3粒子融合過程研究の現状と将来

Ternary Fusion Process: now and future

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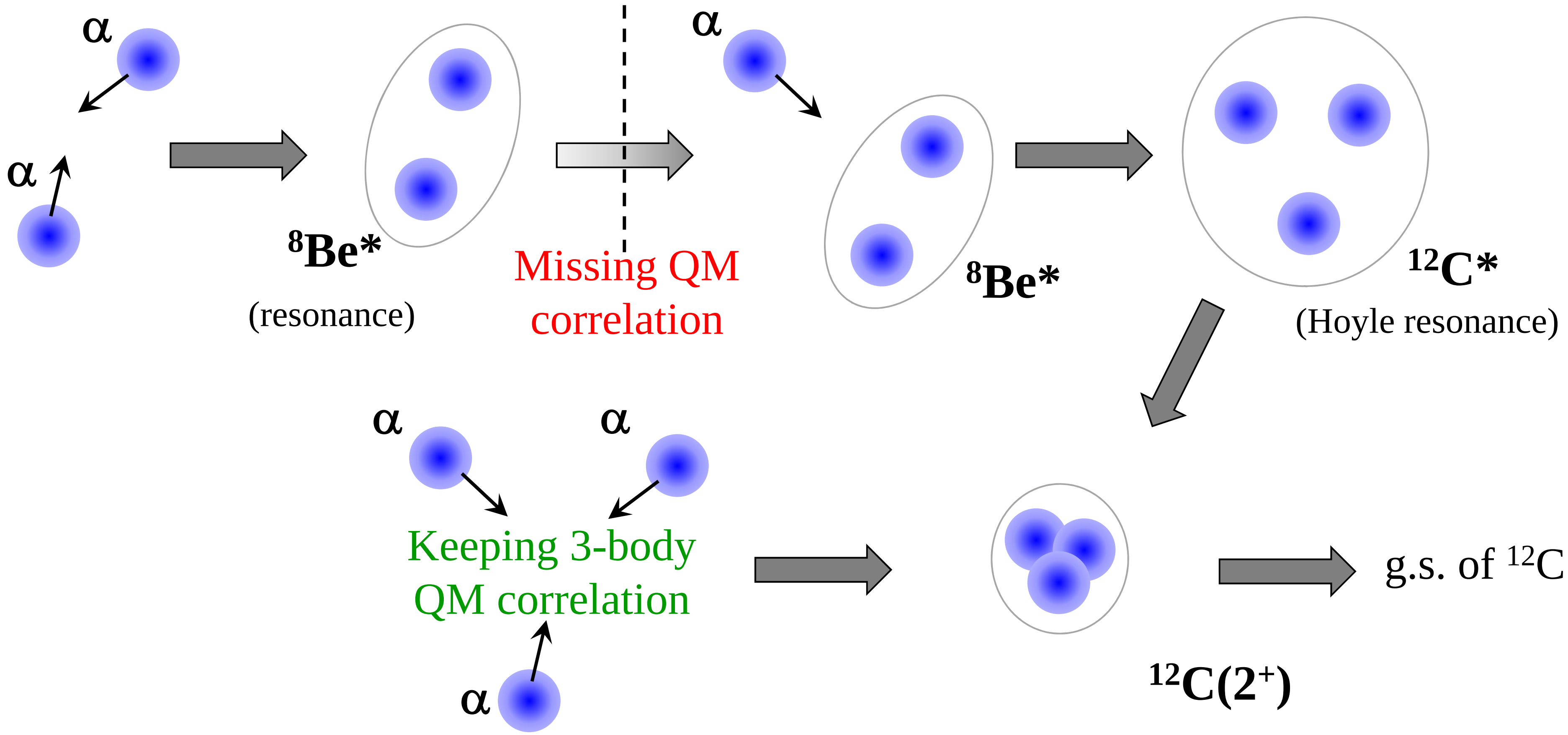
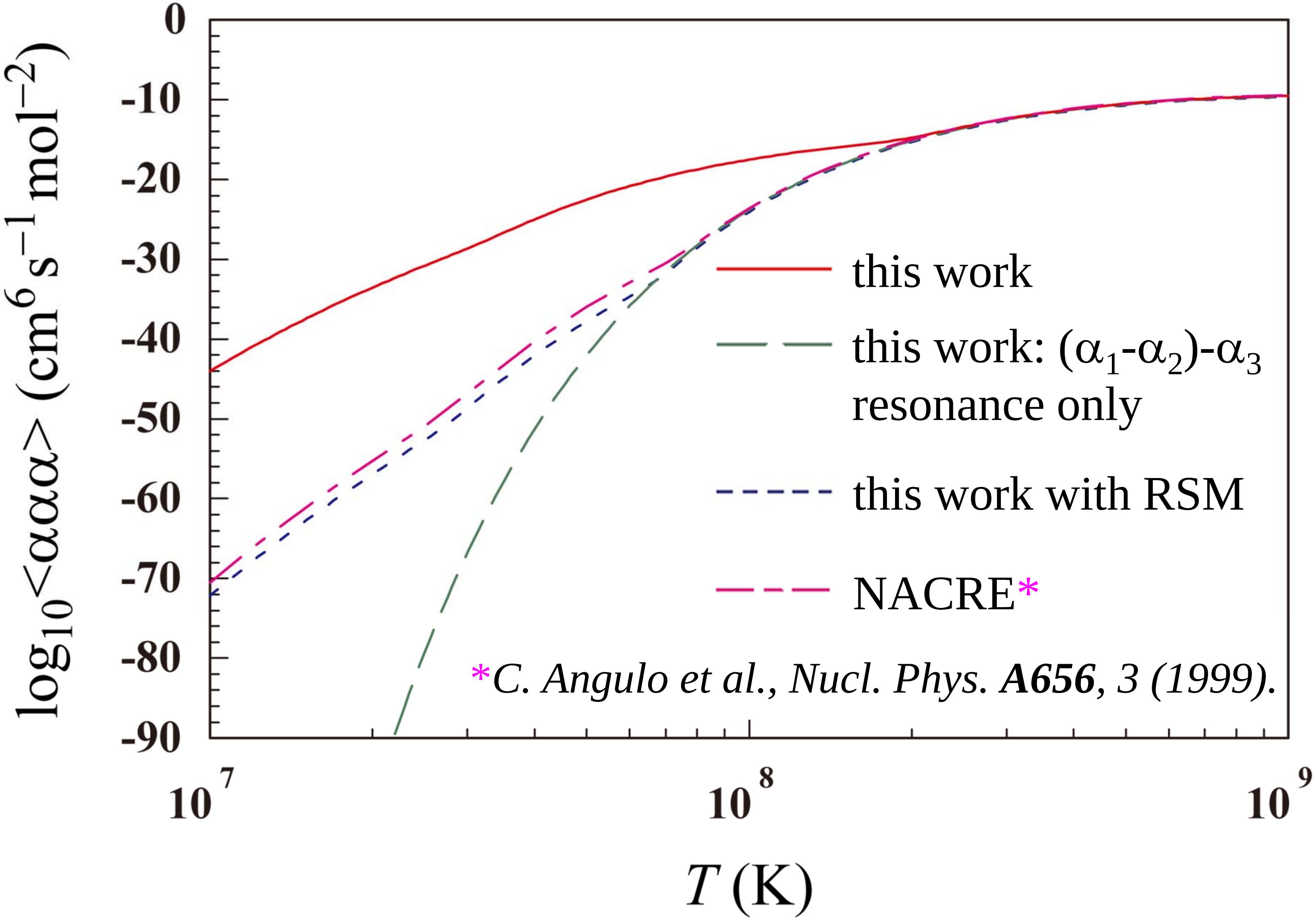
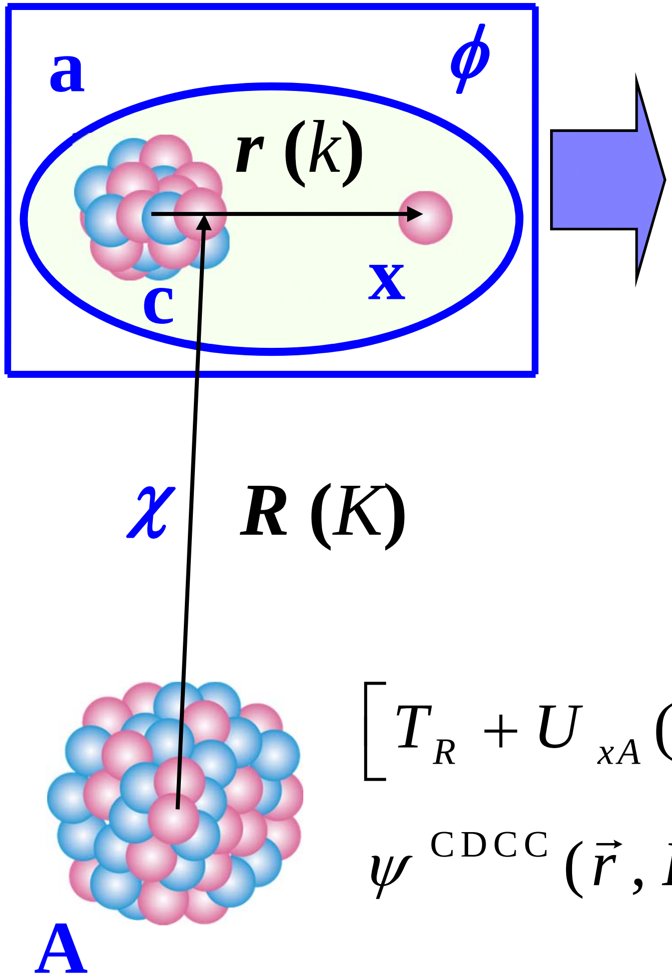


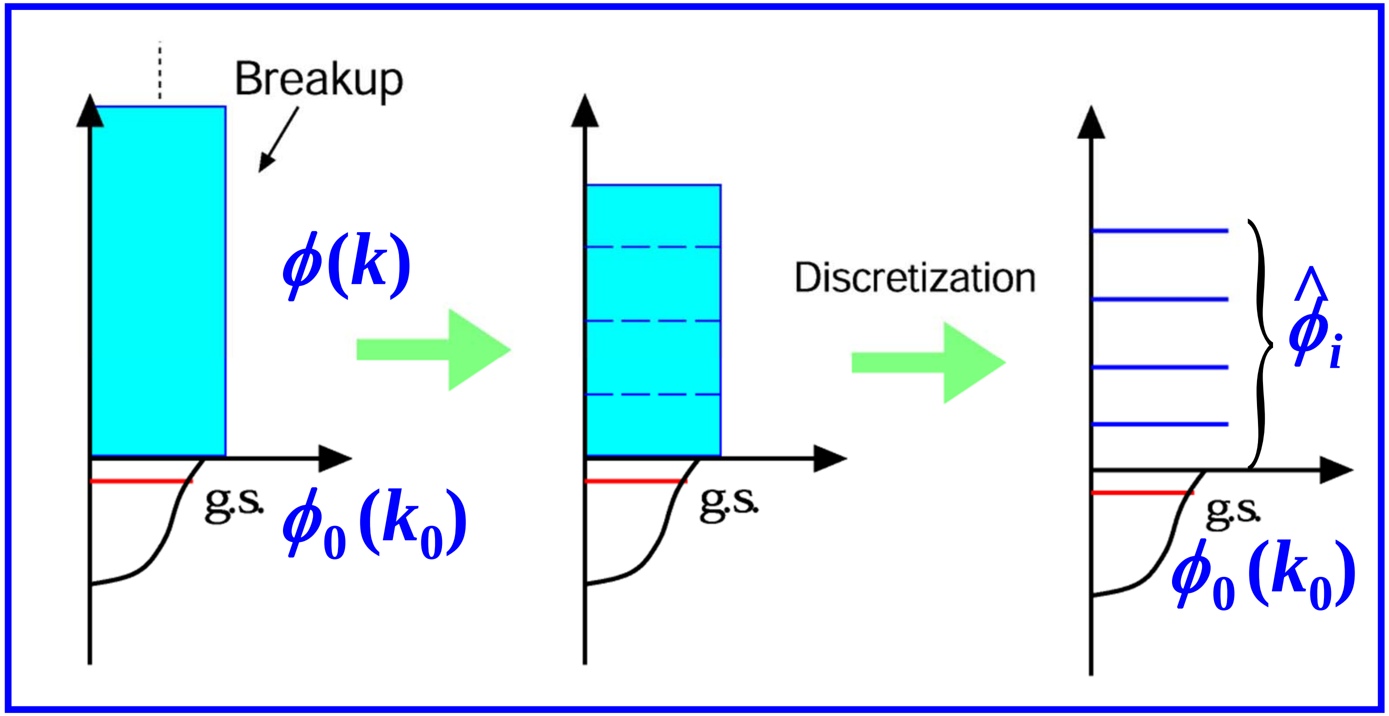
The reaction rate

—— *K.O., M. Kan, and M. Kamimura, Prog. Theor. Phys.* ***122****, 1055 (2009); arXiv:0905.0007 [astro-ph.SR].*



The Continuum-Discretized Coupled Channels method (CDCC)





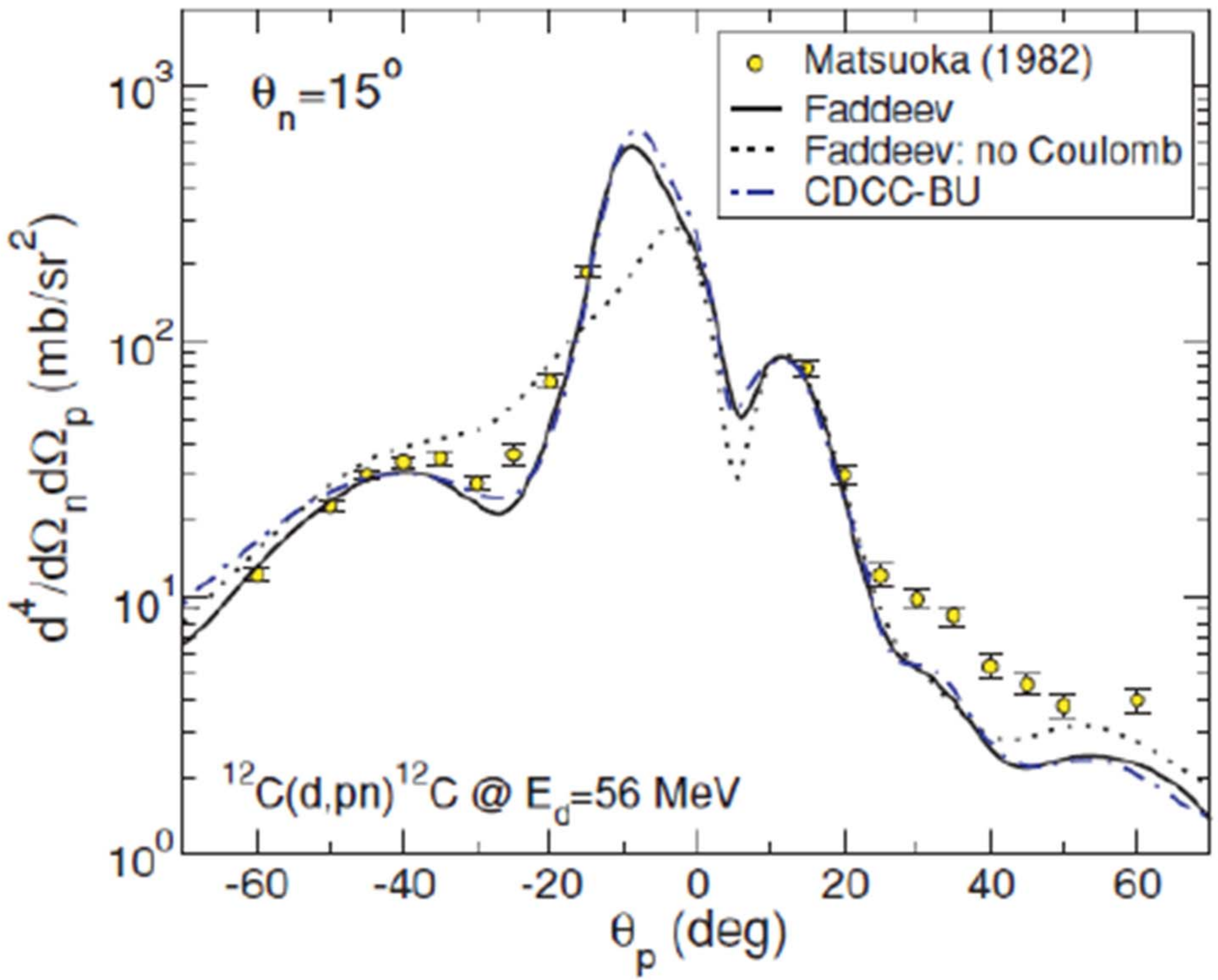
$$\left[T_R + U_{xA}(\vec{r},\vec{R}) + U_{cA}(\vec{r},\vec{R}) + h_a\left(\vec{r}\right) - E\right]\psi^{CDCC}(\vec{r},\vec{R}) = 0,$$

$$\psi^{CDCC}(\vec{r},\vec{R}) = \sum_{i=0}^{i_{max}} \hat{\phi}_i(\vec{r})\hat{\chi}_i(\hat{K}_i,\vec{R}).$$

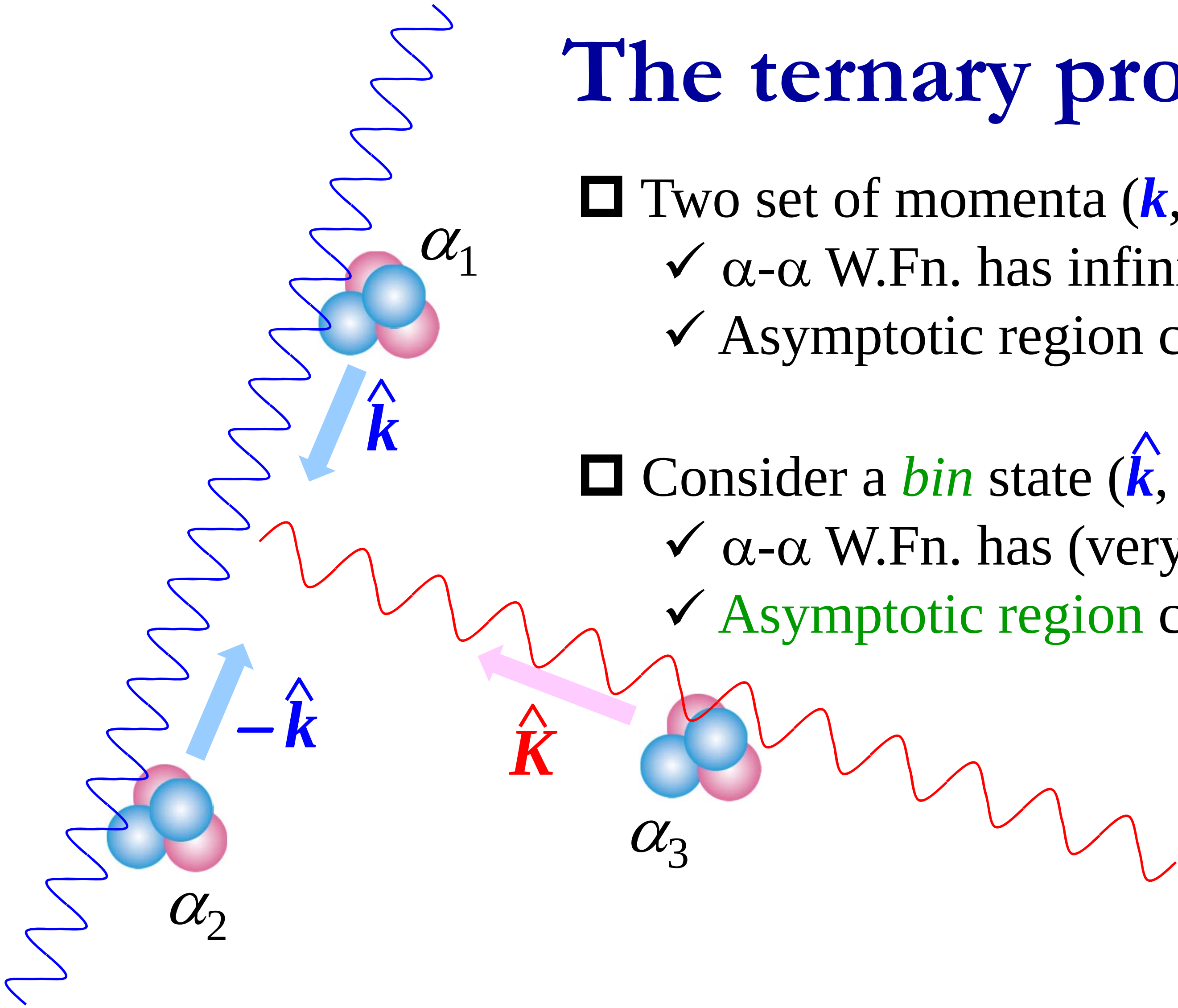
$$\hat{\chi}_i \to U_i^{(-)}\delta_{i0} - \sqrt{K_0 / K_i}\,S_{i0}U_i^{(+)}$$

$^{12}C(d,pn)$ at 56 MeV

A. Deltuva, A. M. Moro, E. Cravo, F. M. Nunes, and A. Fonseca, PRC**76**, 064602 (2007).



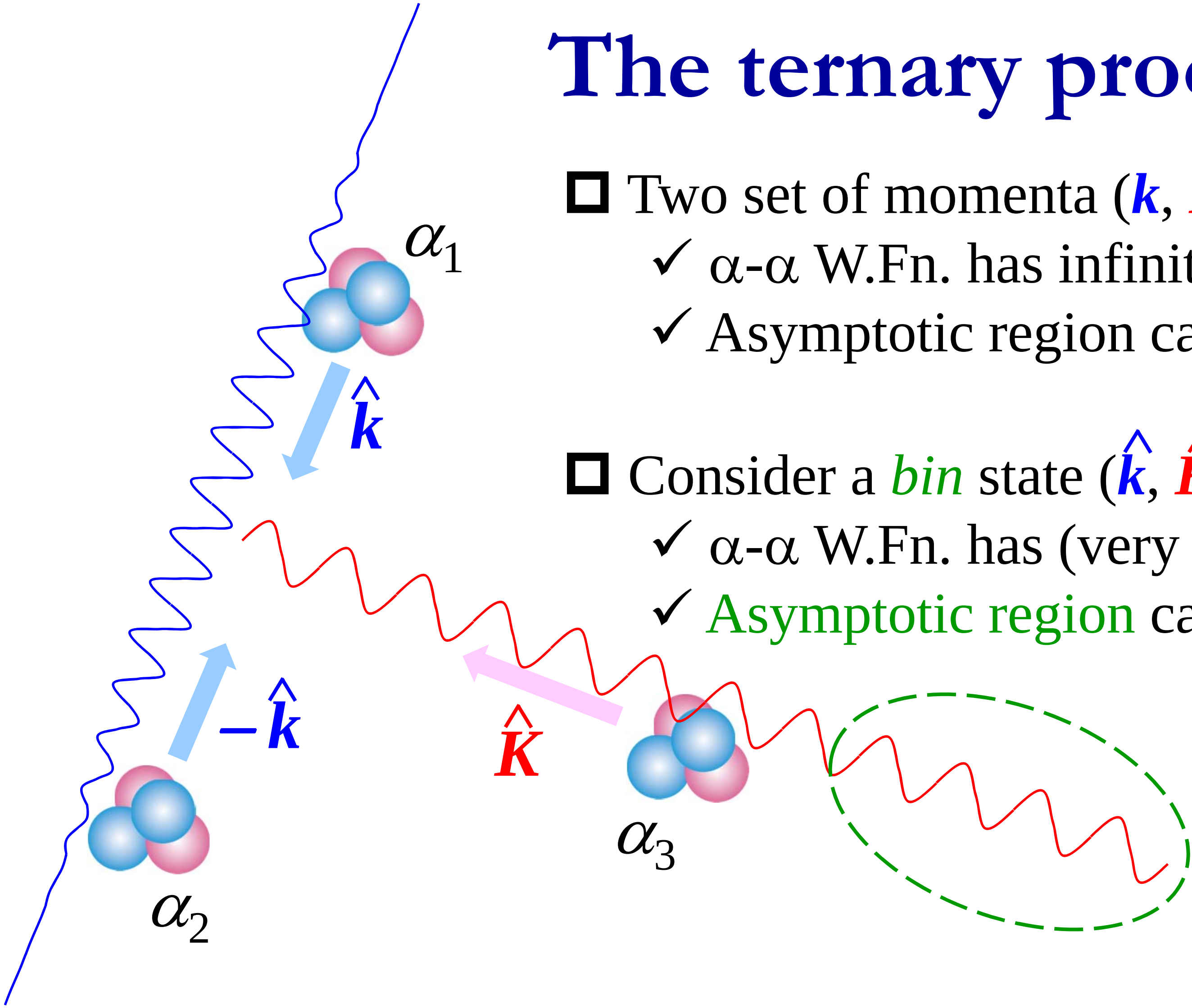
The ternary process



- Two set of momenta (***k***, ***K***) define the incident channel.
  - ✓ α-α W.Fn. has infinite range.
  - ✓ Asymptotic region cannot be defined.

- Consider a *bin* state ($\hat{k}$, $\hat{K}$) as an incident channel.
  - ✓ α-α W.Fn. has (very long but) finite range.
  - ✓ Asymptotic region can be defined.

The ternary process



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Reaction probability (1/3)

- **Low energy 3α wave function**

$$\Psi^{\text{free}}_{3\alpha,\boldsymbol{K},\boldsymbol{k}}(\boldsymbol{R},\boldsymbol{r}) = \frac{1}{\sqrt{(2\pi)^3}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \frac{1}{\sqrt{(2\pi)^3}} e^{i\boldsymbol{K}\cdot\boldsymbol{R}}$$

Binning over *k*, taking s-wave component of each plane wave, and including channel coupling (CC).

αα-α W. Fn.s

$$\Psi^{0^+}_{\hat{k}_{i_0},E}(r,R) = \sqrt{\frac{2}{\pi}} \frac{i}{32\pi^2} e^{i\{\delta(\hat{k}_{i_0})+\pi/2\}} \frac{\sqrt{N_{i_0}}}{\hat{k}_{i_0}\hat{K}_{i_0}} \sum_{i=1}^{i_{\max}} \frac{\hat{u}_i(r)}{r} \frac{\hat{\chi}_i^{(i_0)}(R)}{R}$$

initial condition

(cancel out)

orthonormal αα W. Fn.s

$$\hat{u}_i(r) = \frac{1}{\sqrt{N_i}} \int_{k_i}^{k_{i+1}} f_i(k) u(k,r) dk, \quad N_i = \int_{k_i}^{k_{i+1}} |f_i(k)|^2 dk.$$

weight

Reaction probability (2/3)

- **Normalization of scattering wave in macroscopic space Ω**

$$\int_\Omega d\boldsymbol{r} \int_\Omega d\boldsymbol{R} \left(\frac{C}{(2\pi)^3} e^{i\boldsymbol{k}_0\cdot\boldsymbol{r}} e^{i\boldsymbol{K}_0\cdot\boldsymbol{R}} \right)^* \frac{C}{(2\pi)^3} e^{i\boldsymbol{k}_0\cdot\boldsymbol{r}} e^{i\boldsymbol{K}_0\cdot\boldsymbol{R}} = 1$$

$$C = \frac{(2\pi)^3}{V_\Omega}$$

volume of Ω

- **E2 transition probability (per unit time)**

GEM with rearrangement

$$P_{\hat{k}_{i_0},E} \equiv \frac{4\pi}{75\hbar} \left(\frac{\hbar\omega}{\hbar c} \right)^5 \sum_M \left| \left\langle \Psi_M^{2^+} \middle| O_M^{E2} \middle| \frac{C}{N_{i_0}} \Psi_{\hat{k}_{i_0},E}^{0^+} \right\rangle \right|^2$$

averaged 3a W. Fn.

✓ Probability of E2 transition in Ω

✓ Ω contains 3 alpha particles.

✓ This probability depends on Ω (proportional to V_Ω^{-2}).

Reaction probability (3/3)

- **We need the probability per unit volume when α density is unity.**

$$\bar{P}_{\hat{k}_{i_0},E} = V_\Omega^3 \frac{P_{\hat{k}_{i_0},E}}{V_\Omega}$$

prop. to V_Ω^{-2}

independent of V_Ω!

$$= \frac{4\pi(2\pi)^6}{75\hbar}\left(\frac{\hbar\omega}{\hbar c}\right)^5 \sum_M \left| \left\langle \Psi_M^{2^+} \left| O_M^{\mathrm{E2}} \right| \frac{1}{N_{i_0}} \Psi_{\hat{k}_{i_0},E}^{0^+} \right\rangle \right|^2$$

⇕

$$(\sigma v)_{\hat{k}_{i_0},E} = \frac{2(2\pi)^7}{75\hbar}\left(\frac{\hbar\omega}{\hbar c}\right)^5 \sum_M \left| \left\langle \Psi_M^{2^+} \left| O_M^{\mathrm{E2}} \right| \Psi_{\hat{k}_{i_0},E}^{0^+} \right\rangle \right|^2$$

Reaction rate (1/2)

- **Integration of σv with Maxwell Boltzmann distribution**

$$\langle \alpha\alpha\alpha \rangle (T) \equiv \int\int\int \bar{P}_{\hat{k}_{i_0},E} \quad \varpi (v_1, v_2, v_3)\, d\boldsymbol{v}_1 d\boldsymbol{v}_2 d\boldsymbol{v}_3$$

$$\varpi (v_1, v_2, v_3) \equiv \left[\left(\frac{m_\alpha}{2\pi k_B T} \right)^{3/2} \right]^3 \exp \left[-\frac{m_\alpha}{2k_B T} \left(v_1^2 + v_2^2 + v_3^2 \right) \right]$$

$$\varpi (v_1, v_2, v_3) \longrightarrow \varpi' (v_G, v_r, v_R)$$

$$\langle \alpha\alpha\alpha \rangle (T) = \frac{4}{\pi (k_B T)^3} \int\int \bar{P}_{\hat{k}_{i_0},E} \quad \exp \left[-\frac{\varepsilon_r + \varepsilon_R}{k_B T} \right] \sqrt{\varepsilon_r \varepsilon_R}\, d\varepsilon_r d\varepsilon_R$$

$$= \frac{4}{\pi (k_B T)^3} \int\int \bar{P}_{\hat{k}_{i_0},E} \quad \exp \left[-\frac{E}{k_B T} \right] \sqrt{\varepsilon_r (E - \varepsilon_r)} \frac{\hbar^2 k}{\mu_r} dk dE$$

Reaction rate (2/2)

$$\langle\alpha\alpha\alpha\rangle(T) = \frac{4}{\pi(k_{\mathrm{B}}T)^3}\int\int \bar{P}_{\hat{k}_{i_0},E} \exp\left[-\frac{E}{k_{\mathrm{B}}T}\right]\sqrt{\varepsilon_r(E-\varepsilon_r)}\frac{\hbar^2 k}{\mu_r}dkdE$$

$$\approx \frac{4}{\pi(k_{\mathrm{B}}T)^3}\int\left\{\sum_{i_0}^{i_{\mathrm{open}}}\sqrt{\hat{\varepsilon}_{i_0}(E-\hat{\varepsilon}_{i_0})}\frac{\hbar^2\hat{k}_{i_0}}{\mu_r}\left(\int_{k_{i_0}}^{k_{i_0+1}}\bar{P}_{\hat{k}_{i_0},E}\,dk\right)\right\}\exp\left[-\frac{E}{k_{\mathrm{B}}T}\right]dE$$

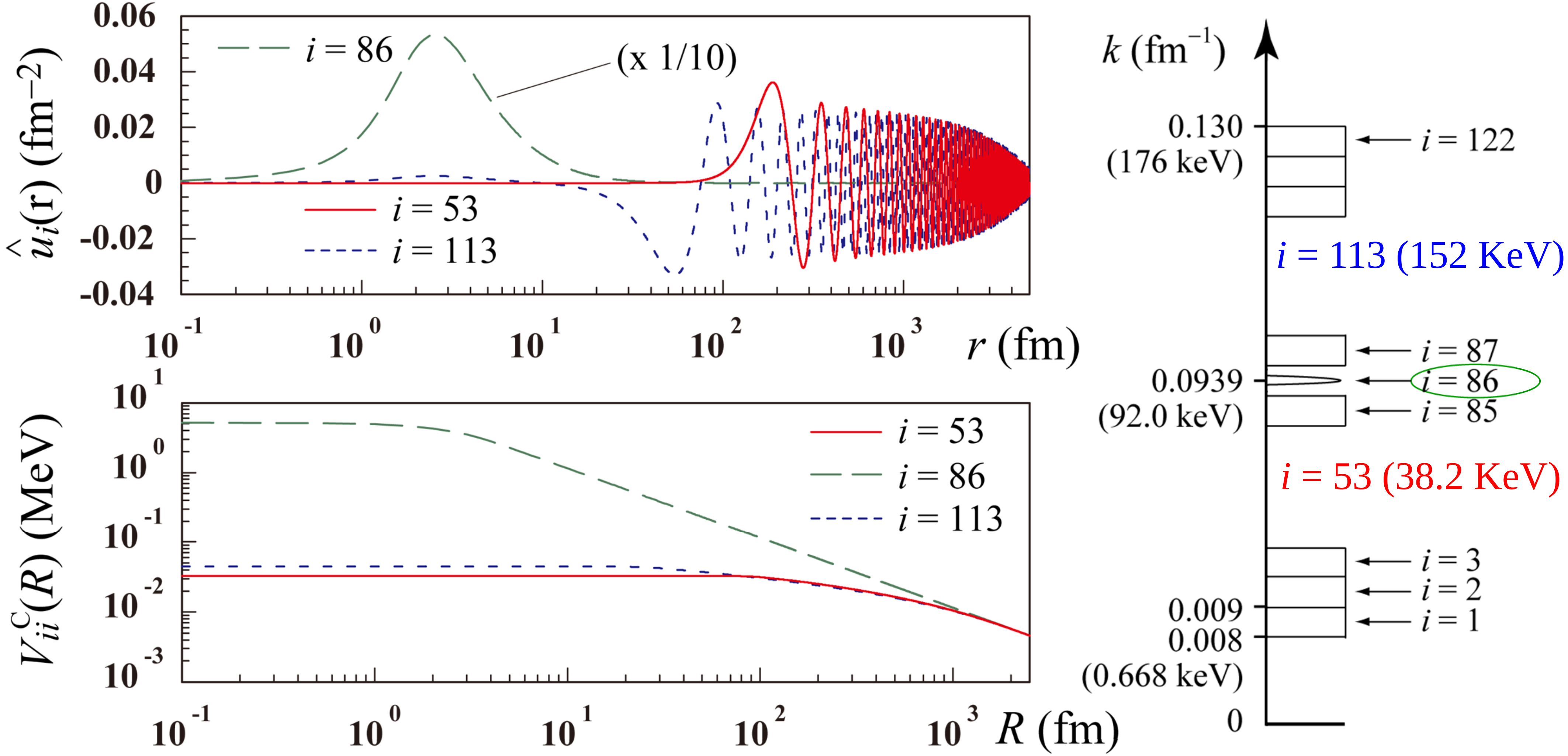
Incoherent sum (integration) over i_0 and E

$$\longrightarrow \bar{P}_{\hat{k}_{i_0},E}N_{i_0}$$

$$\langle\alpha\alpha\alpha\rangle(T) = 3N_{\mathrm{A}}^2\frac{4}{\pi(k_{\mathrm{B}}T)^3}\int\left\{\sum_{i_0=1}^{i_{\mathrm{open}}} w_{i_0}(\sigma v)_{\hat{k}_{i_0},E}\right\}\exp\left(-\frac{E}{k_{\mathrm{B}}T}\right)dE$$

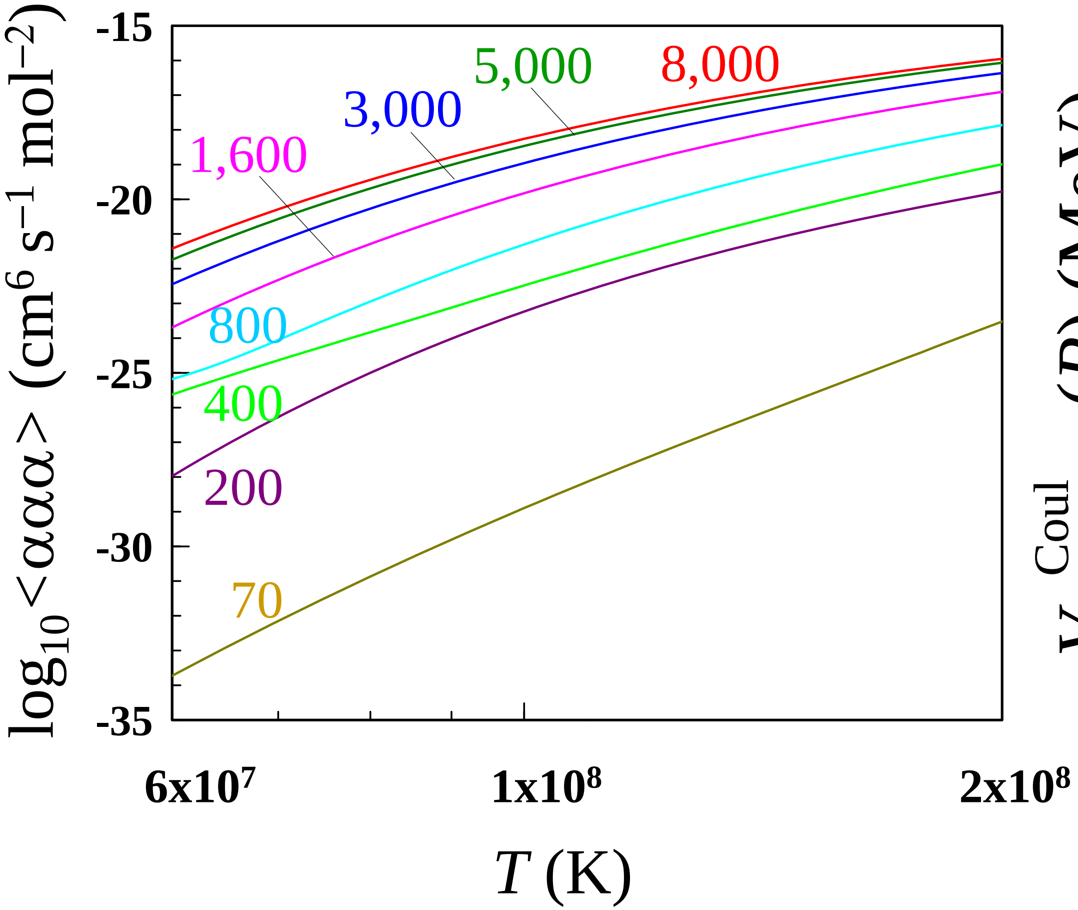
$$w_{i_0} = \frac{2\hat{\epsilon}_{12,i_0}}{\hat{k}_{i_0}}\sqrt{\hat{\epsilon}_{12,i_0}(E-\hat{\epsilon}_{12,i_0})}$$

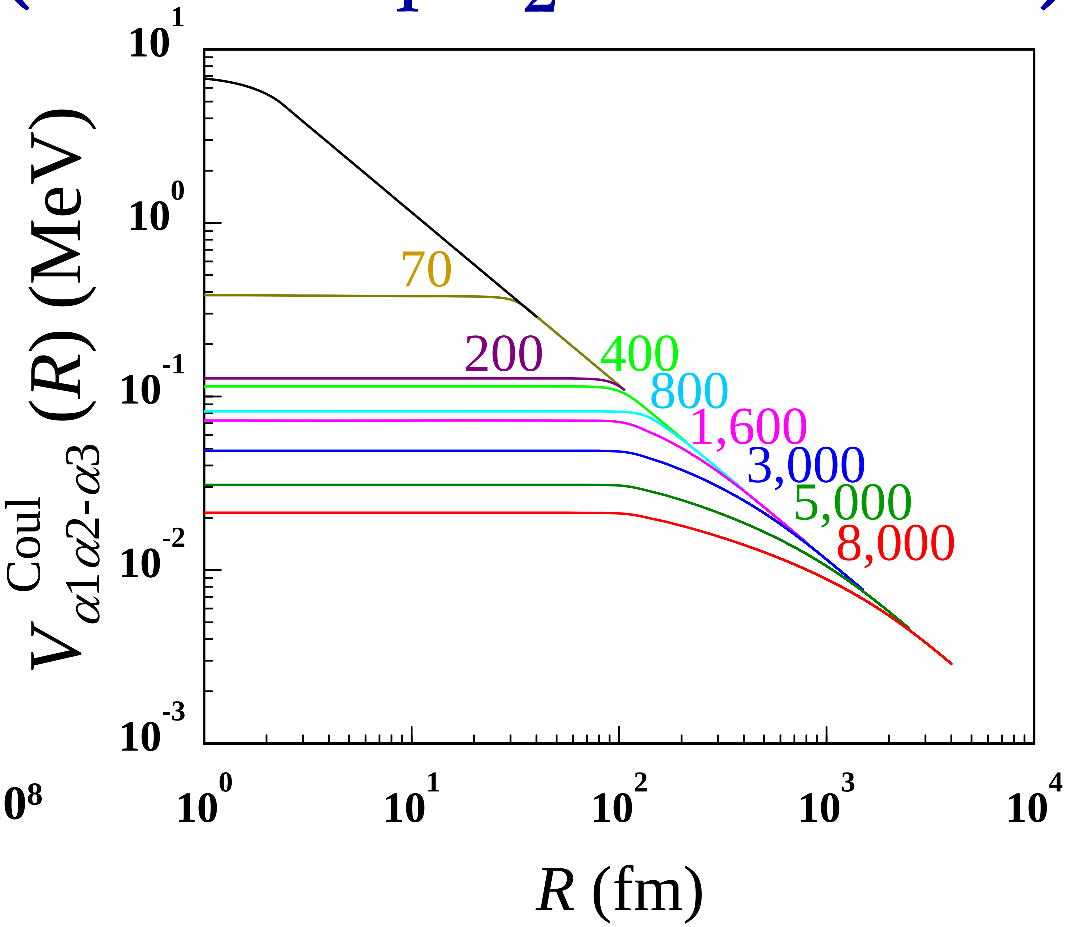
α_1-α_2 W.Fn. and (α_1-α_2)-α_3 Coulomb pot.



$$V_{ii}^{C}(R) = \left\langle \phi_i(\mathbf{r}) \middle| V_{\alpha 1 \alpha 3}^{C}(R_1) + V_{\alpha 2 \alpha 3}^{C}(R_2) \middle| \phi_i(\mathbf{r}) \right\rangle_{\mathbf{r}} .$$

Convergence with r_{max} (below α_1-α_2 resonance)





- ✓ Crude description of the αα continuum gives much smaller reaction rate (in good agreement with NACRE).
- ✓ 1ch calculation never converges.
- ✓ Adiabatic calculation does not work at all.

Summary

- **The three-body triple-α reaction rate is evaluated.**
  - ✓ The ternary fusion process (TFP) is formulated by CDCC.
  - ✓ The resonant and nonresonant processes are described on the same footing.
  - ✓ The α_1-α_2 nonresonant states below the resonance are essentially important.
  - ✓ The $(\alpha_1$-$\alpha_2)$-α_3 Coulomb barrier in the nonresonant capture process is much lower than that in the resonant process.
  - ✓ We obtain a markedly larger reaction rate than NACRE below 10^8 K.
  - ✓ Our rate is restricted to the density below about 3×10^5 g/cm^3.

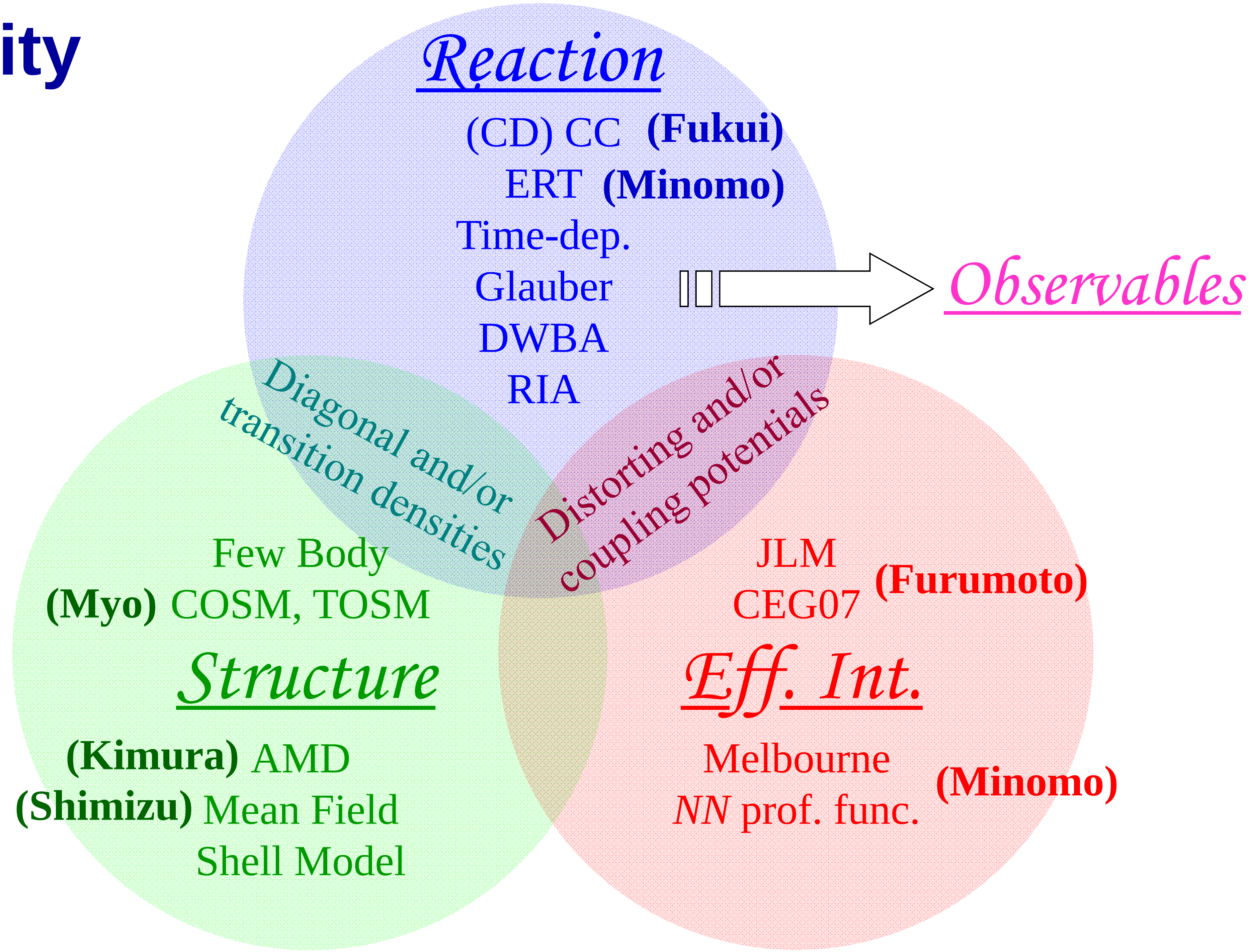
- **The previous method for the triple-α reaction is examined**
  - ✓ The Resonance Shift Method (used in many studies including NACRE) is shown *a very crude approximation* to the present three-body calculation.
  - ✓ The use of the triple α rate of NACRE implicitly assumes the reaction rate is just a “free” input parameter for astrophysics.

Future Perspective

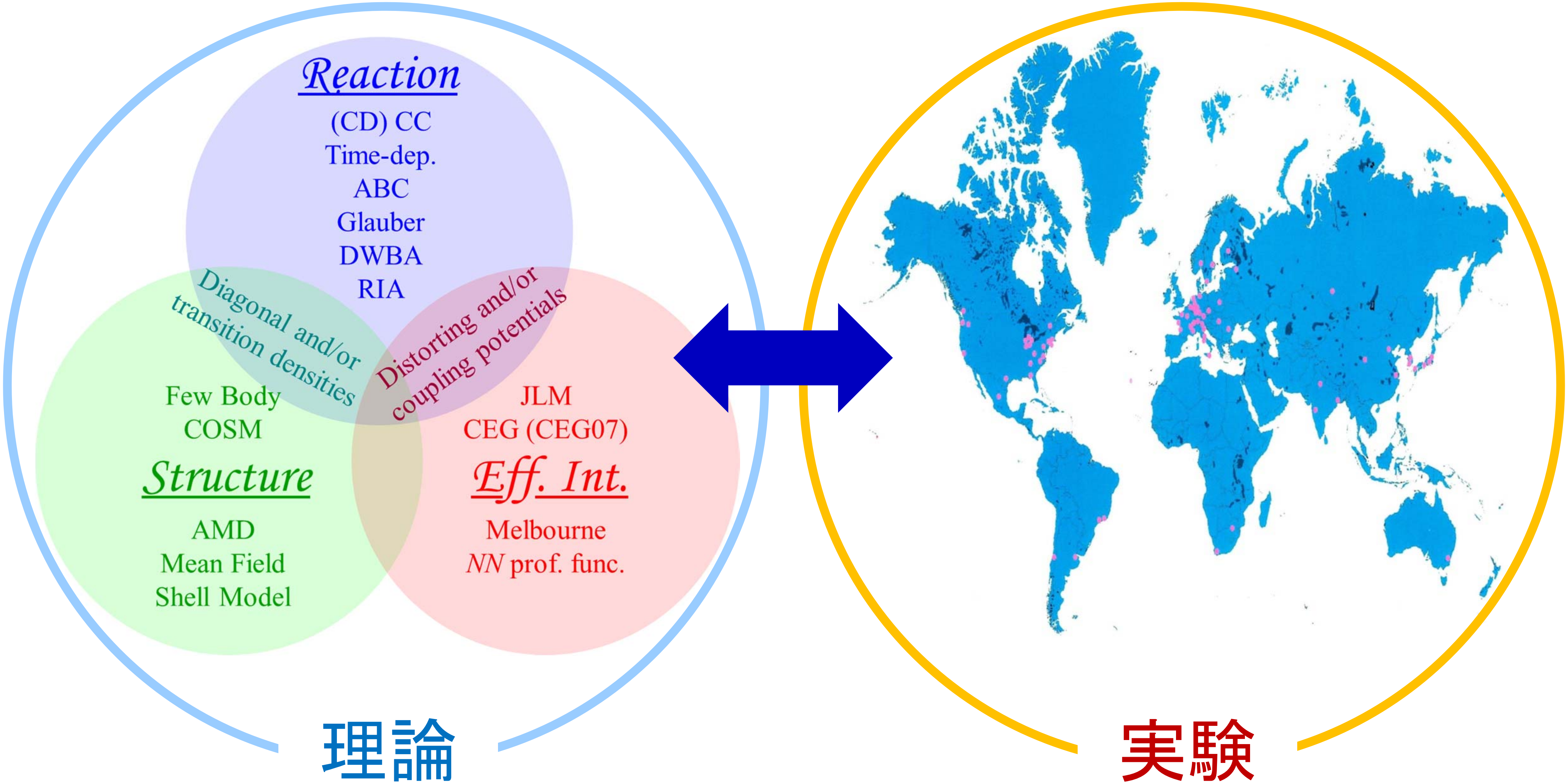
- **Rearrangement Channels**
  - ✓ Differential method will be more appropriate.
  - ✓ Inclusion of closed channels (compact W. Fn.) in the framework.
  - ✓ Important also for nonperturbative transfer calculation.

- **Understanding of the TFP nucleosynthesis**
  - ✓ $\alpha(\alpha n,\gamma)^9$Be, $n(p\alpha,{}^6$Li) etc.
  - ✓ 2p processes.
  - ✓ Experimental verification of TFP

Trinity


Reaction
(CD) CC (Fukui)
ERT (Minomo)
Time-dep.
Glauber
DWBA
RIA
Observables
Diagonal and/or
transition densities
Distorting and/or
coupling potentials
Few Body
(Myo) COSM, TOSM
Structure
(Kimura) AMD
(Shimizu) Mean Field
Shell Model
JLM
CEG07 (Furumoto)
Eff. Int.
Melbourne
NN prof. func. (Minomo)

実証的原子核物理学



※2012年3月初旬にRCNP研究会の開催を計画・申請中