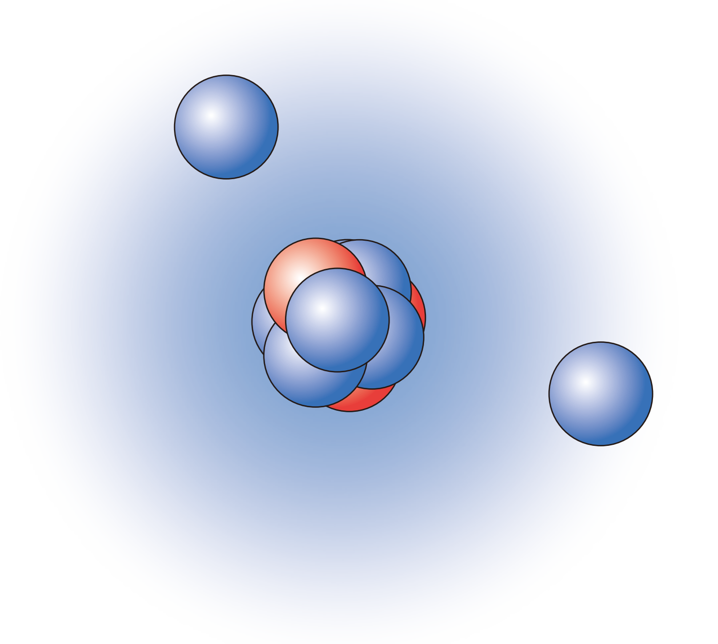
Description of scattering states using complex scaling method and its application to the breakup reaction of two-neutron halo nuclei

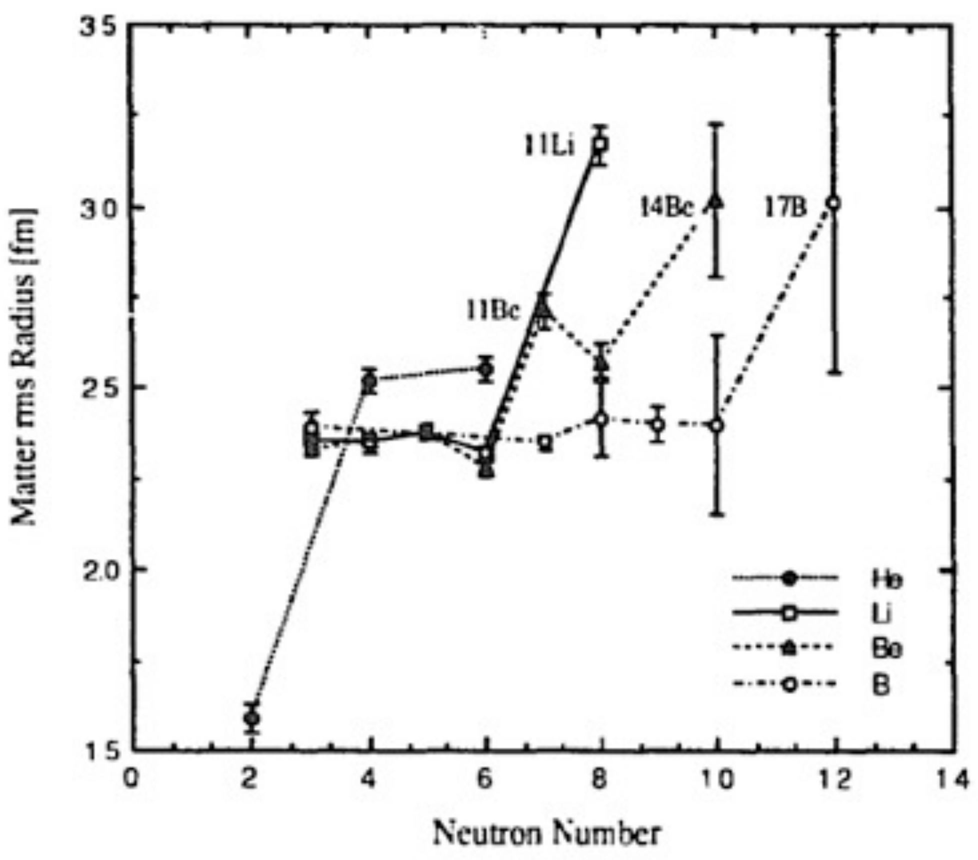
Yuma Kikuchi (RCNP)

In collaboration with
T. Myo, M. Takashina, K. Katō, and K. Ikeda

Exotic structures in neutron-rich nuclei

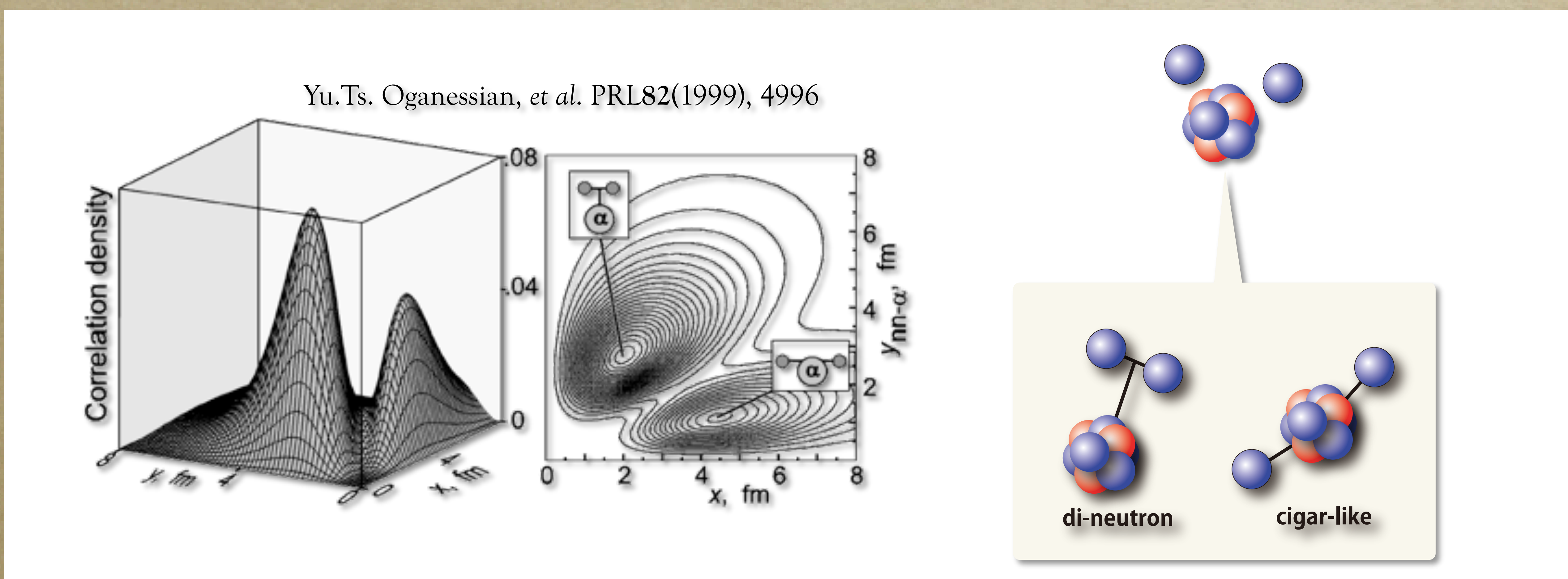
- Neutron halo nuclei
  - Neutron halo structure caused by weakly-bound neutrons is one of the most interesting topics in neutron-rich nuclei.
    - A few valence neutrons are weakly bound and spread beyond the core nucleus.
    - Enormous large matter radii are observed in experiments.





Two-neutron halo nuclei and dineutron correlation

- In two-neutron halo nuclei, their structures and binding mechanisms have been studied by using the core+n+n three-body model.
  - From the results of the core+n+n calculations, it has been suggested that
    - the correlation between the two halo neutrons is important in reproducing the observed binding energies and matter radii, and
    - the n-n correlation in halo nuclei is characterized as the spatially-correlated n-n pair, the so-called “dineutron”.



Yu.Ts. Oganessian, *et al.* PRL82(1999), 4996

To investigate the neutron halo nuclei

- The neutron halo nuclei are easily broken up with neutron emission, since a few valence neutrons are weakly bound.
  - They have only a few bound states, and most of their states are observed as resonances and continuum states.
  - Theoretically, to investigate their structures, it is required to describe the resonances and continuum states of core + neutrons.

Complex scaling method (CSM) is one of powerful tools to investigate the resonances and continuum states of many-body systems.

Complex scaling method

and

Complex-scaled solutions of

the Lippmann-Schwinger equation

Complex scaling method

- CSM enables us to find the resonances in similar manner to the bound states.
  - CSM is one of the methods to solve the eigenvalue problem with outgoing boundary conditions.
  - In CSM, the relative coordinates and momenta are transformed as follows:

$$U(\theta) : \mathbf{r} \to \mathbf{r}e^{i\theta}, \qquad \mathbf{k} \to \mathbf{k}e^{-i\theta}$$

and then, we obtain the complex-scaled Schrödinger equation as

$$\hat{H}\chi(\mathbf{r}) = E\chi(\mathbf{r}) \to \hat{H}^{\theta}\chi^{\theta}(\mathbf{r}) = E^{\theta}\chi^{\theta}(\mathbf{r})$$

where

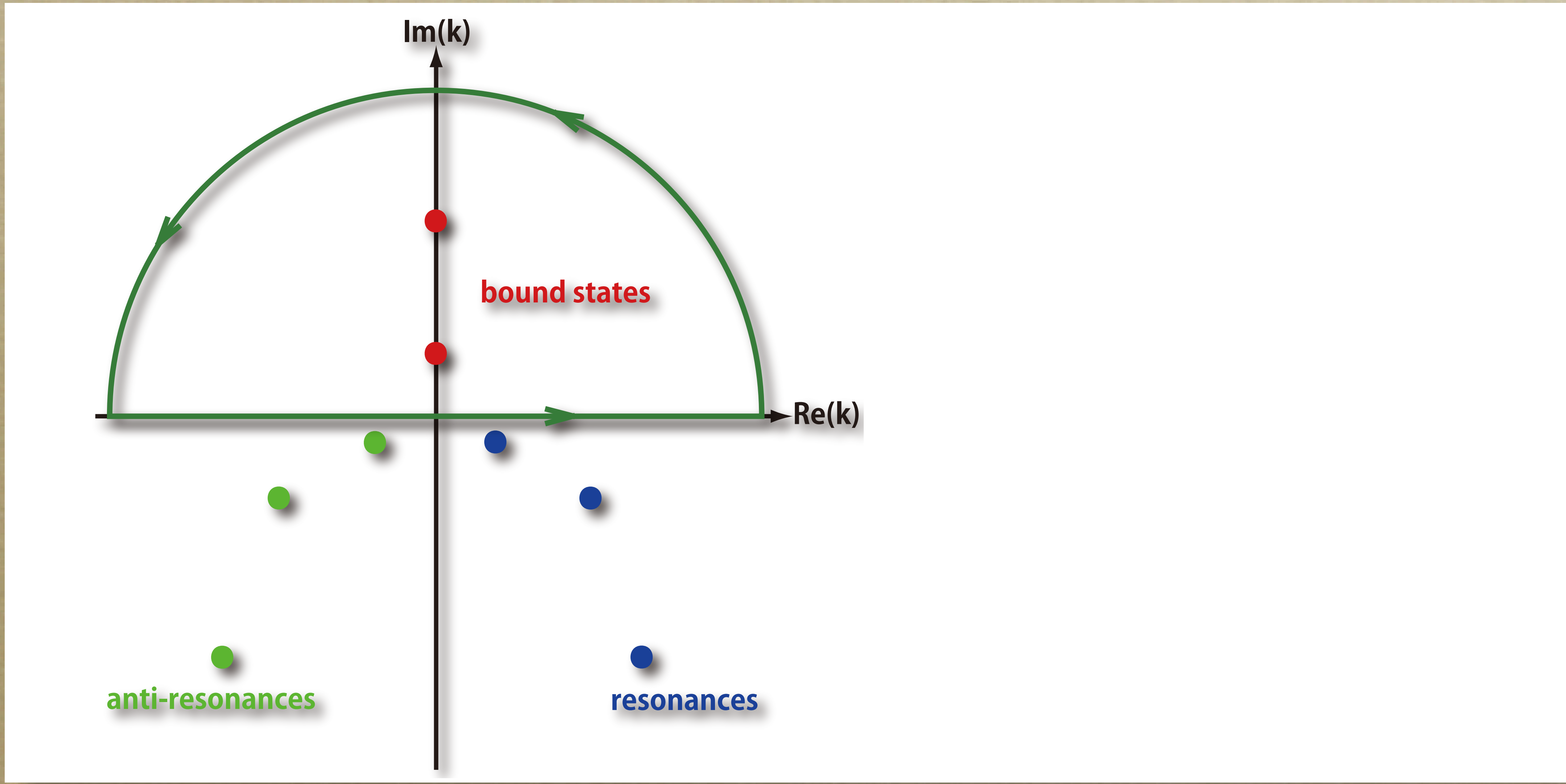
$$\chi^{\theta}(\mathbf{r}) = U(\theta)\chi(\mathbf{r}) = e^{\frac{3}{2}i\theta}\chi(\mathbf{r}e^{i\theta})$$

$$\hat{H}^{\theta} = U(\theta)\hat{H}U^{-1}(\theta)$$

S. Aoyama et al., PTP 116, 1 (2006).

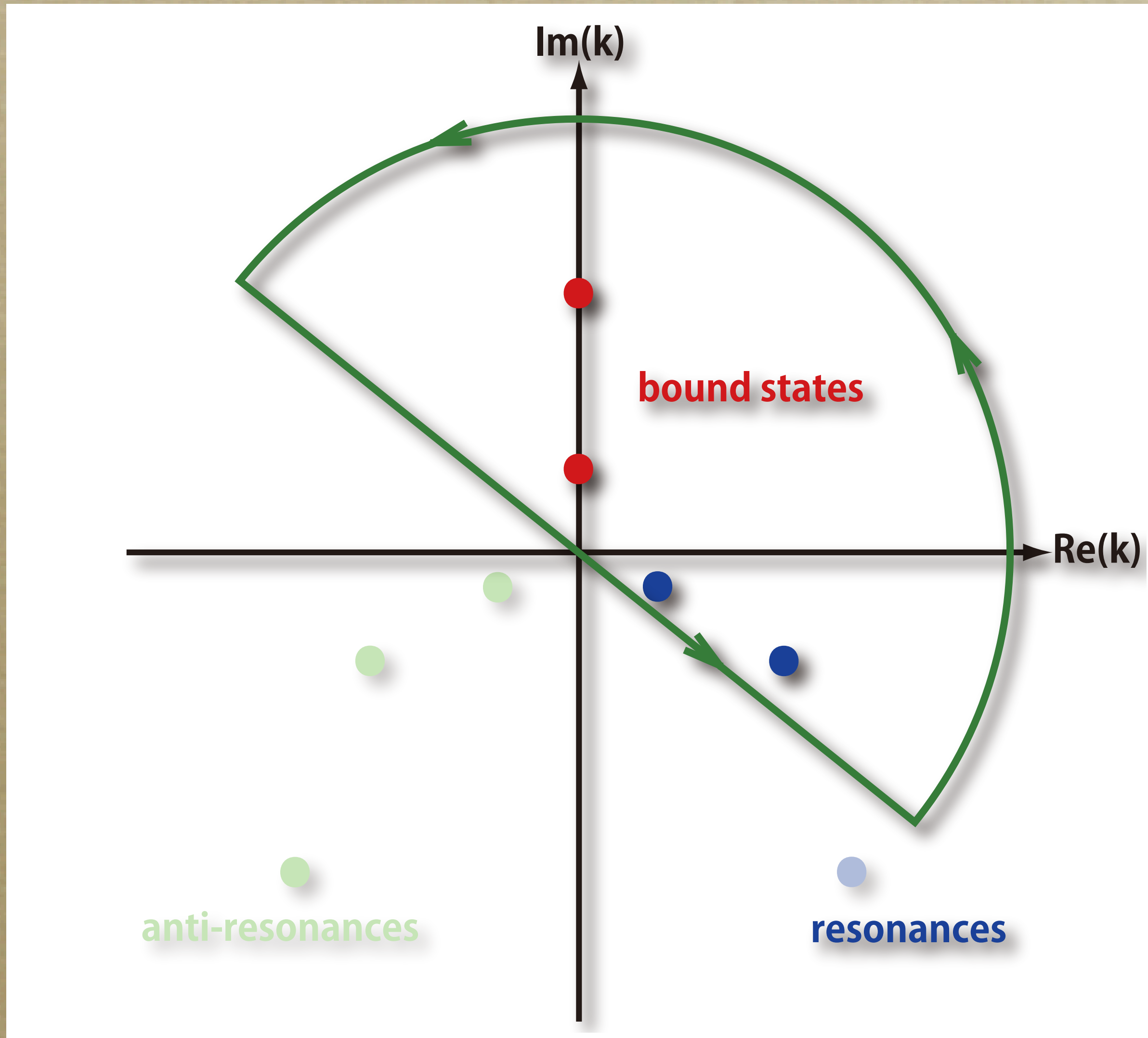
Complex scaling method

- CSM enables us to find the resonances in similar manner to the bound states.
  - Under this transformation, the contour of the integral path in the momentum space are rotated, and then, we can find the resonance poles in S-matrix as residues.
    - The resonance poles are isolated from the continuum ones.
    - We can solve the resonances by using L^2-type basis functions.



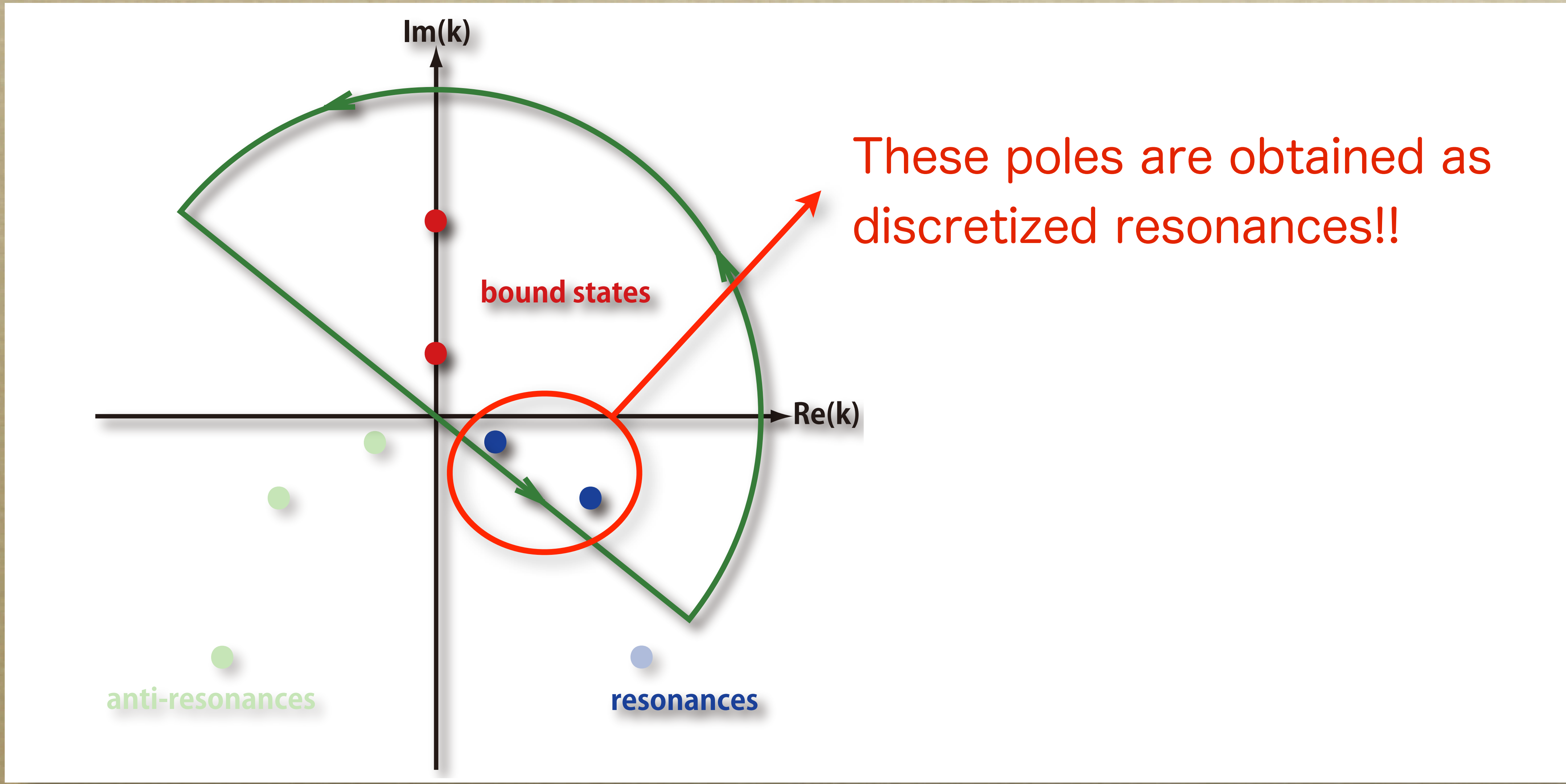
Complex scaling method

- CSM enables us to find the resonances in similar manner to the bound states.
  - Under this transformation, the contour of the integral path in the momentum space are rotated, and then, we can find the resonance poles in S-matrix as residues.
    - The resonance poles are isolated from the continuum ones.
    - We can solve the resonances by using L^2-type basis functions.



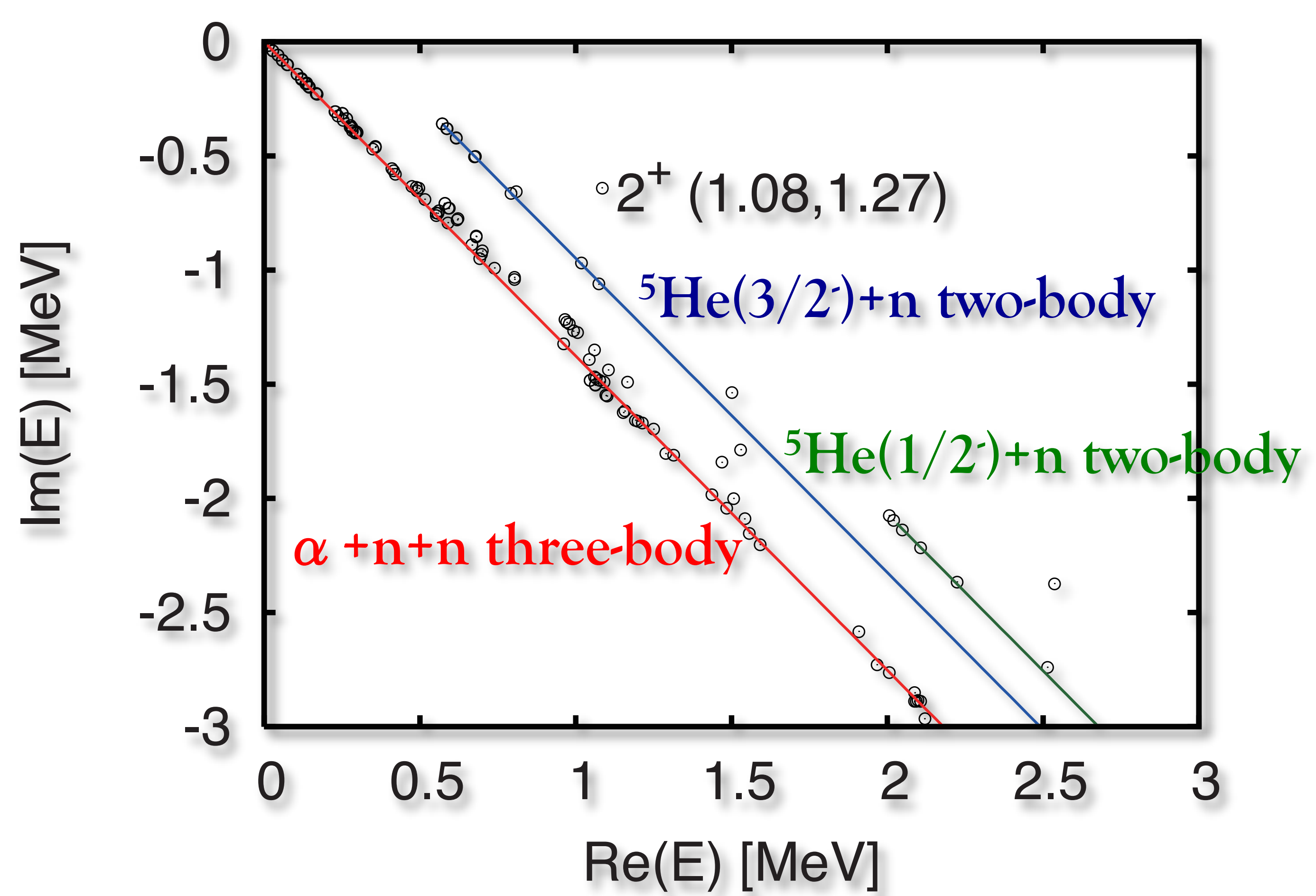
Complex scaling method

- CSM enables us to find the resonances in similar manner to the bound states.
  - Under this transformation, the contour of the integral path in the momentum space are rotated, and then, we can find the resonance poles in S-matrix as residues.
    - The resonance poles are isolated from the continuum ones.
    - We can solve the resonances by using L^2-type basis functions.



Complex scaling method

- In CSM, it is noted that the obtained energy eigenvalues are complex numbers, and impose an outgoing boundary condition for each decay channel.
  - The resonance pole has a energy of E-iΓ/2.
  - The continuum poles are located on the 2θ-lines starting with the thresholds, and classified into a family of each decay channel.



ex) obtained spectra of 2^+ states of ^{6}He

Complex-scaled solutions of the LS equation

- To describe the scattering states, we combine the complex-scaled Green's function with the Lippmann-Schwinger equation.
  - We start from the formal solution of the Lippmann-Schwinger equation.

$$\Psi^{(\pm)} = \Phi_0 + \lim_{\varepsilon \to 0} \frac{1}{E - \hat{H} \pm i\varepsilon} \hat{V} \Phi_0$$

  - We replace the Green's function with the complex-scaled one, and obtain the complex-scaled solutions of the Lippmann-Schwinger equation (CSLS).

$$\mathcal{G}^\theta(E; \boldsymbol{\xi}, \boldsymbol{\xi}') = \left\langle \boldsymbol{\xi} \left| \frac{1}{E - \hat{H}^\theta} \right| \boldsymbol{\xi}' \right\rangle = \sum_n \frac{\chi_n^\theta(\boldsymbol{\xi}) \tilde{\chi}_n^\theta(\boldsymbol{\xi}')}{E - E_n^\theta}$$

T. Myo et al., PRC63, 054313 (2001).

Complex-scaled solutions of the LS equation

- To describe the scattering states, we combine the complex-scaled Green's function with the Lippmann-Schwinger equation.
  - We start from the formal solution of the Lippmann-Schwinger equation.

$$\Psi^{(\pm)} = \Phi_0 + \lim_{\varepsilon \to 0} \frac{1}{E - \hat{H} \pm i\varepsilon} \hat{V} \Phi_0$$

  - We replace the Green's function with the complex-scaled one, and obtain the complex-scaled solutions of the Lippmann-Schwinger equation (CSLS).

$$\mathcal{G}^\theta(E; \boldsymbol{\xi}, \boldsymbol{\xi}') = \left\langle \boldsymbol{\xi} \left| \frac{1}{E - \hat{H}^\theta} \right| \boldsymbol{\xi}' \right\rangle = \sum_n \frac{\chi_n^\theta(\boldsymbol{\xi}) \tilde{\chi}_n^\theta(\boldsymbol{\xi}')}{E - E_n^\theta}$$

Outgoing boundary conditions are taken into account via imaginary parts of the energy eigenvalues.

T. Myo et al., PRC63, 054313 (2001).

Complex-scaled solutions of the LS equation

- To describe the scattering states, we combine the complex-scaled Green's function with the Lippmann-Schwinger equation.
  - We start from the formal solution of the Lippmann-Schwinger equation.

$$\Psi^{(\pm)} = \Phi_0 + \lim_{\varepsilon \to 0} \frac{1}{E - \hat{H} \pm i\varepsilon} \hat{V} \Phi_0$$

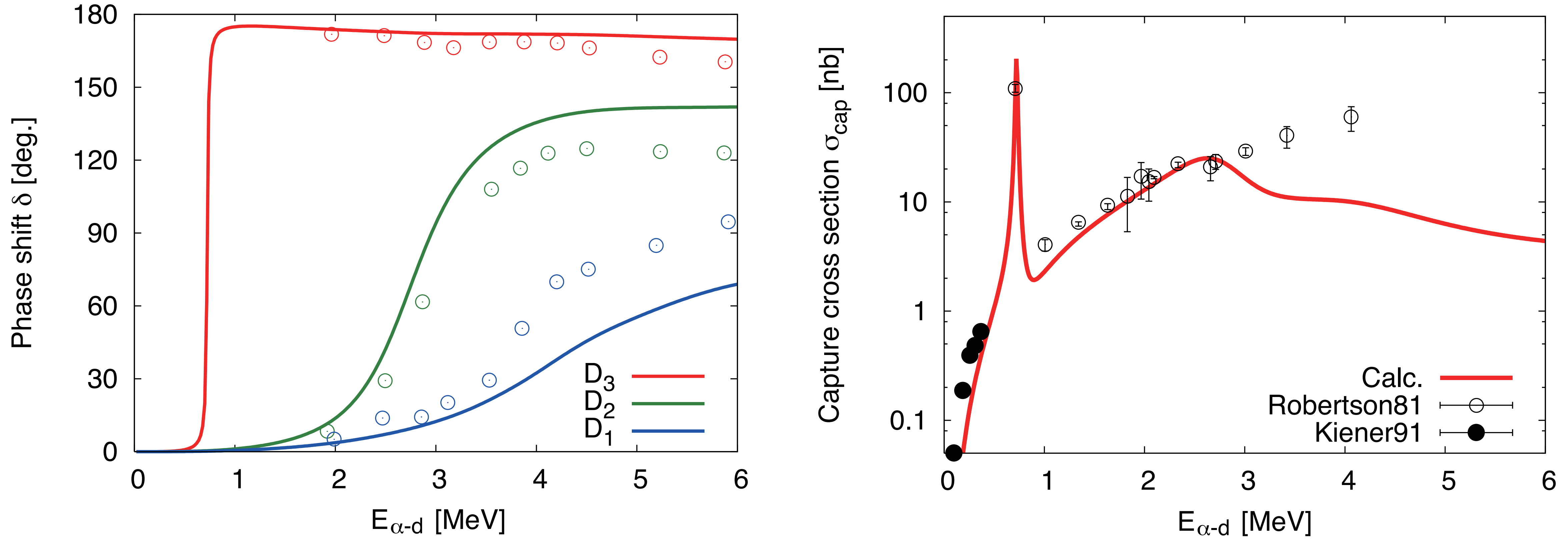
  - We replace the Green's function with the complex-scaled one, and obtain the complex-scaled solutions of the Lippmann-Schwinger equation (CSLS).

$$\langle \Psi^{(+)} | = \langle \Phi_0 | + \sum\!\!\!\!\!\!\int_n \langle \Phi_0 | \hat{V} U^{-1}(\theta) | \chi_n^\theta \rangle \frac{1}{E - E_n^\theta} \langle \tilde{\chi}_n^\theta | U(\theta)$$

    - Advantages in CSLS
      - We can solve the scattering problem by using L^2-type basis functions.
      - It is not necessary to consider boundary conditions explicitly.
      - It is not necessary to solve the coupled-channel equation.

Examples: Application of CSLS

- Application to the α+d elastic scattering and the radiative capture
  - CSLS well describes the α+d elastic phase shift and the radiative capture cross section.
  - CSLS is capable of investigating the scattering problems of the three-body systems.



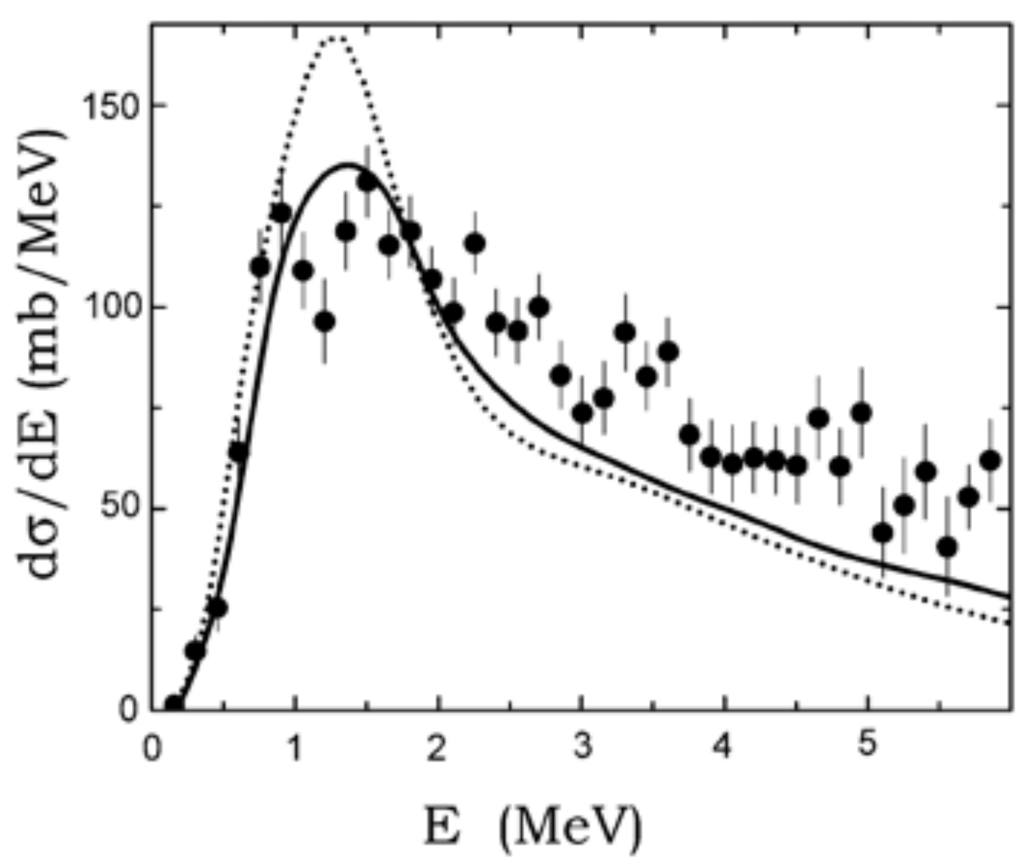
YK et al., submitted to PRC.

Application to Coulomb breakup of ^{6}He

Coulomb breakup reactions for 2n halo nuclei

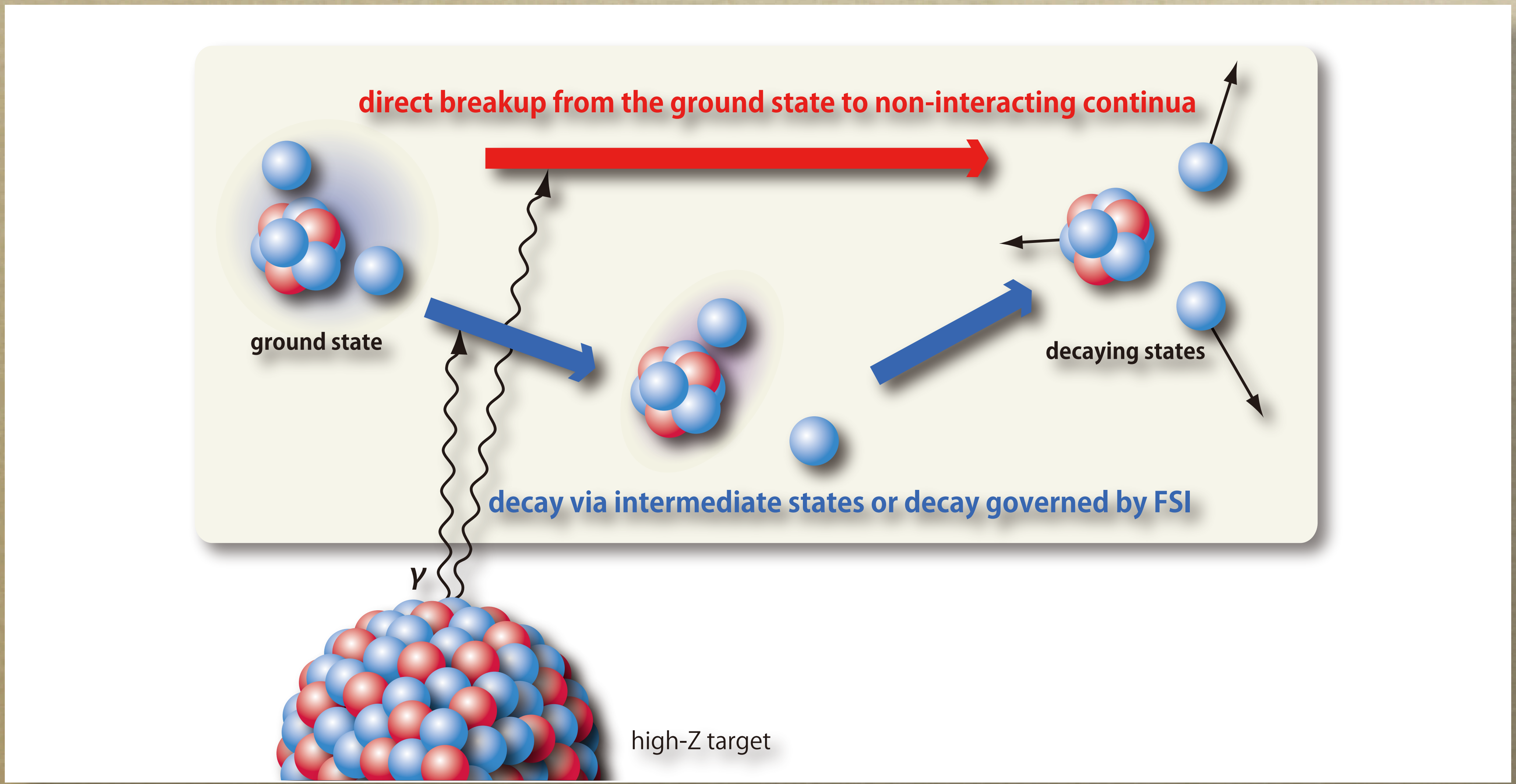
- For two-neutron halo cases, low-lying enhancement in the cross section is also observed.
  - Is this enhancement a possible tool to investigate the halo structures in the ground states?
  - If the breakup process is dominated by the direct breakup as similar to the one-neutron halo case, it can be true.

^{6}He breakup: T. Aumann et al., PRC 59, 1252 (1999).



Problems

- Are final states in the Coulomb breakup of the two-neutron halo also described by a plane wave?
  - In core+n+n systems, binary subsystems of core-n and n-n can form the resonances and/or virtual states.
  - The structure of the continuum states is a key in investigating the Coulomb breakup reaction of two-neutron halo nuclei.



In this talk

- To investigate the breakup mechanism of two-neutron halo nuclei, we calculate the core+n+n three-body continuum states accurately, and examine how the ground-state properties of halo nuclei can be extracted from the observed cross section.
  - To describe the three-body continuum states, we employ the core+n+n cluster model and the complex-scaled solutions of LS equation.
  - We calculate the Coulomb breakup cross section and invariant mass spectra for ^{6}He, and show the reliability of our method.
  - We discuss the what kinds of structure of continuum states play key roles in reproducing the low-lying enhancement in the Coulomb breakup cross section.

α+n+n three-body model

- Hamiltonian

$$\hat{H} = \sum_{i=1}^{3} t_i - T_{\rm cm} + \sum_{i=1}^{2} V_{\alpha\text{-}n}(\mathbf{r}_i) + V_{n\text{-}n} + V_{\alpha nn} + \lambda|\Phi_{\rm PF}\rangle\langle\Phi_{\rm PF}|$$

$V_{\alpha\text{-}n}$: KKNN potential

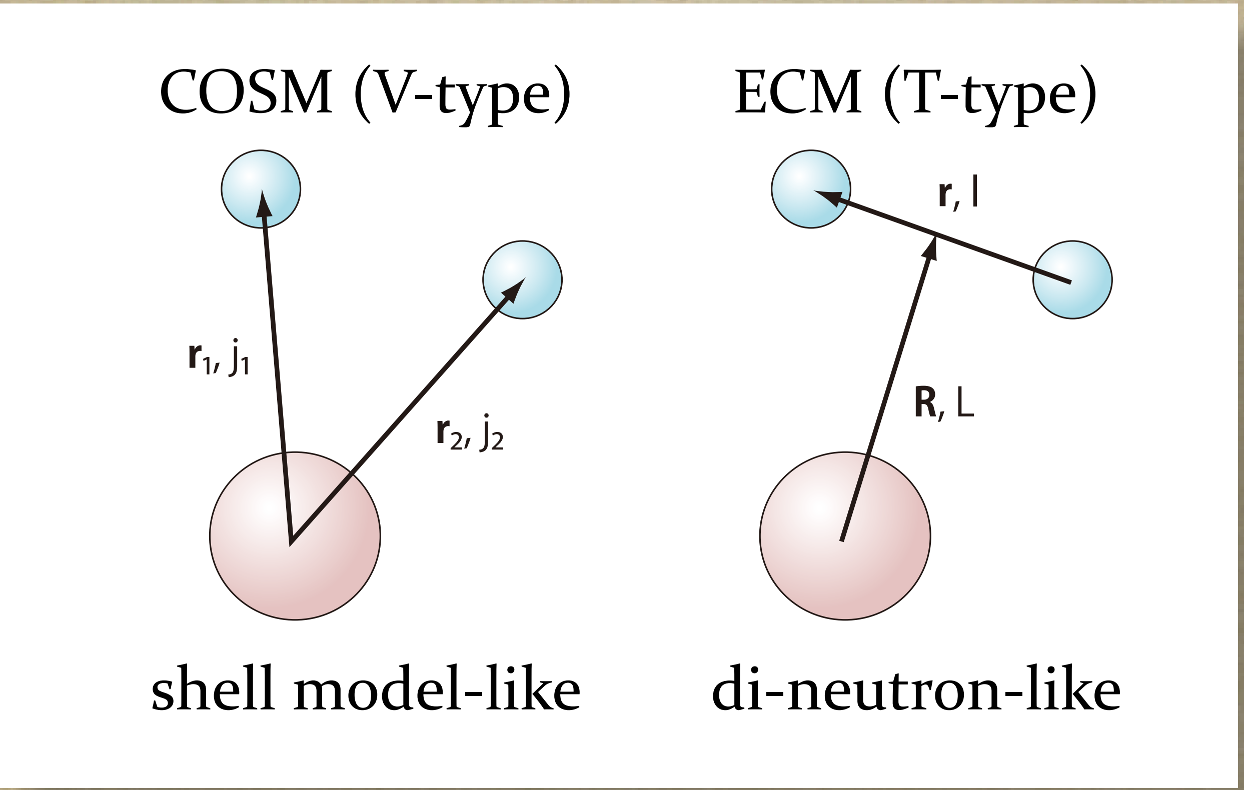
$V_{n\text{-}n}$: Minnesota force

$V_{\alpha nn}$: effective three-body αnn potetial

- Wave function

$$\chi(nn) = \chi_V(\mathbf{r}_1, \mathbf{r}_2) + \chi_T(\mathbf{r}, \mathbf{R})$$

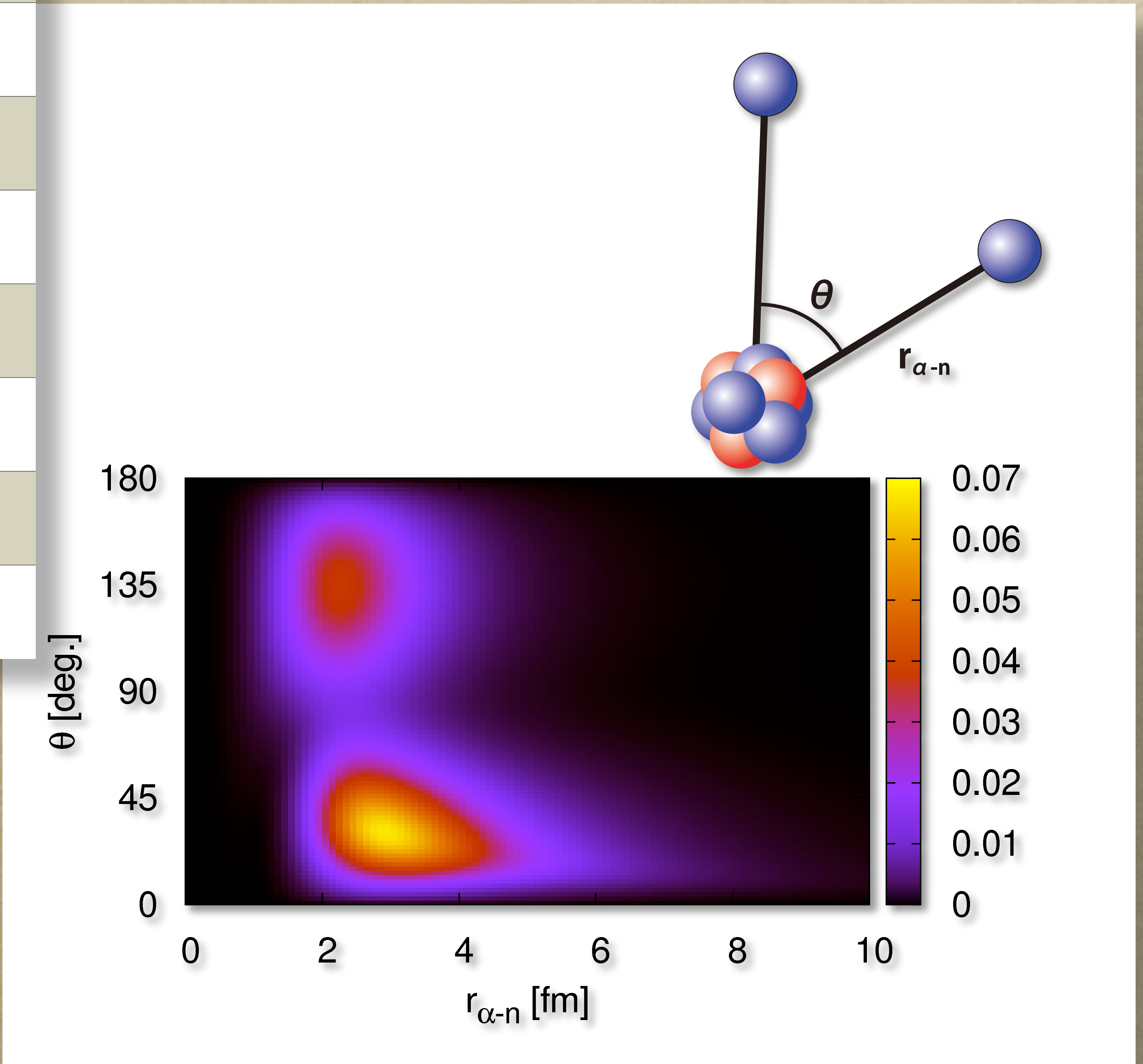
S. Aoyama et al., PTP 93, 99 (1995).



Obtained properties in the present calculation

- Obtained ground-state properties and two-neutron density in ^{6}He

	calc.	exp.
S_{2n} (MeV)	0.975	0.975
R_m (fm)	2.46	2.48±0.03
		2.33±0.04
		2.5
R_{ch} (fm)	2.04	2.068(11)
$R_{c\text{-}2n}$ (fm)	3.49	
$R_{n\text{-}n}$ (fm)	4.7	
θ (deg.)	67.9	



B(E1) strength and cross section in CSLS

- Using CSLS, we can calculate the Coulomb breakup cross section.
  - Here, we calculate the cross section by using the E1 strength distribution and equivalent photon method.
    - Cross section in equivalent photon method

$$\frac{d^6\sigma}{d\mathbf{k}d\mathbf{K}} = \frac{16\pi^3}{9\hbar c} \cdot N_{E1}(E_\gamma) \cdot \frac{d^6 B(E1)}{d\mathbf{k}d\mathbf{K}}$$

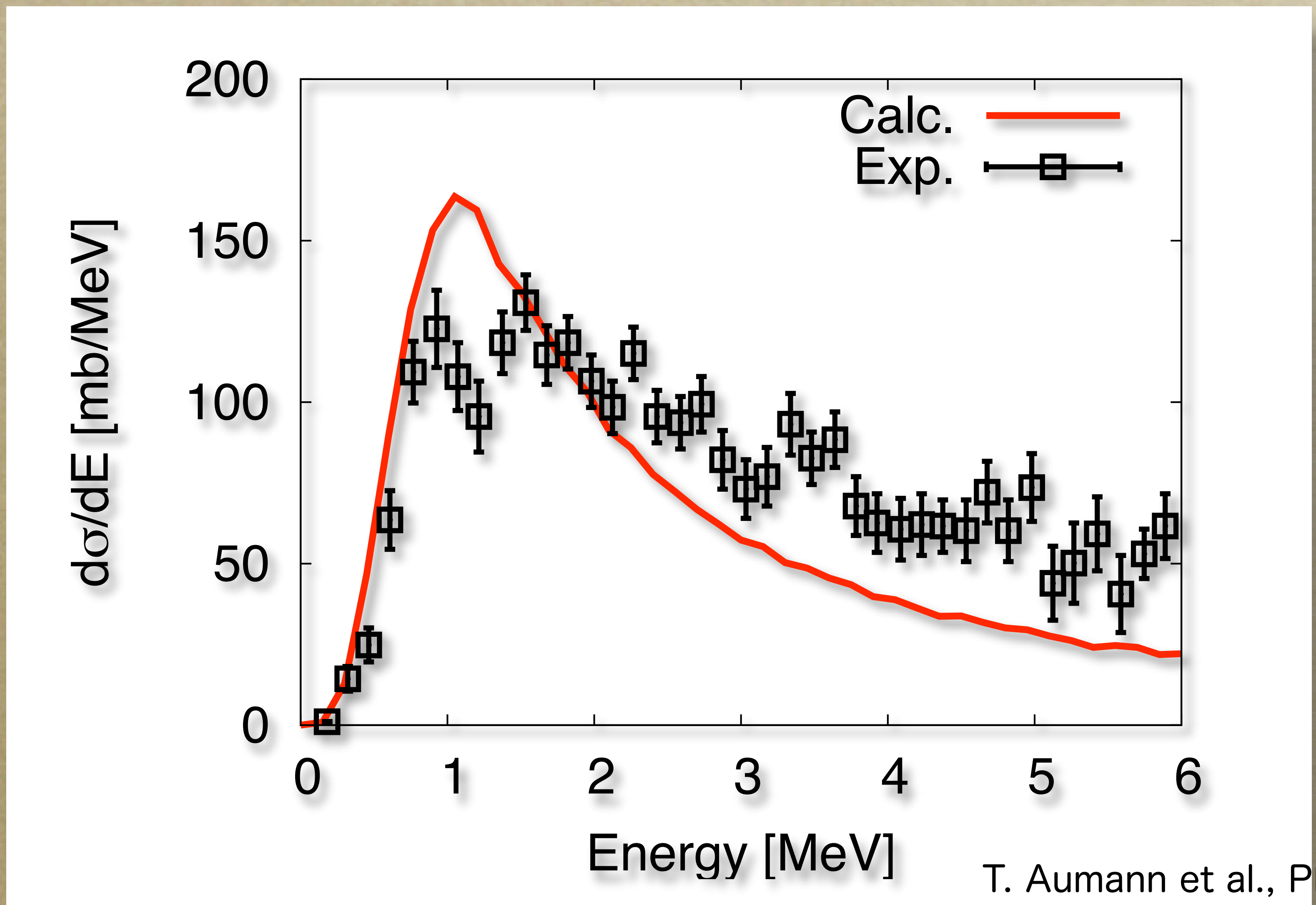
$$\frac{d^6 B(E1)}{d\mathbf{k}d\mathbf{K}} = \frac{1}{2J_{\rm gs}+1}\left|\langle\Psi^{(+)}(\mathbf{k},\mathbf{K})||\hat{O}(E1)||\Phi_{\rm gs}\rangle\right|^2$$

    - E1 matrix element in CSLS

$$\langle\Psi^{(+)}(\mathbf{k},\mathbf{K})|\hat{O}(E1)|\Phi_{\rm gs}\rangle = \langle\Phi_0(\mathbf{k},\mathbf{K})|\hat{O}(E1)|\Phi_{\rm gs}\rangle + \sum\!\!\!\!\!\!\int_n \langle\Phi_0(\mathbf{k},\mathbf{K})|\hat{V}U^{-1}(\theta)|\chi_n^\theta\rangle \frac{1}{E-E_n^\theta}\langle\tilde{\chi}_n^\theta|U(\theta)\hat{O}(E1)|\Phi_{\rm gs}\rangle$$

Calculated cross section

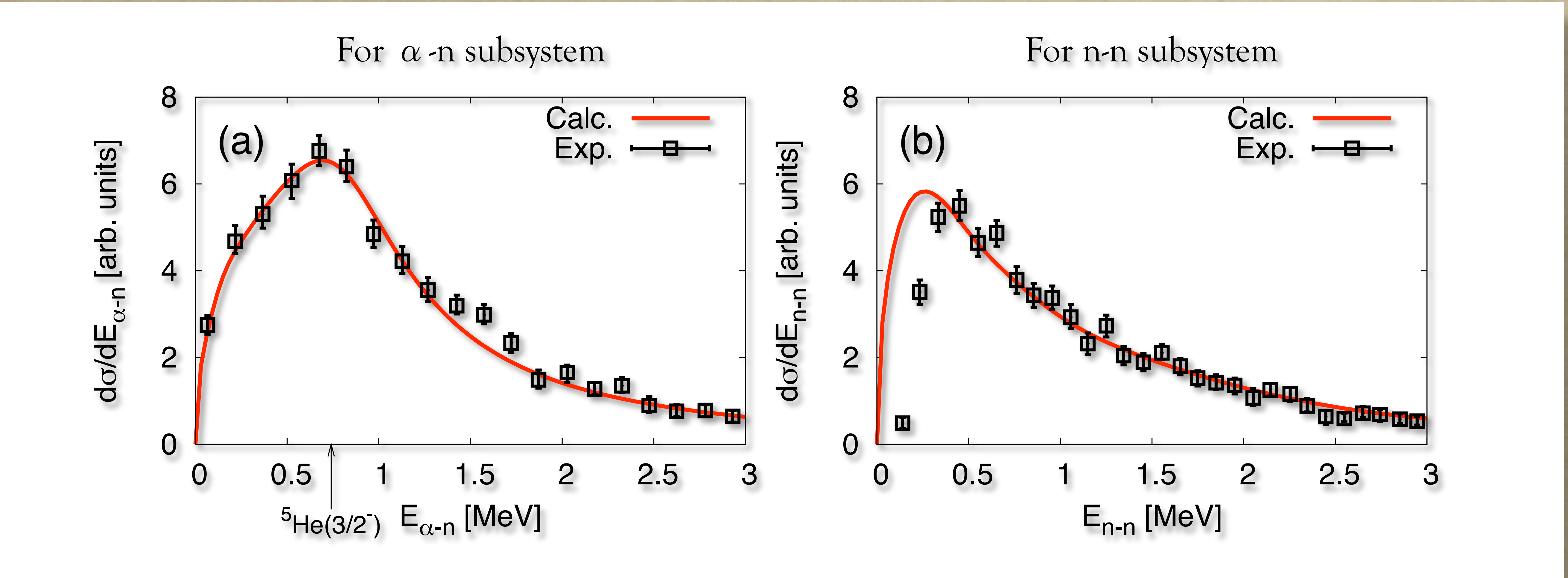
- Calculated cross section in comparison with the observed data.
  - Our result well reproduces the Coulomb breakup cross section especially in low-energy region below 2 MeV.
  - It is confirmed that the low-lying enhancement in the cross section can be reproduced in CSLS.



T. Aumann et al., PRC 59, 1252 (1999).

Calculated invariant mass spectra

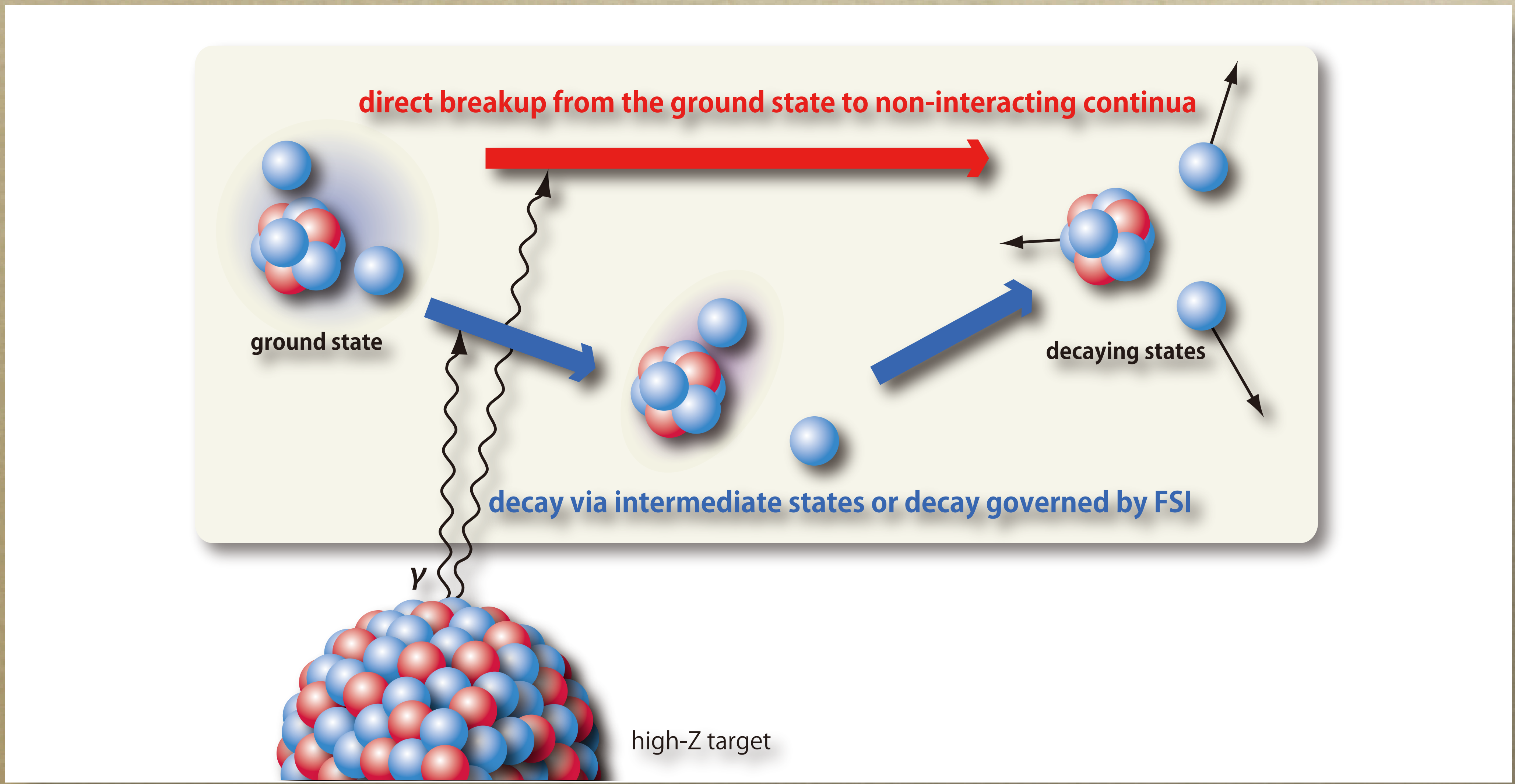
- Calculated invariant mass spectra for α-n and n-n subsystems.
  - Shapes of the invariant mass spectra are well reproduced by using CSLS.
  - CSLS enables us to discuss the structures and correlations not only of the total system but also of the binary subsystems.



How the ground-state information survive?

- In CSLS, the breakup component is given as the first term in the E1 transition matrix element.

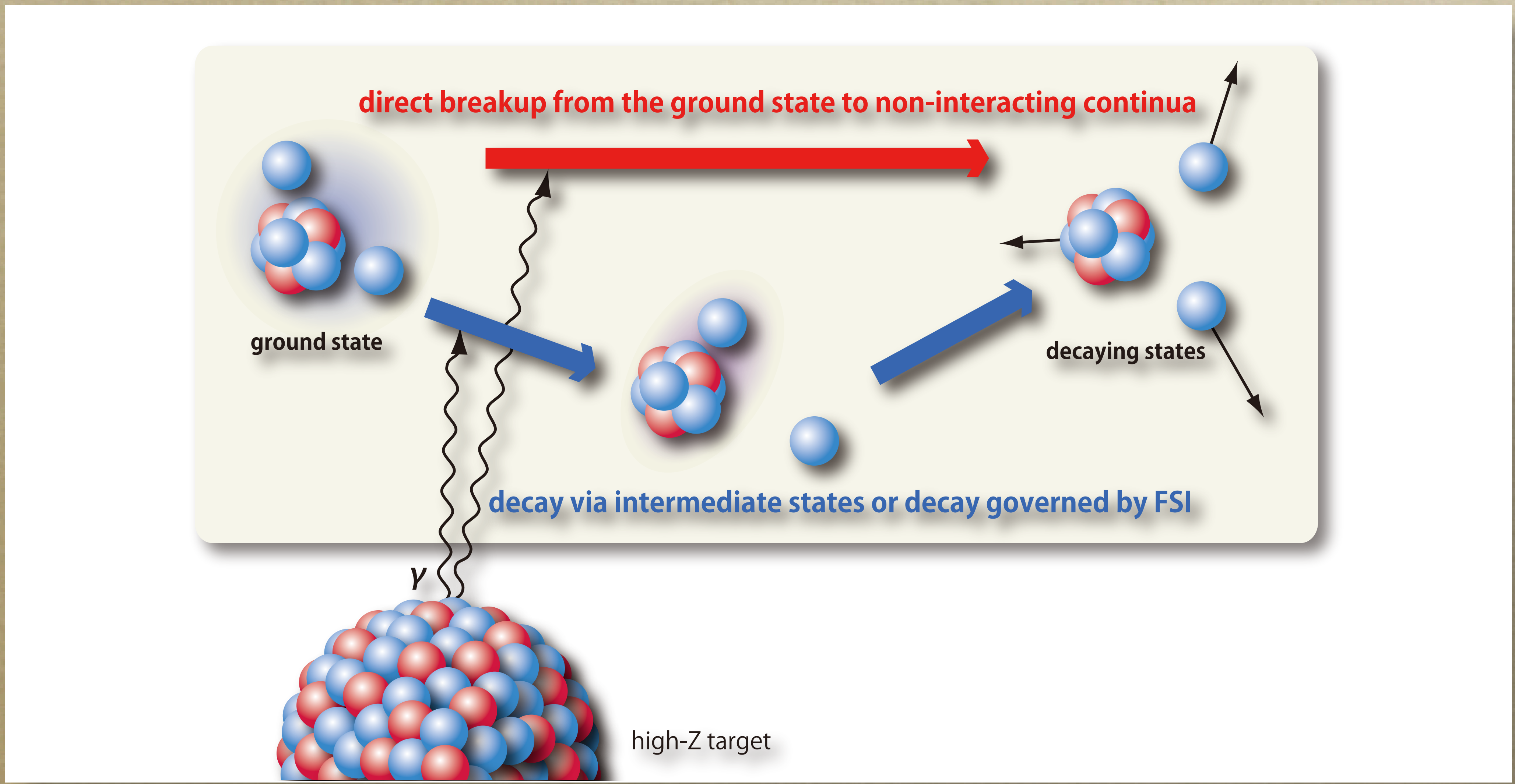
$$\langle \Psi^{(+)}(\mathbf{k},\mathbf{K})|\hat{O}(E1)|\Phi_{\rm gs}\rangle = \langle \Phi_0(\mathbf{k},\mathbf{K})|\hat{O}(E1)|\Phi_{\rm gs}\rangle + \sum\!\!\!\!\!\!\int_n \langle \Phi_0(\mathbf{k},\mathbf{K})|\hat{V}U^{-1}(\theta)|\chi_n^\theta\rangle \frac{1}{E-E_n^\theta}\langle\tilde{\chi}_n^\theta|U(\theta)\hat{O}(E1)|\Phi_{\rm gs}\rangle$$



How the ground-state information survive?

- In CSLS, the breakup component is given as the first term in the E1 transition matrix element.

$$\langle \Psi^{(+)}(\mathbf{k},\mathbf{K})|\hat{O}(E1)|\Phi_{\rm gs}\rangle = \underline{\langle \Phi_0(\mathbf{k},\mathbf{K})|\hat{O}(E1)|\Phi_{\rm gs}\rangle} \quad \text{direct breakup component}$$
$$+ \sum_n\!\!\!\!\!\!\int \langle \Phi_0(\mathbf{k},\mathbf{K})|\hat{V}U^{-1}(\theta)|\chi_n^\theta\rangle \frac{1}{E-E_n^\theta}\langle\tilde{\chi}_n^\theta|U(\theta)\hat{O}(E1)|\Phi_{\rm gs}\rangle$$



How the ground-state information survive?

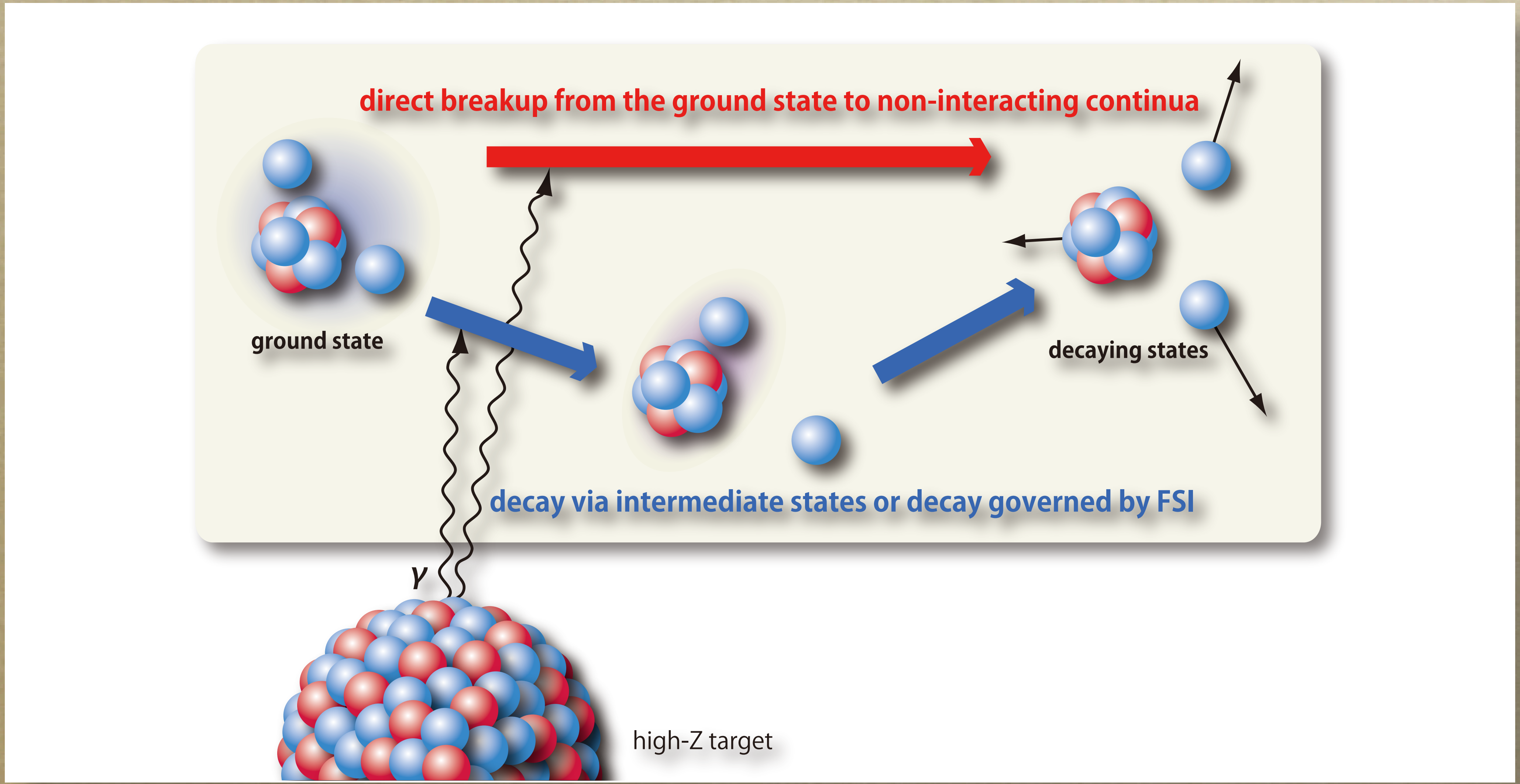
- In CSLS, the breakup component is given as the first term in the E1 transition matrix element.

$$\langle \Psi^{(+)}(\mathbf{k},\mathbf{K})|\hat{O}(E1)|\Phi_{\rm gs}\rangle = \langle \Phi_0(\mathbf{k},\mathbf{K})|\hat{O}(E1)|\Phi_{\rm gs}\rangle$$

direct breakup component

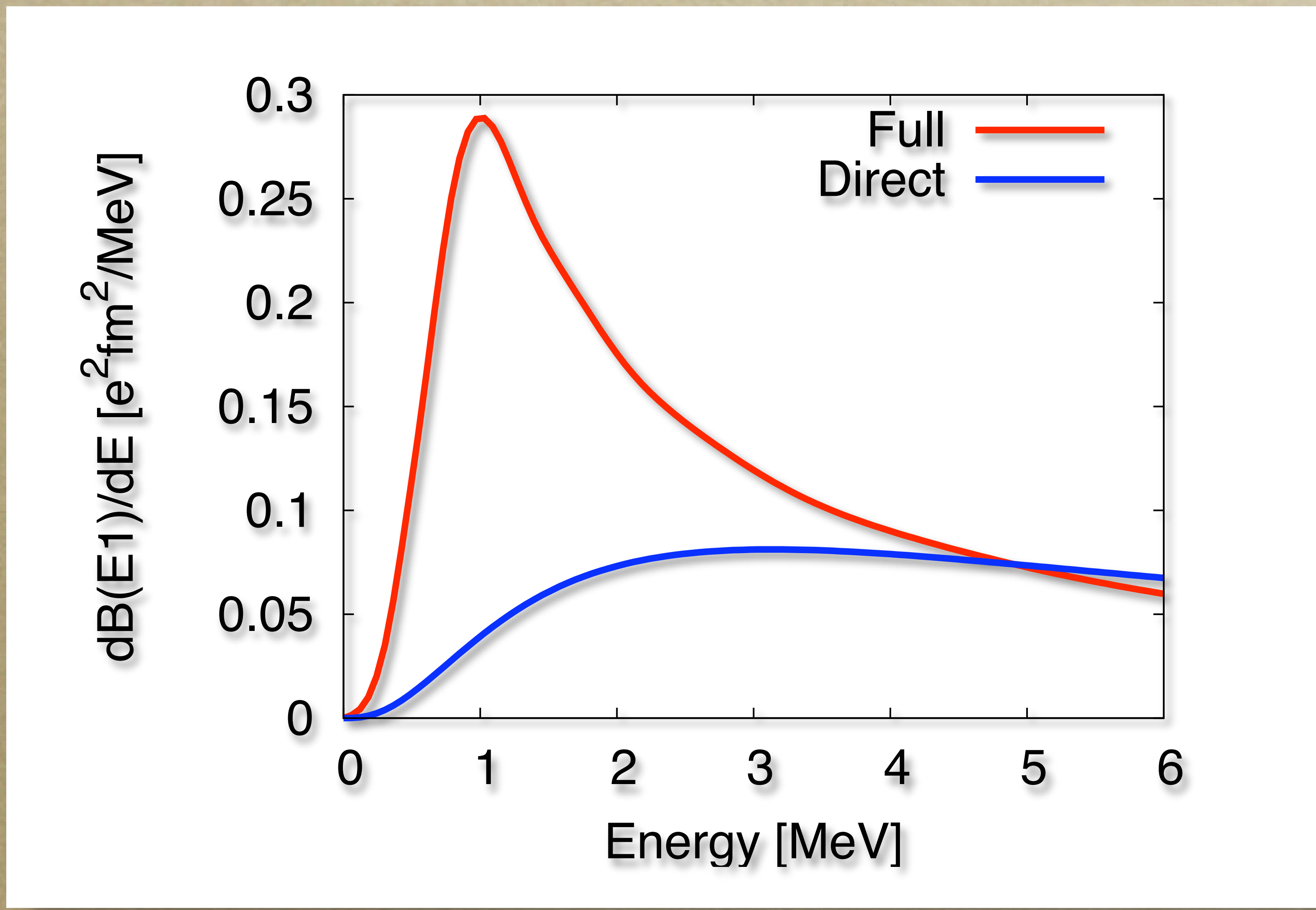
$$+ \sum_n \langle \Phi_0(\mathbf{k},\mathbf{K})|\hat{V}U^{-1}(\theta)|\chi_n^\theta\rangle \frac{1}{E-E_n^\theta}\langle\tilde{\chi}_n^\theta|U(\theta)\hat{O}(E1)|\Phi_{\rm gs}\rangle$$

effect of sequential decay and final state interaction



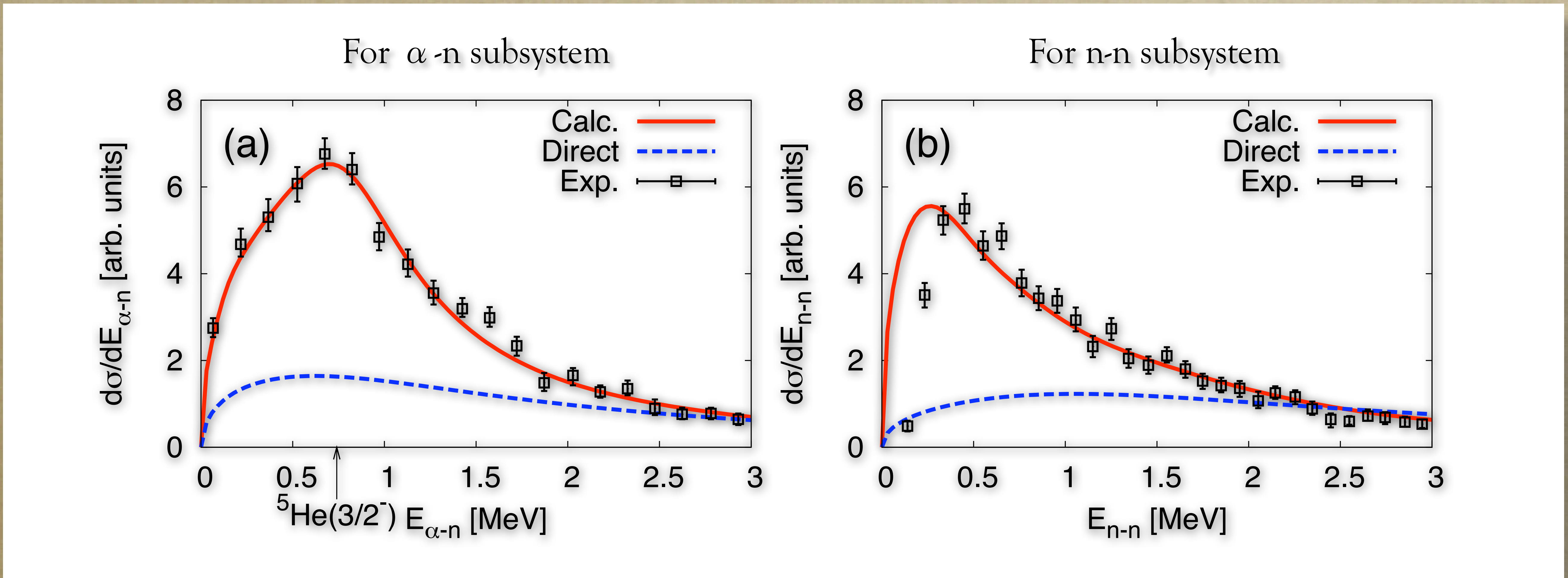
FSI effect in the cross section

- Comparison - direct breakup component vs final state interaction
  - It is shown that the low-lying enhancement in the cross section cannot be reproduced by direct breakup component.
  - FSI plays a key role in the Coulomb breakup reaction of ^{6}He, and hence, the information on the ground state might be masked by FSI.



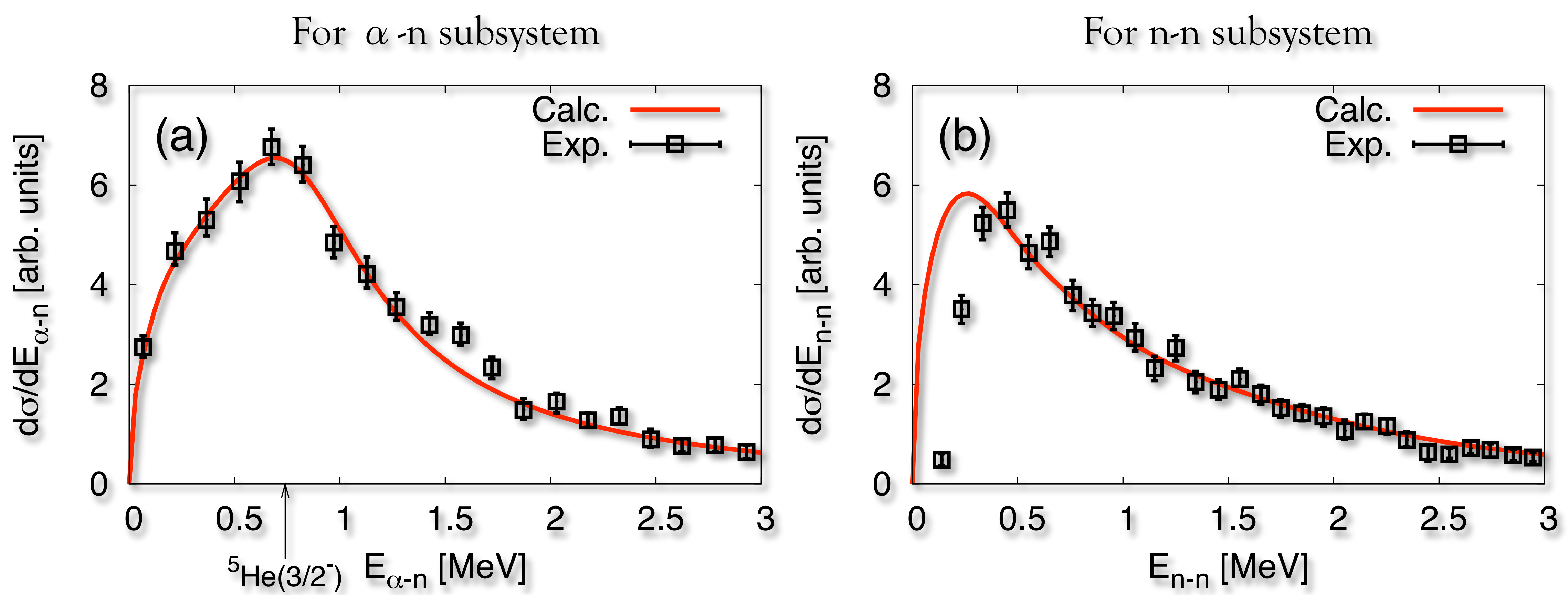
FSI effect in the invariant mass spectra

- Comparison - direct breakup component vs. final state interaction
  - Similarly to the cross section, FSI plays a key role in reproducing the spectra.
  - Peaks in the cross section and invariant mass spectra comes from the structures of three-body continuum states of ^{6}He.



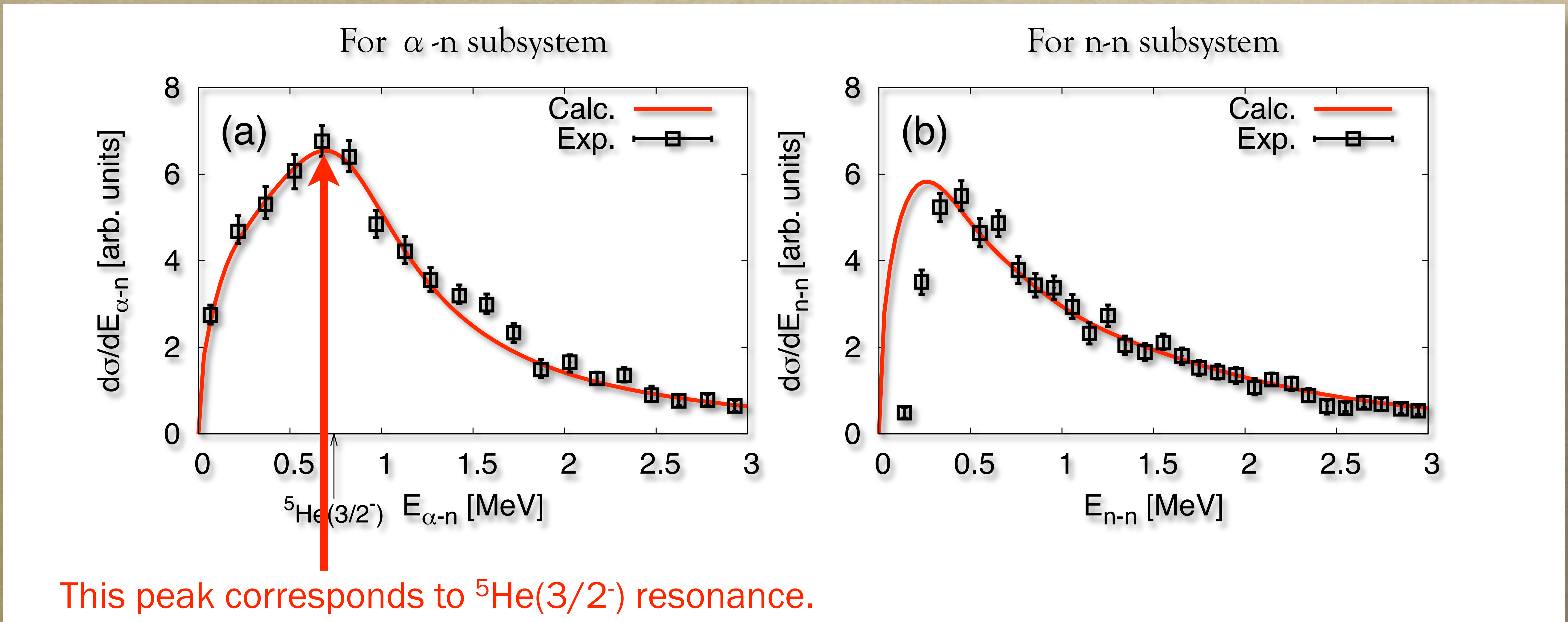
What kinds of FSI are important?

- From the invariant mass spectra, we can see what kinds of FSI are important in the Coulomb breakup reaction of ^{6}He.
  - For α-n subsystem, the $^{5}He(3/2^{-})$ resonance is important.
  - For n-n subsystem, the peak indicates the importance of n-n virtual states.



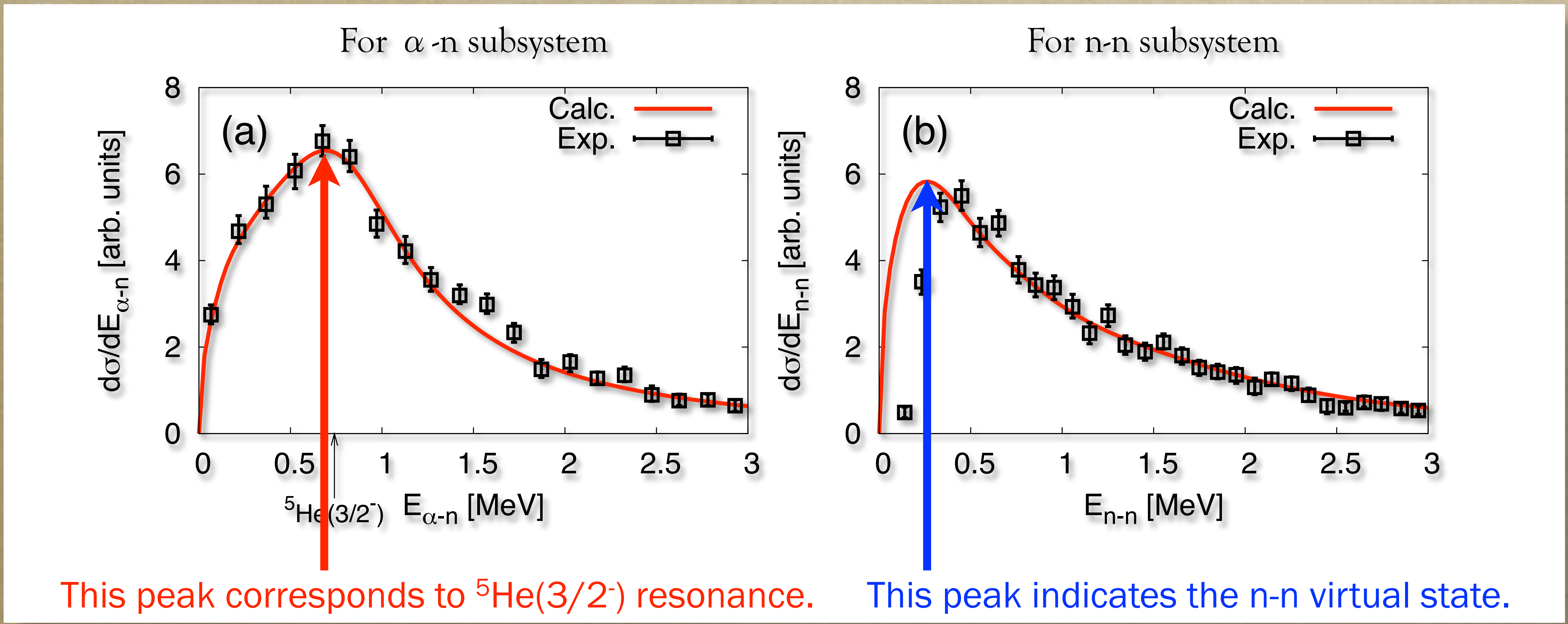
What kinds of FSI are important?

- From the invariant mass spectra, we can see what kinds of FSI are important in the Coulomb breakup reaction of ^{6}He.
  - For α-n subsystem, the ^{5}He(3/2$^-$) resonance is important.
  - For n-n subsystem, the peak indicates the importance of n-n virtual states.



What kinds of FSI are important?

- From the invariant mass spectra, we can see what kinds of FSI are important in the Coulomb breakup reaction of ^{6}He.
  - For α-n subsystem, the ^{5}He(3/2$^-$) resonance is important.
  - For n-n subsystem, the peak indicates the importance of n-n virtual states.



Summary

- We calculate the Coulomb breakup cross section and invariant mass spectra for ^{6}He, and discuss the structure of continuum states in final states.
  - To calculate the continuum states, we employ the core+n+n cluster model and complex-scaled solutions of Lippmann-Schwinger equation.
  - By using the CSLS, the calculated cross section and invariant mass spectra well reproduce the observed data.
  - From the obtained results, the direct breakup component, which corresponds to the Fourier transform of the ground-state wave function, has no significant contribution to the cross section and spectra.
    ➡ The information with respect to the ground-state structure is masked by the strong FSI.
  - The FSI of ^{5}He($3/2^-$) and n-n virtual state play key roles in reproducing the observed cross section and spectra.