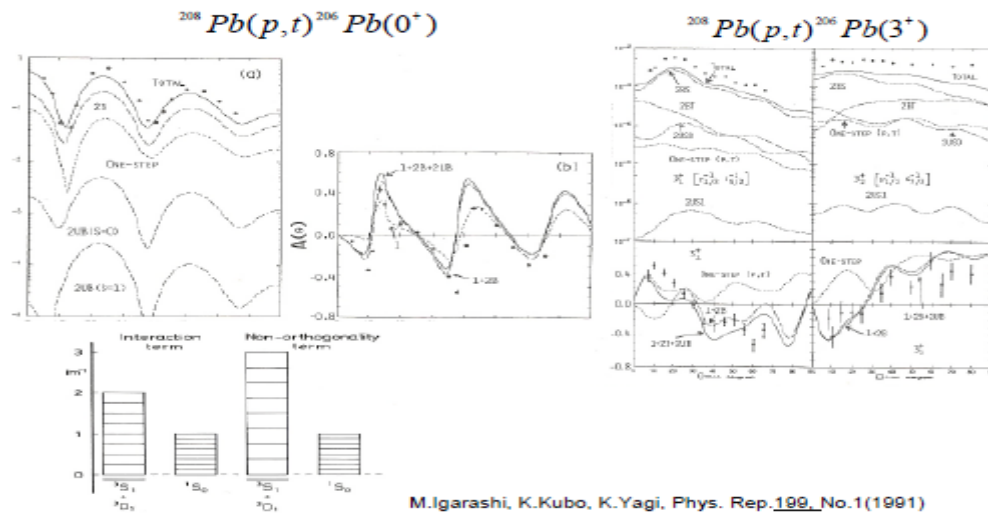
- Examples of one-step and successive transfer processes in (p,t) reaction

(1) (p,t) on ^{48}Ca

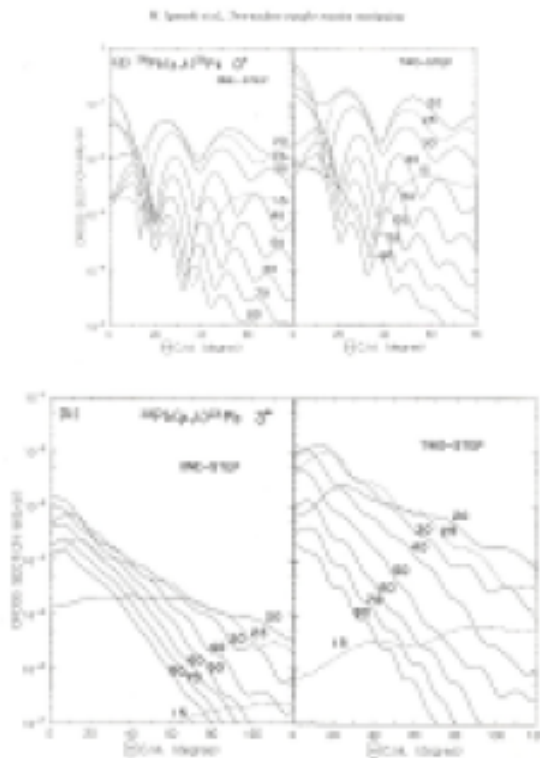
(2) (p,t) on ^{116}Sn in BCS state



(3) contribution of break up p-d* - t processes



Energy dep. of 1- and 2-step cross sections



DWBA and NO cross sections

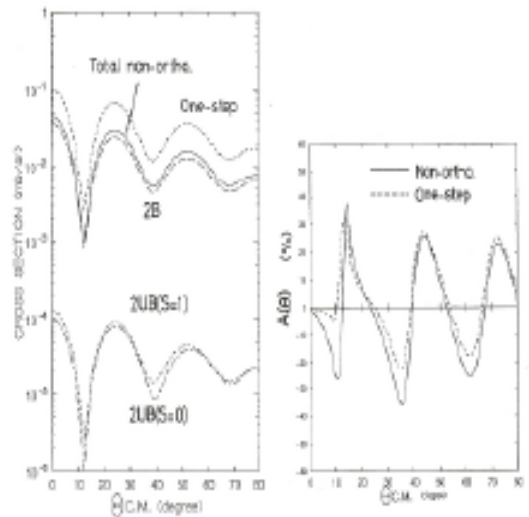
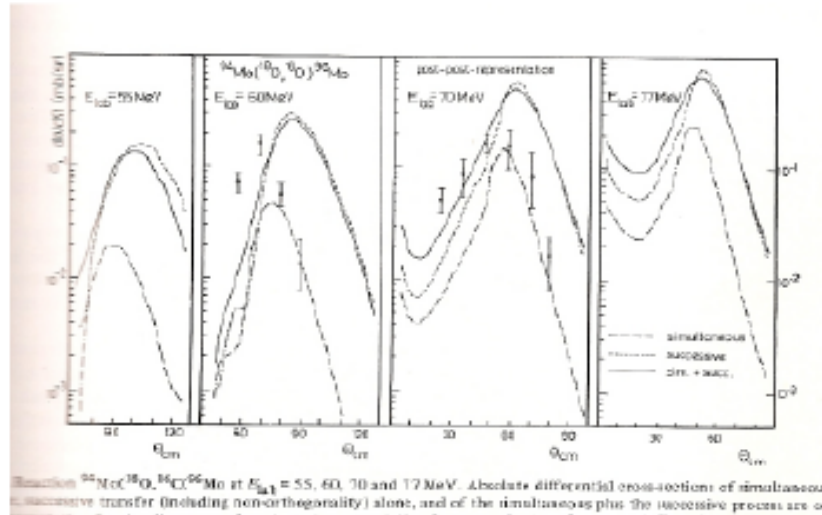


Fig. 412. One-step and non-ortho cross sections and their ratio for the $^{208}\text{Pb}(p,t)^{206}\text{Pb}(0^+)$ reaction at $E_p = 10 \text{ MeV}$.

Fig. 414. The angular distribution of the cross sections for the $^{208}\text{Pb}(p,t)^{206}\text{Pb}(0^+)$ reaction at $E_p = 10 \text{ MeV}$. The solid line shows the total cross section, the dashed line shows the one-step cross section, and the dotted line shows the non-ortho cross section.

Dominance of successive transfer processes in

$(^{18}\text{O}, ^{16}\text{O})$ on ^{94}Mo at $E_{\text{lab}} = 60\text{MeV}$



U. Goetz, M. Ichimura, R. A. Broglia, Phys. Rep. **16C**, No. 3

\$ Channel coupling potential for composite projectile channels

\$\$ Projectile break up

- $c(a+b) + A \leftrightarrow c^*(a+b) + A$: c^* = break up state of c

for which $\Phi_c = \phi_c \phi_A$, $\Phi_{c^*} = \phi_{c^*} \phi_A$; $\bar{V}_c = \bar{V}_{aA} + \bar{V}_{bA}$

$$U_{cc^*}^{(c)} = \frac{\langle \phi_c | U_{aA} + U_{bA} | \phi_{c^*} \rangle}{-}$$

where $U_{aA} = \langle \phi_A | \bar{V}_{aA} | \phi_A \rangle$, $U_{bA} = \langle \phi_A | \bar{V}_{bA} | \phi_A \rangle$:

$$\therefore \langle \Phi_c | \hat{V}_c | \Phi_{c^*} \rangle = \langle \phi_c \phi_A | \bar{V}_{aA} + \bar{V}_{bA} - U_c | \phi_{c^*} \phi_A \rangle$$

\$\$ One particle transfer:

- $$a(b+x) + A \leftrightarrow b + B(A+x)$$

for which $\Phi_a = \phi_a \phi_A$, $\Phi_b = \phi_a \phi_B$; $\bar{V}_b = \bar{V}_{aA} + \bar{V}_{bx}$

- $$U_{ab}^{(b)} = \underbrace{\langle \Phi_a | \bar{V}_{bx} | \Phi_b \rangle} + \underbrace{\langle \Phi_a | \bar{V}_{bA} - U_b | \Phi_b \rangle}$$

$\therefore U_{ab}^{(b)} = \langle \Phi_a | \hat{V}_b | \Phi_b \rangle = \langle \Phi_a | \bar{V}_{bx} + \bar{V}_{bA} - U_b | \Phi_b \rangle,$

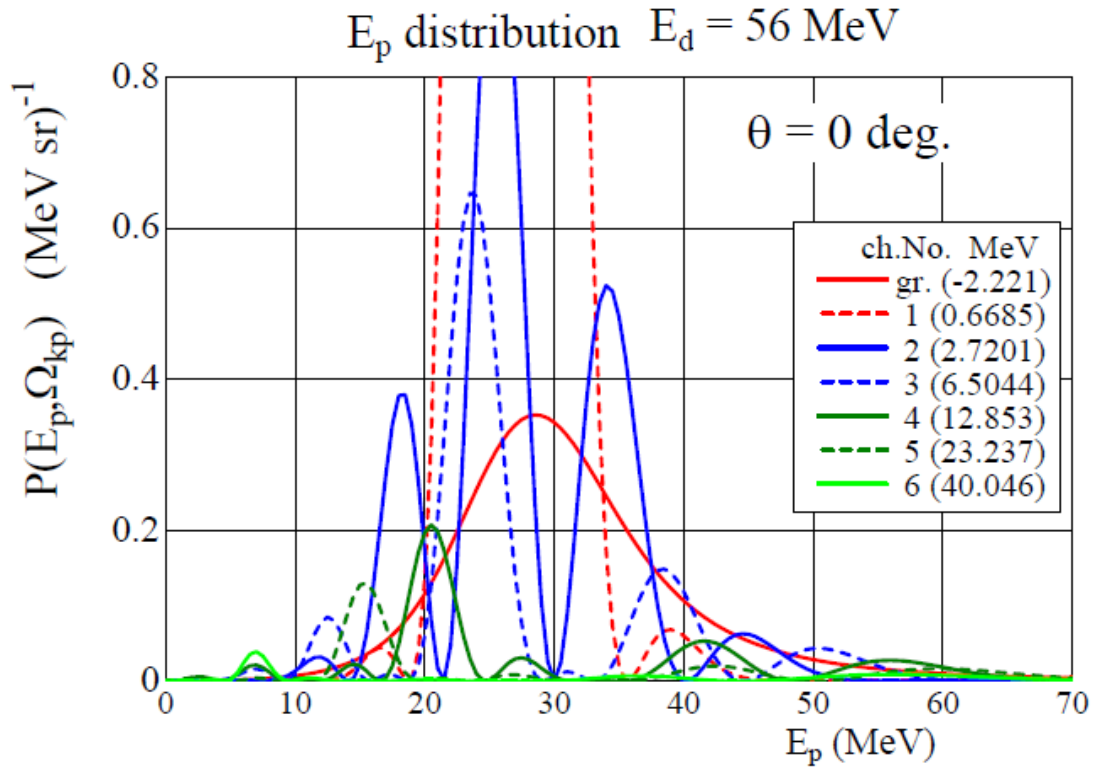
* Usually, $\langle \Phi_a | \bar{V}_{bA} - U_b | \Phi_b \rangle$ is neglected. since

$$\langle \Phi_a | \bar{V}_{bA} - U_b | \Phi_b \rangle \approx \langle \phi_a | \{ \langle \phi_A | \bar{V}_{bA} | \phi_A \rangle - U_b \} \langle \phi_A | \phi_B \rangle | \phi_b \rangle$$

and $\langle \phi_A | \bar{V}_{bA} | \phi_A \rangle - U_b$ is presumably small unless A is too small.*

\$\$ Problem of energy dependence of \bar{V}_c .

Effective potentials used in microscopic calculations depend on the energy of relative motion of the individual constituents of the composite particles. For instance, \bar{V}_{aA} in $U_{cc^*}^{(c)}$ depends on the energy of relative motion of a and A , E_{aA} . But E_{aA} is not unique because of the internal motion of a in c . Conventionally, it is fixed at $E_{aA} = E_c/2$. for all states of d^* . As the figure below shows, this is entirely unjustifiable, in particular for the d^* at high excitation energies. Energy independent effective interaction will be very useful.



Y. Iseri, private communication

[Some future subjects of
Microscopic Theory of Nuclear Reactions (MST)]

Discovery and elucidation of new reaction mechanisms are the purpose of nuclear reaction studies in general and precise microscopic description thereof is the aim of MST in particular. For further developments of MST as a precision theory, still more refinements of the theory and its application to wider range of actual reactions are necessary.. Examples of such refinement are those concerning the effective NN interaction in nuclear medium, τ , which is one of the most important ingredients of MTS. At present the G-matrix is most widely used for τ . Question then is if G-matrix is the best for it. For example, are different theories developed in nuclear structure theory not useful for MTS? Is the local density nuclear matter approximation justifiable for τ in finite nuclei? If G-matrix is used, refinements such as exact calculation of Pauli operator*, effects of relativity and coupling to

Δ - resonance etc. need be pursued. In the actual applications of MST to the Distorted Wave theory of direct reactions, it should be able to describe not only optical potentials, but also distorting potentials including non-diagonal matrix elements with respect to channels.

Besides the effective interaction τ discussed above there are and will be many interesting subjects of investigation for MST.

* K.Suzuki, R.Okamoto, M.Kohno and S.Nagata, Nucl. Phys. A665 92 (2000)