

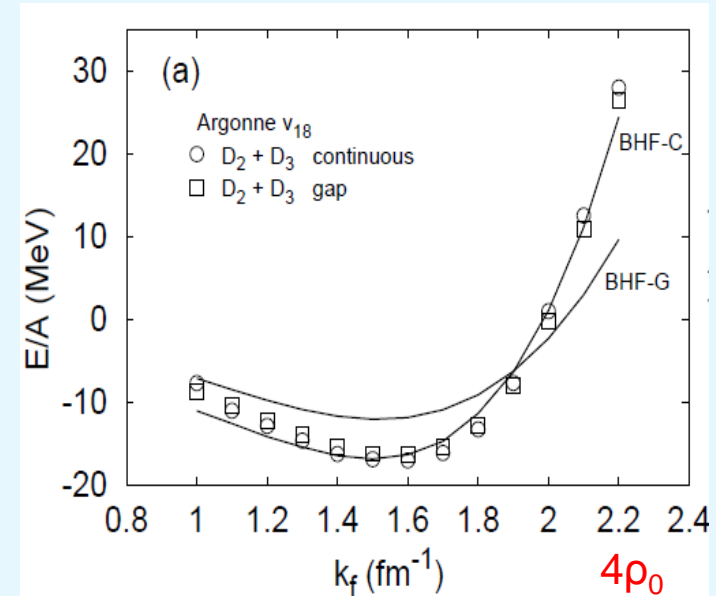
# ESC模型に基づく3体力

Y. Yamamoto (理研・肥山研)  
T.A. Rijken

1. Neutron-star matter and TBR
2. ESC-based TBF

# G行列理論に基づく nuclear saturation

LOBT with continuous choice  
is reliable up to high density



Role of Three-Body Interaction  
(TBA+TBR) is essential for  
saturation problem

- Attraction at low densities
- Repulsion at high densities → neutron-star matter

# TNI (Three Nucleon Interactions)

Lagaris-Pandharipande (Nucl.Phys.A359(1981) 349)

”unknown parts” for nuclear saturation property

⇒ phenomenological three nucleon interaction

⇒ effective density-dependent two-body interaction

(1) repulsion at high densities (TNR)

(2) attraction at low densities (TNA)

phenomenological

$$\tilde{V}_{TNI}(r; \rho) = \tilde{V}_{TNR}(r; \rho) + \tilde{V}_{TNA}(r; \rho)$$

$$\tilde{V}_{TNR}(r; \rho) = V_1 \exp(-(r/\lambda_1)^2) (1 - \exp(-\gamma_1 \rho))$$

$$\tilde{V}_{TNA}(r; \rho) = V_2 \exp(-(r/\lambda_2)^2) \rho (1 - \exp(-\gamma_2 \rho)) (\vec{\tau}_1 \cdot \vec{\tau}_2)^2$$

with  $NN$  interaction (RSC)

TNI2 :  $\kappa = 250$  MeV,    TNI3 :  $\kappa = 300$  MeV

## Phenomenological TBF (Urbana IX)

$$V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^R$$

$$V_{ijk}^{2\pi} = A \sum_{\text{cyc}} \left[ \{X_{ij}, X_{jk}\} \{\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j, \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k\} + \frac{1}{4} [X_{ij}, X_{jk}] [\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j, \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k] \right]$$

$$V_{ijk}^R = U \sum_{\text{cyc}} T^2(m_\pi r_{ij}) T^2(m_\pi r_{jk}) .$$

$$X_{ij} = Y(m_\pi r_{ij}) \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + T(m_\pi r_{ij}) S_{ij}$$

$$A = -0.0293 \text{ MeV and } U = 0.0048 \text{ MeV} \quad \text{variation}$$

$$A = -0.0333 \text{ MeV and } U = 0.00038 \text{ MeV} \quad \text{BHF}$$

### Effective two-body force

$$\bar{V}_{ij}^{\text{pheno}}(\mathbf{r}) = (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \left[ (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) V_C^{2\pi}(r) + S_{ij}(\hat{\mathbf{r}}) V_T^{2\pi}(r) \right] + V^R(r)$$

$$\bar{V}_{ij}^{\text{micro}}(\mathbf{r}) = (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) V_C^{\tau\sigma}(r) + (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) V_C^\sigma(r) + V_C(r) \\ + S_{ij}(\hat{\mathbf{r}}) \left[ (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) V_T^\tau(r) + V_T(r) \right] .$$

later

# G-matrix approach to hyperonic neutron-star matter with Nijmegen YN interaction models

M. Baldo, G.F.Burgio and H.-J. Schulze  
P.R. C61 (2000) 055801

**NSC89**

I. Vidana, A. Polls, R. Ramos, L. Engvik and M. Hjorth-Jensen  
P.R. C62 (2000) 035801

**NSC97**

S. Nishizaki, Y. Yamamoto and T. Takatsuka  
P.T.P. 105 (2001) 607; 108 (2002) 703

**NHC-Dm**

**Hyperon-mixing による最大質量の著しい減少(同時発見)**

From TNI (Three Nucleon Interaction)  
to TBI (Three Baryon Interaction)

TNI(Three Nucleon Interaction)=  
TNR(repulsion: phenomenological) +TNA(attraction:  $2\pi$ -exchange)

Assume :

phenomenological three-body interactions (TNR)  
work among all baryons (nucleons and hyperons)

$\Rightarrow$  TBR (Three-Baryon Repulsion)

**Universal TBR (NYT papers)**

# Maximum-mass problem of neutron stars

714

*S. Nishizaki, Y. Yamamoto and T. Takatsuka*

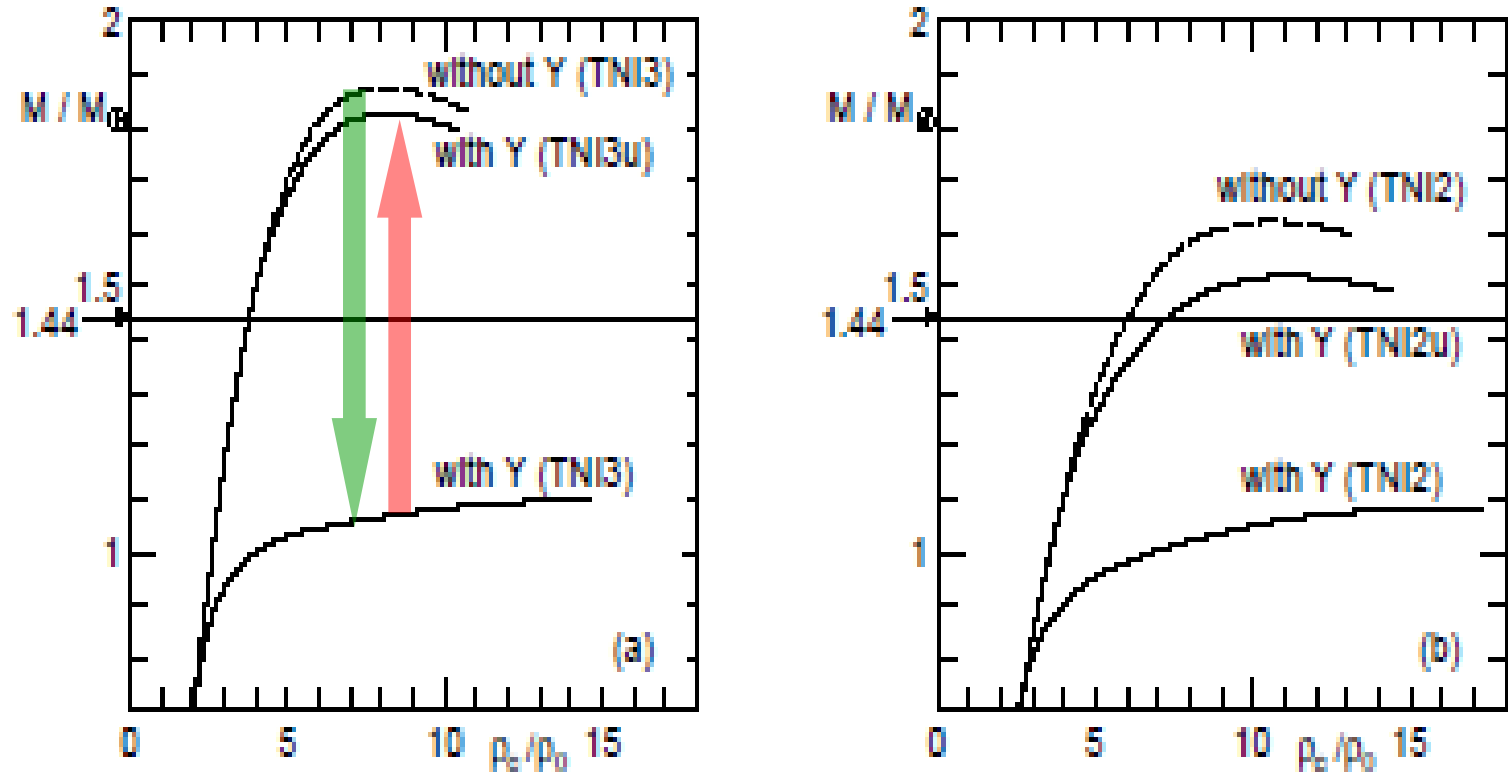
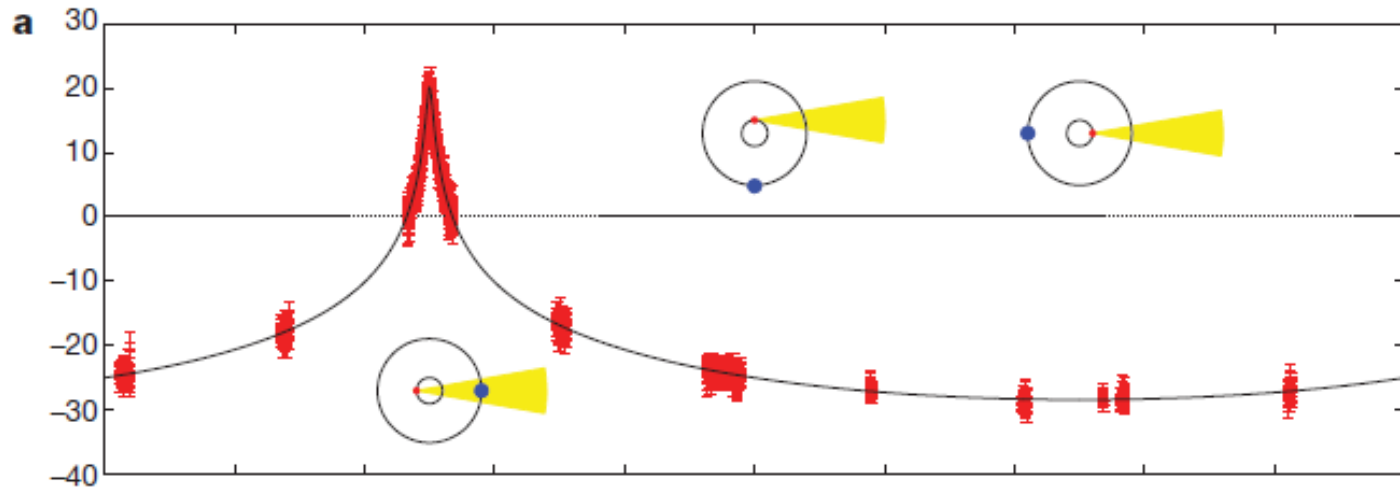


Fig. 9. The mass of a neutron star in units of the solar mass  $M_{\odot}$  as functions of the central baryon density  $\rho_c$  with use of (a) TNI3 and (b) TNI2. The notation here is the same as in Fig. 8.

## Importance of universal TBR

# Shapiro delay measurement for PSR J1614-2230



$$(1.97 \pm 0.04)M_{\odot}$$

714

*S. Nishizaki, Y. Yamamoto and T. Takatsuka*

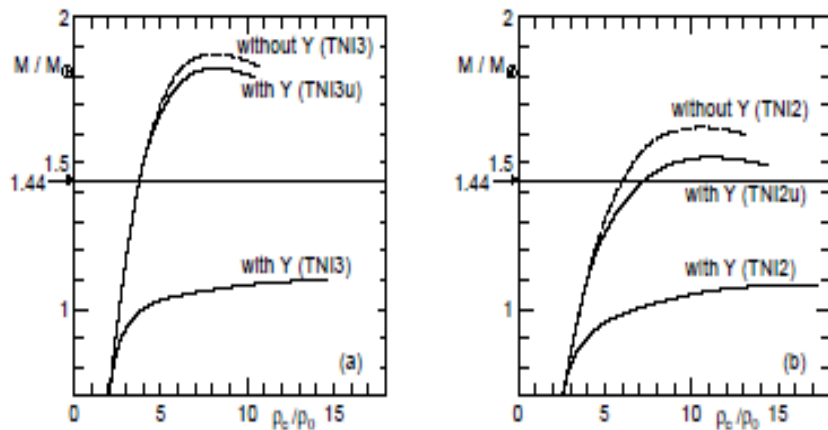


Fig. 9. The mass of a neutron star in units of the solar mass  $M_{\odot}$  as functions of the central baryon density  $\rho_c$  with use of (a) TNI3 and (b) TNI2. The notation here is the same as in Fig. 8.

もっと強い3体斥力効果が必要！  
その起源は？



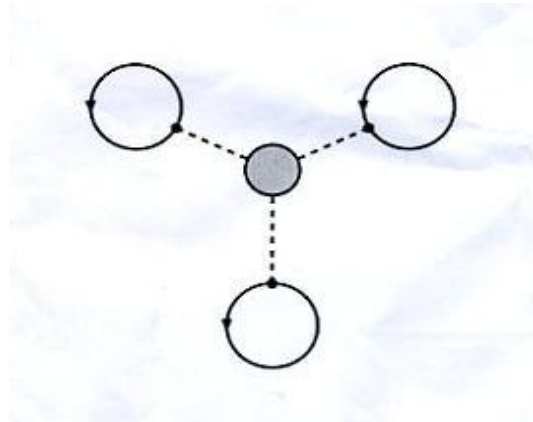
**universal**

**TBRは実在するとして**

**Saturation curve (incompressibility)に  
不可欠なTNRは、universal TBRとして  
中性子星の最大質量問題で重要な役割**

**様々な模型的(実体論的)検討・整理が必要**

# Phenomenological modeling of Three-Body Repulsion in ESC model



Necessary for maximum mass  
of neutron star

Universal among  
NNN, NNY, NYY...

Three-body force due to triple-meson correlation



Reduction of meson mass in medium

$$M_V(\rho) = M_V \exp(-\alpha\rho) \quad \text{for vector mesons}$$

**Medium-Induced Repulsion**

よいモデル的なモデルへ : multi-pomeron coupling model

## 思いがけない発展

# 核-核散乱におけるTNRの発現

FDAの成立:  $G(r; \rho)$  with  $\rho = \rho_1 + \rho_2$   
高密度 ( $\leq 2\rho_0$ ) におけるTNRの重要性

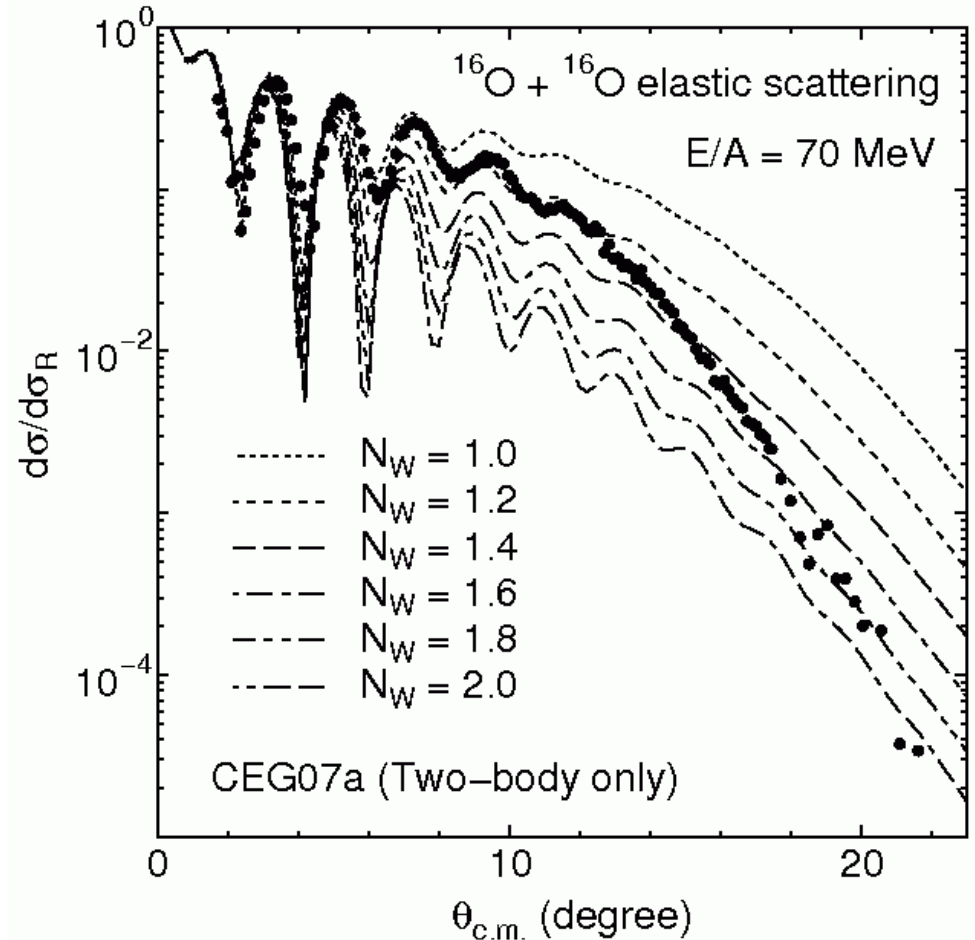
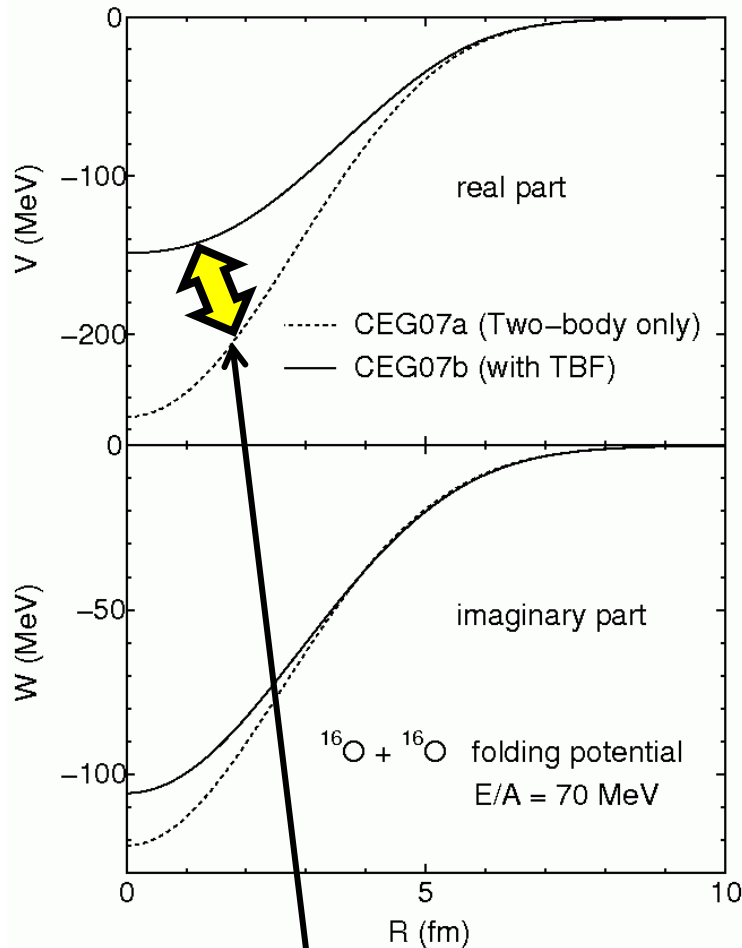
なぜTNRを導入したか

中性子星にはTNRの支えが必要であるという認識

実際、 $2\rho_0$ ではTNRのcontributionが圧倒的である実感

by Furumoto, Sakuragi, Y.Y.

# $^{16}\text{O} + ^{16}\text{O}$ elastic scattering $E/A = 70 \text{ MeV}$



**Effect of three-body force**

$$U_{DFM} = V_{DFM} + iN_w W_{DFM}$$

microscopic  
Consistent three-nucleon forces

P.Grange, A.Lejeune, M.Martzolff, J.F.Mathot  
P.R. C40 (1989) 1040

Paris

W.Zuo, A.Lejeune, U.Lombard, J.F.Mathot  
N.P. A706 (2002) 418

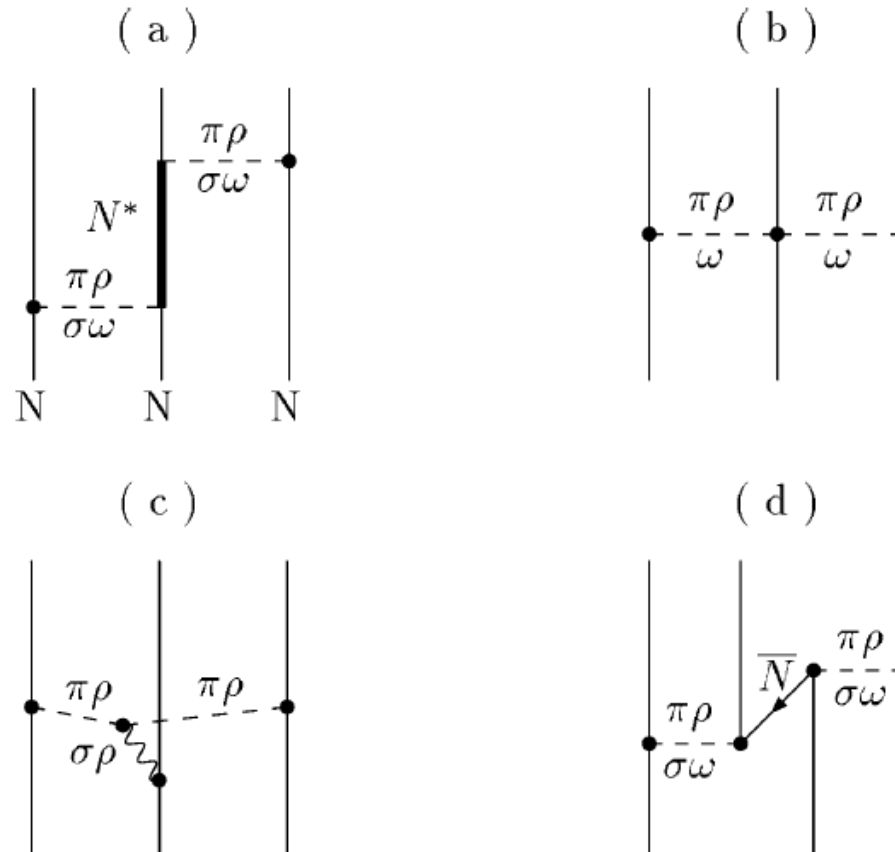
Av18

Z.H.Li, U.Lombardo, H.J.Schulze. W.Zuo  
P.R. C77 (2008) 034316

Bonn

中性子星の内核に関して高塚学派と対立

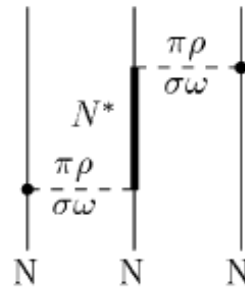
# Microscopic TBF ( $\pi \rho \sigma \omega$ )



ESC model では 全てMeson-Pair Exchange TBF に取り込まれる

AV18 version (Zuo et al., N.P. A706 (2002) 418)

①  $\pi, \rho$  exchange

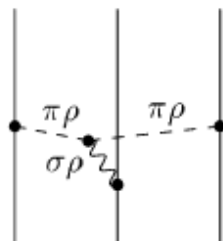


2  $\pi$  exchange : Fujita-Miyazawa contribution involving  $\Delta$

All order  $\pi$  and  $\rho$  exchange : Tucson-Melbourne TBF

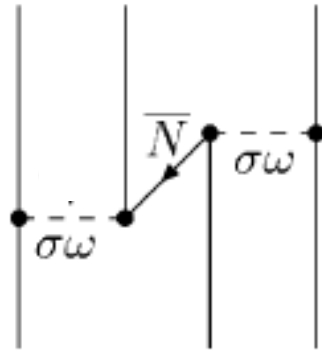
TM TBF のparameterizationを  
NN potential の選択に応じて変える

Isobar  $\Delta$  (1232) and Roper  $N^*$  (1440) play the major role



small contribution (ここでは考えない)

②  $(\sigma, \omega) - (\bar{N}N) 3BF$



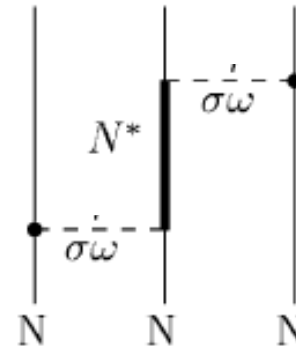
TNRのオリジン  
DBHF

$$V_I^{\sigma\sigma}(r) = \left( \frac{g_{\sigma NN}^2 m_\sigma}{4\pi} \right)^2 \frac{1}{m_N^3} \sum_3 \overline{\phantom{x}} \\ \times \left[ \left( \frac{3}{5} k_F^2 + h_x^2 + h_y^2 + 2z_r h_x h_y \right) Z_x^\sigma Z_y^\sigma \right. \\ \left. - \frac{m_\sigma^2}{4} (Z_x^\sigma Y_y^\sigma + Z_y^\sigma Y_x^\sigma + z_r G_x^\sigma G_y^\sigma) \right],$$

$$V_I^{\sigma\omega}(r) = \frac{g_{\sigma NN}^2 g_{\omega NN}^2 m_\sigma m_\omega}{(4\pi)^2} \frac{m_\omega^2}{4m_N^3} \sum_3 \overline{\phantom{x}} \\ \times [Z_x^\sigma Y_y^\omega + Z_y^\sigma Y_x^\omega],$$



③  $\sigma, \omega$ -Roper 3BF



$$\begin{aligned}
 W_I^{\mu\mu}(r) & \qquad \qquad \qquad (\mu = \sigma, \omega) \\
 &= \left( \frac{g_{\mu NN} g_{\mu NR}}{4\pi} \right)^2 \frac{m_\mu^2}{2m_N^3} \frac{m_N}{m_R - m_N} C^{\mu\mu} \cdot \overline{\sum}_3 \\
 & \times \left[ Z_x^\mu Z_y^\mu, \right. \\
 & \quad \left| Z_x^\mu Z_y^\mu \left( \frac{3}{5} k_F^2 + h_x^2 + h_y^2 + 2z_r h_x h_y \right), \right. \\
 & \quad \left| Z_x^\mu \tilde{G}_y^\mu (h_y + z_r h_x) + Z_y^\mu \tilde{G}_x^\mu (h_x + z_r h_y), \right. \\
 & \quad \left| Z_x^\mu \tilde{Y}_y^\mu + Z_y^\mu \tilde{Y}_x^\mu, \right. \\
 & \quad \left. \left| \tilde{G}_x^\mu \tilde{G}_y^\mu z_r \right] \right.
 \end{aligned}$$

$g_{\sigma NR}, g_{\omega NR}$  (and the corresponding form factors), on which very little experimental or theoretical information is available.

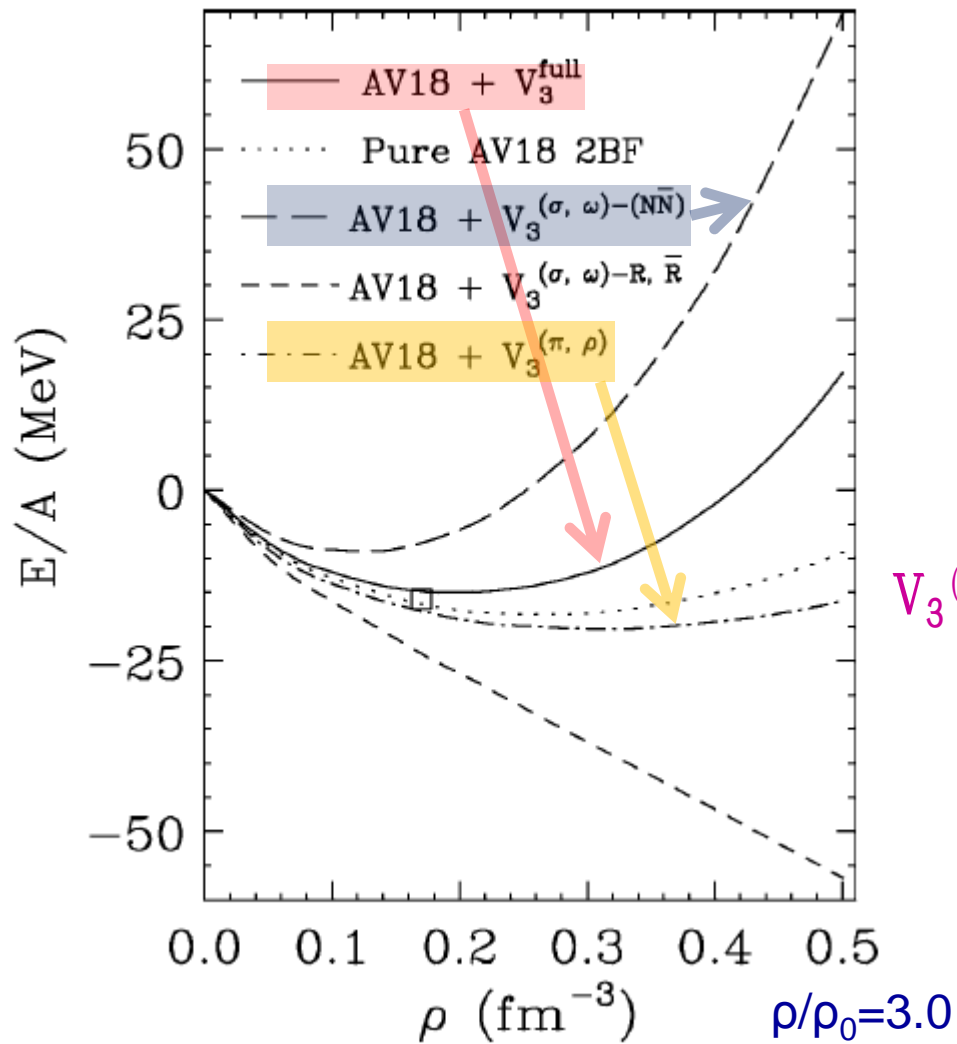
Large uncertainties

## Effective density-dependent two-body interaction

$$\bar{V}_{ij}(\mathbf{r}) = \rho \int d^3r_k \sum_{\sigma_k, \tau_k} g(r_{ik})^2 g(r_{jk})^2 V_{ijk}$$

$$\begin{aligned} \bar{V}_{ij}(\mathbf{r}) = & (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)V_C(r) + (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)V_S(r) + V_I(r) \\ & + S_{ij}(\hat{\mathbf{r}})[(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)V_T(r) + V_Q(r)], \end{aligned}$$

$$S_{ij}(\hat{\mathbf{r}}) = 3(\boldsymbol{\sigma}_i \cdot \hat{\mathbf{r}}_{ij})(\boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ij}) - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$



$V_3^{(\pi, \rho)}$ の寄与小さい

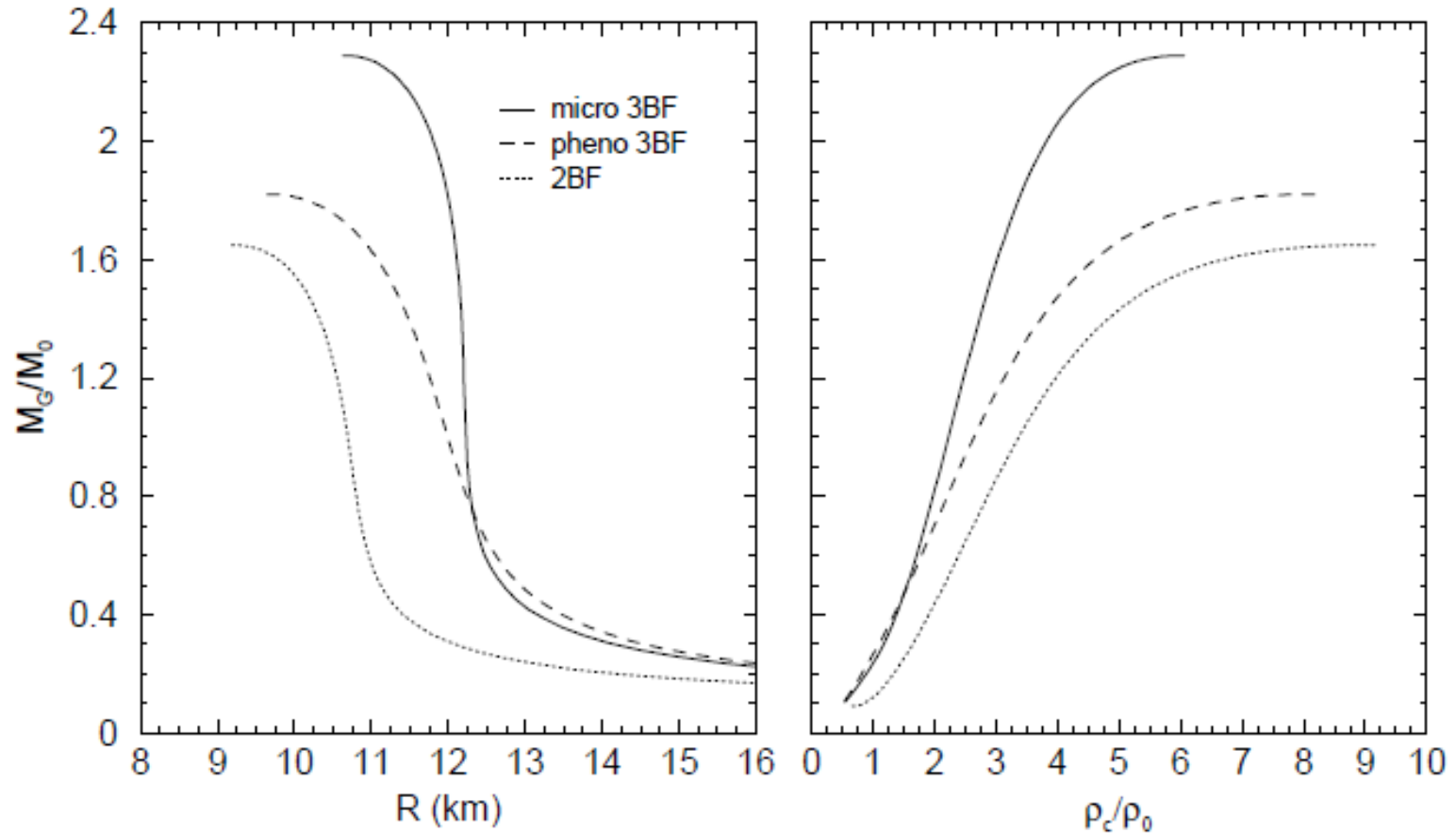
$\sigma, \omega$ -Roper 3BF attractive

$\sigma\omega-(N\bar{N})$  3BF repulsive

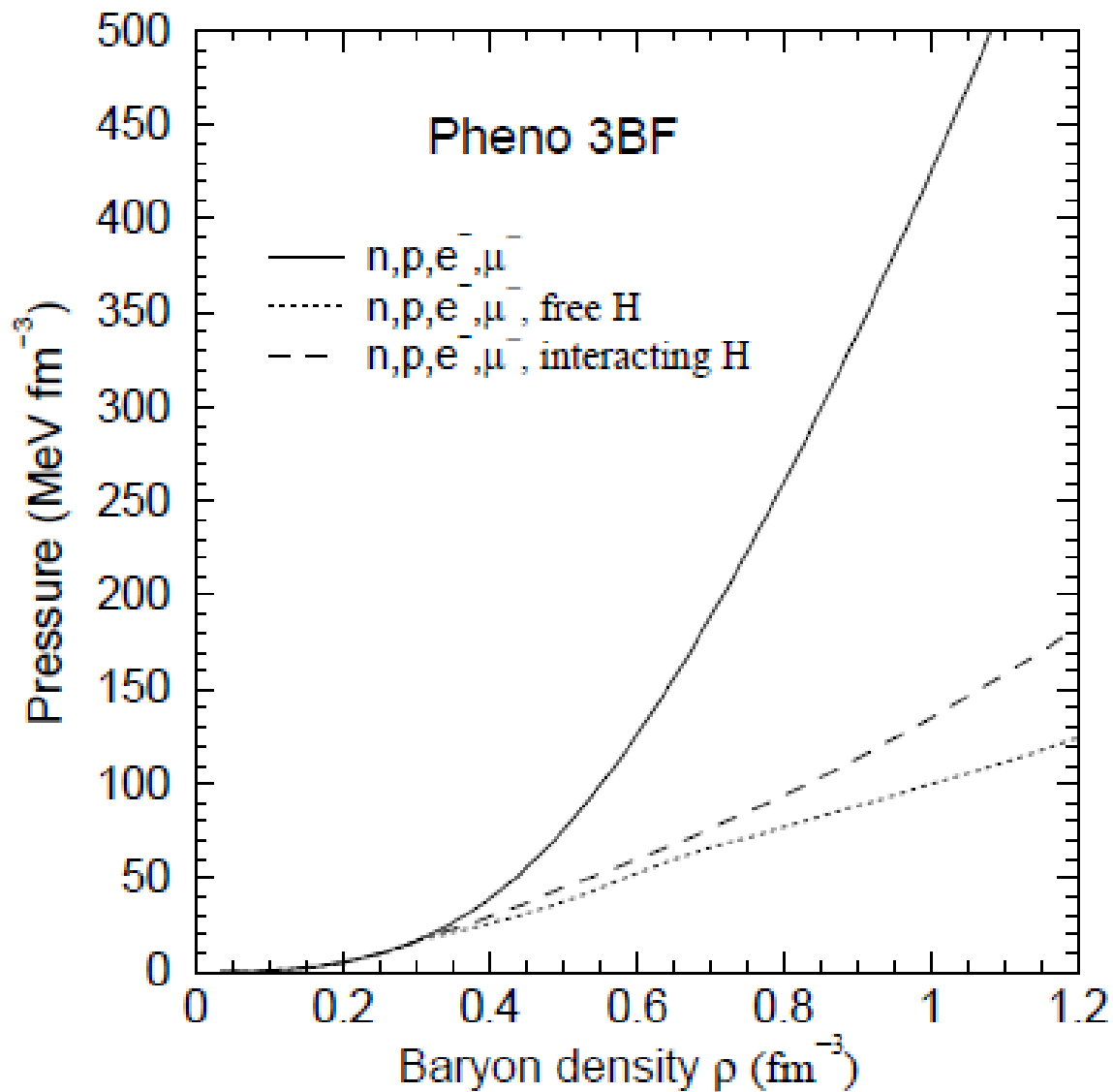
cancellationでsaturationが与えられる

ESC-MPEでは消える(立場)!

# Neutron star structure



*Figure 5.* The neutron star gravitational mass (in units of solar mass  $M_\odot$ ) is displayed vs. the radius (left panel) and the normalized central baryon density  $\rho_c$  ( $\rho_0 = 0.17 \text{ fm}^{-3}$ ) (right panel).



Hyperon mixingによるEOSの著しいソフト化

Universal TBR cannot be derived from their model

For maximum-mass problem of neutron star  
they introduce another model (quark-matter core)

Our approach

ESC-based TBF

Multi-pomeron coupling TBF

Meson-pair exchange TBF

# Phenomenological Pomeron

For high energy scattering of particles such as pp,  
the exchange is made by whole families of related particles (reggeon)  
ex.  $\rho$ -family :  $\rho$  (770),  $\rho_3$  (1690),  $\rho_5$  (2350),  $\dots$

In Regge theory, the exchange effect of members of  
such a family is given in terms of Regge trajectory  $\alpha(t)$

Trajectories of all of dominant meson-exchange families  
lie close to  $\alpha(t) = \alpha_0 + \alpha' t = 0.55 + 0.86 t$   
Then, energy dependence of total c.s. is  $s^{\alpha_0-1} = s^{-0.45}$   
It decreases with increasing energy

Experimentally, total c.s. at first flattens out and  
begins to rise slowly

Another Regge trajectory to produce a rising cross-section  
It must be such that  $\alpha_0 = 1 + \epsilon$  with  $\epsilon$  positive

**This is a pomeron**



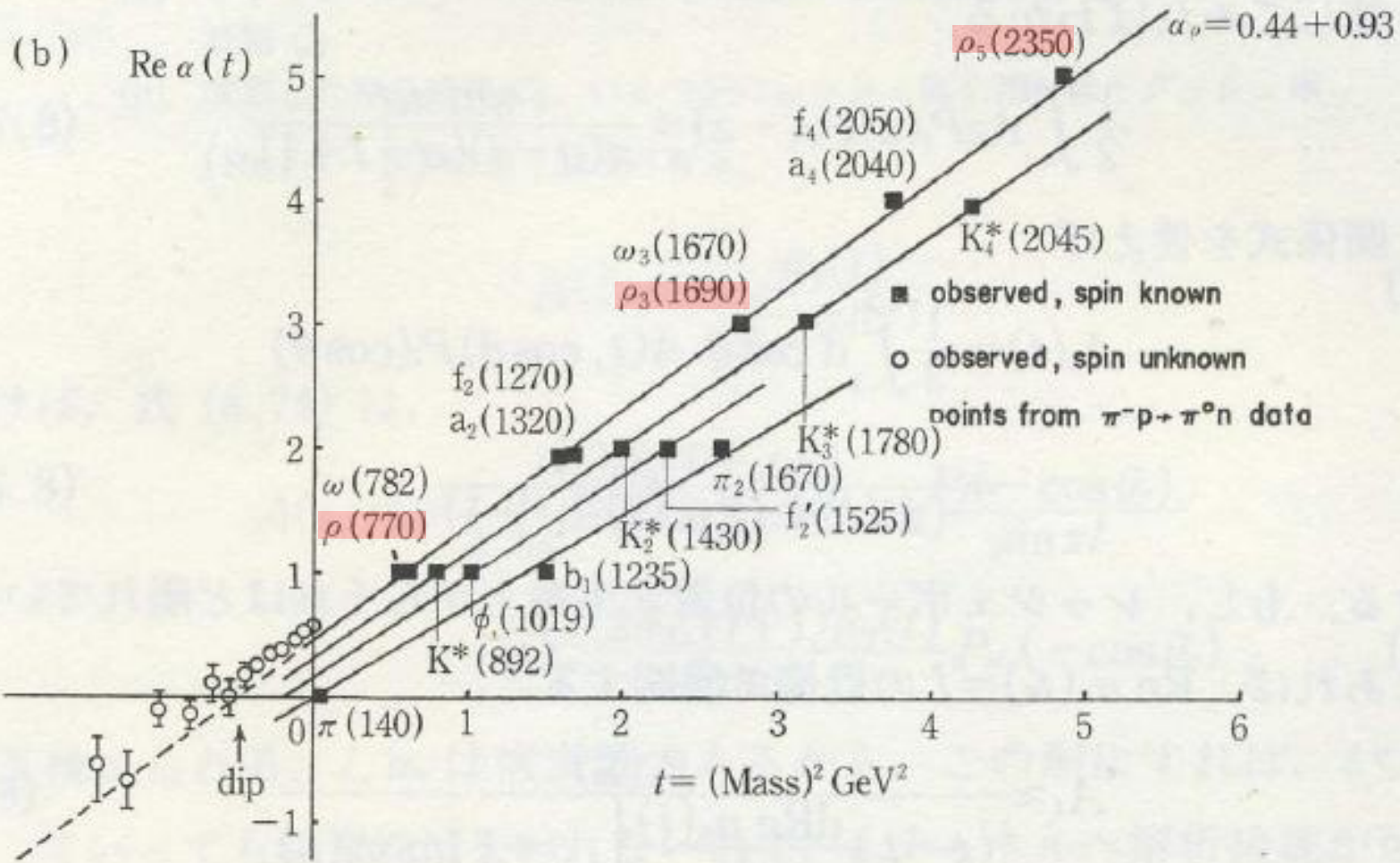


図 8.18 チュー-フラウチ (Chew-Frautschi) のプロット  
 レッジ軌跡の  $\text{Re } l$  をエネルギーの関数として描く。

# NN potential and Reggeon exchange

by young Rijken

For the low-energy s-channel region, the Reggeon exchange can be approximated very well by the exchange of the lowest mass boson on the Regge trajectory

The Reggeon exchange model reduces at low energies in the NN channel to an OBE model (not the traditional one)

New contribution due to  $J=0$  component of Pomeron trajectory  
Pomeron exchange potential is identical to that of scalar exchange except for a (-) sign and a Gaussian  $t$ -dependence instead of a Yukawian

The  $J=0$  component from Pomeron give at low energies an appreciable repulsive potential in all baryon-baryon channels due to the dominant  $SU(3)$ -singlet character

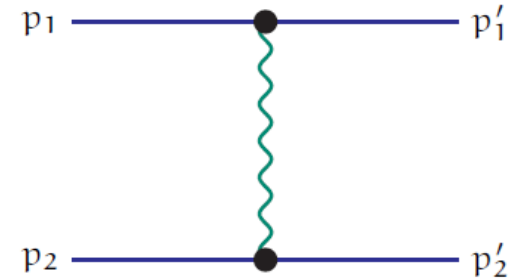
Pomeron mass and coupling constant obtained from Regge-pole fit to the high-energy scattering data are in agreement with the fitted values in the Reggeon exchange model to low-energy NN scattering

# Two-body Potential from Pomeron-exchange

Lagrangian & Propagator

$$\mathcal{L}_{\text{PNN}} = g_{\text{P}} \bar{\psi}(\mathbf{x}) \psi(\mathbf{x}) \sigma_{\text{P}}(\mathbf{x})$$

$$\Delta_{\text{F}}^{\text{P}}(k^2) = + \exp(-\mathbf{k}^2/4m_{\text{P}}^2) / \mathcal{M}^2$$



scaling mass  $\mathcal{M} = 1\text{GeV}$ .

$$-iM_{\text{P}}(p_1', p_2'; p_1, p_2) = (+i)^2 g_{\text{P}}^2 [\bar{u}(p')u(p)] [\bar{u}(-p')u(-p)] \cdot i\Delta_{\text{F}}^{\text{P}}[(p' - p)^2]$$

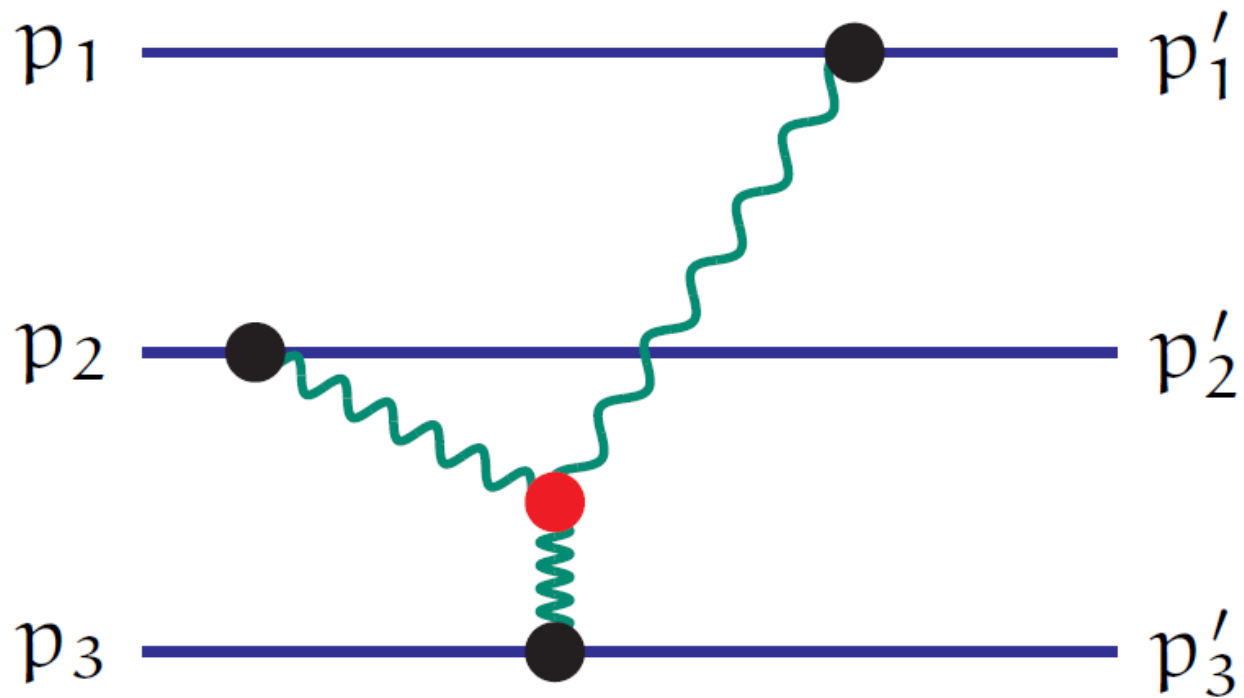
$$M_{\text{P}}(p_1', p_2'; p_1, p_2) = g_{\text{P}}^2 [\bar{u}(p')u(p)] [\bar{u}(-p')u(-p)] \cdot \Delta_{\text{F}}^{\text{P}}[(p' - p)^2]$$

$$\approx g_{\text{P}}^2 \exp(-\mathbf{k}^2/4m_{\text{P}}^2) / \mathcal{M}^2,$$

$$V_{\text{P}}(r) = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} M_{\text{P}}(\mathbf{p}' - \mathbf{p}) \delta(\mathbf{k} - \mathbf{p}' + \mathbf{p})$$

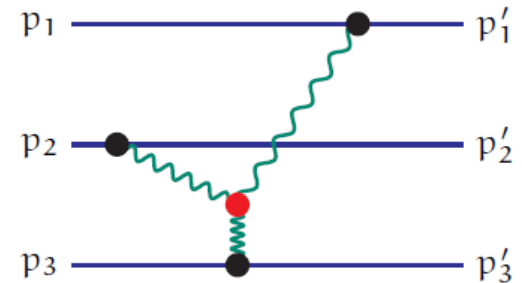
$$= \frac{g_{\text{P}}^2}{4\pi} \frac{4}{\sqrt{\pi}} \frac{m_{\text{P}}^3}{\mathcal{M}^2} \exp(-m_{\text{P}}^2 r_{12}^2).$$

# Multi-Pomeron Couplings and the Universal Repulsion in Nuclear/Hyperonic Matter



# Three-body Potential from the Triple-pomeron vertex

$$\mathcal{L}_{\text{PPP}} = g_{3\text{P}} \mathcal{M} \sigma_{\text{P}}^3(\mathbf{x}) / 3!$$



$$\begin{aligned} M_{3\text{P}}(p'_1, p'_2, p'_3; p_1, p_2, p_3) &= g_{3\text{P}} g_{\text{P}}^3 \prod_{i=1}^3 \{ [\bar{u}(p'_i) u(p_i)] \Delta_{\text{F}}^{\text{P}}[(p'_i - p_i)^2] \} \\ &\approx g_{3\text{P}} g_{\text{P}}^3 \prod_{i=1}^3 \Delta_{\text{F}}^{\text{P}}[(p'_i - p_i)^2]. \end{aligned}$$

$$\begin{aligned} V(\mathbf{x}'_1, \mathbf{x}'_2, \mathbf{x}'_3; \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) &= \int \prod_{i=1}^3 \frac{d^3 p'_i}{(2\pi)^3} \frac{d^3 p_i}{(2\pi)^3} \cdot \\ &\quad \times \prod_{i=1}^3 e^{-i(p'_i \cdot \mathbf{x}'_i - p_i \cdot \mathbf{x}_i)} \cdot \delta \left( \sum \mathbf{p}'_i - \sum \mathbf{p}_i \right) \\ &\quad \times M_{3\text{P}}(p'_1, p'_2, p'_3; p_1, p_2, p_3). \end{aligned}$$

$$V(\mathbf{x}'_1, \mathbf{x}'_2, \mathbf{x}'_3; \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) \equiv V(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) \delta(\mathbf{x}'_1 - \mathbf{x}_1) \delta(\mathbf{x}'_2 - \mathbf{x}_2) \delta(\mathbf{x}'_3 - \mathbf{x}_3)$$

$$\begin{aligned} V(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) &= g_{3\text{P}} g_{\text{P}}^3 \prod_{i=1}^3 \int \frac{d^3 k_i}{(2\pi)^3} \prod_{i=1}^3 e^{-i\mathbf{k}_i \cdot \mathbf{x}_i} \cdot (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \cdot \\ &\quad \times \exp(-\mathbf{k}_1^2 / 4m_{\text{P}}^2) \exp(-\mathbf{k}_2^2 / 4m_{\text{P}}^2) \exp(-\mathbf{k}_3^2 / 4m_{\text{P}}^2) \cdot \mathcal{M}^{-5}, \end{aligned}$$

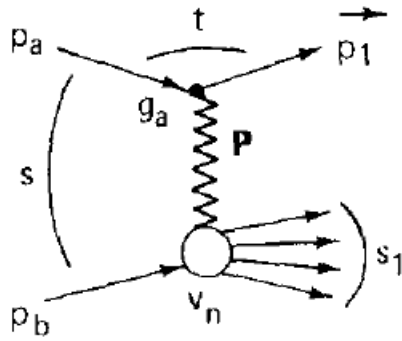
## The effective two-body potential

$$V_{\text{eff}}(\mathbf{x}_1, \mathbf{x}_2) = \rho_{\text{NM}} \int d^3x_3 V(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$$

$$\begin{aligned} V_{\text{eff}}(\mathbf{x}_1, \mathbf{x}_2) &= g_{3\text{P}} g_{\text{P}}^3 \frac{\rho_{\text{NM}}}{\mathcal{M}^5} \cdot \int \frac{d^3k_1}{(2\pi)^3} \int \frac{d^3k_2}{(2\pi)^3} e^{-i\mathbf{k}_1 \cdot \mathbf{x}_1} e^{-i\mathbf{k}_2 \cdot \mathbf{x}_2} \cdot \\ &\quad \times (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2) \exp\left(-\mathbf{k}_1^2/4m_{\text{P}}^2\right) \exp\left(-\mathbf{k}_2^2/4m_{\text{P}}^2\right) \\ &= g_{3\text{P}} g_{\text{P}}^3 \frac{\rho_{\text{NM}}}{\mathcal{M}^5} \cdot \int \frac{d^3k_1}{(2\pi)^3} e^{-i\mathbf{k}_1 \cdot (\mathbf{x}_1 - \mathbf{x}_2)} \cdot \exp\left(-\mathbf{k}_1^2/2m_{\text{P}}^2\right) \\ &= g_{3\text{P}} g_{\text{P}}^3 \frac{\rho_{\text{NM}}}{\mathcal{M}^5} \cdot \frac{1}{4\pi} \frac{4}{\sqrt{\pi}} \left(\frac{m_{\text{P}}}{\sqrt{2}}\right)^3 \exp\left(-\frac{1}{2} m_{\text{P}}^2 r_{12}^2\right) \cdot \end{aligned}$$

# Diffractive (multipomeron) production of heavy showers of particles

Kaidalov et al., N.P. B75(1974) 471



Dissociation of a nucleon at high energy  
due to pomeron exchange

$$\frac{d^2\sigma}{d\xi_1 d^2k_\perp} = \frac{1}{2\pi} g_a^2(k_\perp^2) |G_P(\xi', k_\perp^2)|^2 \sigma_{PN}^{\text{tot}}(s_1, k_\perp^2) \quad \xi_1 = \ln s_1 / m^2$$

$$\simeq 4 g_N(0) g_a^2(k_\perp^2) r(k_\perp^2) \left(\frac{s}{s_1}\right)^{2[\alpha_P(k_\perp^2) - 1]} \quad \text{At high } s_1 \gg m_N^2$$

$r(t)$  : triple-pomeron vertex

$g_N$  : pomeron-nucleon coupling

Based on the experimental data on the  $pp \rightarrow pX$  process

$$r(k_\perp^2) \simeq r(0) \simeq g_N(0)/40$$

## Four-body Potential from the Quadruple-pomeron vertex

$$\mathcal{L}_{\text{PPPP}} = g_{4\text{P}} \sigma_{\text{P}}^4(\mathbf{x})/4!$$

$$\begin{aligned} M_{4\text{P}}(p'_1, p'_2, p'_3, p'_4; p_1, p_2, p_3, p_4) &= g_{4\text{P}} g_{\text{P}}^4 \prod_{i=1}^4 \{ [\bar{u}(p'_i) u(p_i)] \Delta_{\text{F}}^{\text{P}}[(p'_i - p_i)^2] \} \\ &\approx g_{4\text{P}} g_{\text{P}}^4 \prod_{i=1}^4 \Delta_{\text{F}}^{\text{P}}[(p'_i - p_i)^2]. \end{aligned}$$

$$\begin{aligned} V(\mathbf{x}'_1, \mathbf{x}'_2, \mathbf{x}'_3, \mathbf{x}'_4; \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4) &= \int \prod_{i=1}^4 \frac{d^3 p'_i}{(2\pi)^3} \frac{d^3 p_i}{(2\pi)^3} \\ &\times \prod_{i=1}^4 e^{-i(p'_i \cdot \mathbf{x}'_i - p_i \cdot \mathbf{x}_i)} \cdot \delta \left( \sum \mathbf{p}'_i - \sum \mathbf{p}_i \right) \cdot \\ &\times M_{4\text{P}}(p'_1, p'_2, p'_3, p'_4; p_1, p_2, p_3, p_4). \end{aligned}$$

$$V(\mathbf{x}'_1, \mathbf{x}'_2, \mathbf{x}'_3, \mathbf{x}'_4; \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4) \equiv V(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4) \prod_{i=1}^4 \delta(\mathbf{x}'_i - \mathbf{x}_i),$$

$$\begin{aligned} V(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4) &= g_{4\text{P}} g_{\text{P}}^4 \prod_{i=1}^4 \left\{ \int \frac{d^3 k_i}{(2\pi)^3} e^{-i\mathbf{k}_i \cdot \mathbf{x}_i} \right\} \cdot (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4) \cdot \\ &\times \exp(-\mathbf{k}_1^2/4m_{\text{P}}^2) \exp(-\mathbf{k}_2^2/4m_{\text{P}}^2) \exp(-\mathbf{k}_3^2/4m_{\text{P}}^2) \exp(-\mathbf{k}_4^2/4m_{\text{P}}^2) \cdot \mathcal{M}^{-8} \end{aligned}$$



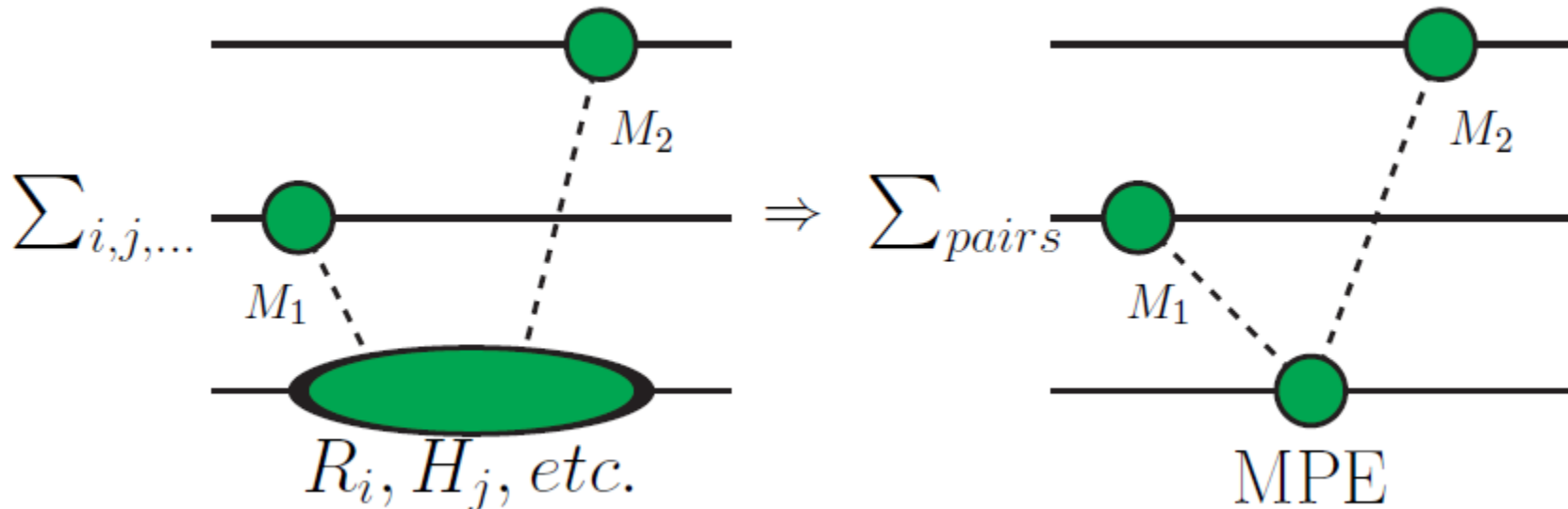
# The effective two-body potential

$$V_{\text{eff}}(\mathbf{x}_1, \mathbf{x}_2) = \rho_{\text{NM}}^2 \int d^3x_3 \int d^3x_4 V(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$$

$$\begin{aligned} V_{\text{eff}}(\mathbf{x}_1, \mathbf{x}_2) &= g_{4\text{P}} g_{\text{P}}^4 \frac{\rho_{\text{NM}}^2}{\mathcal{M}^8} \cdot \int \frac{d^3k_1}{(2\pi)^3} \int \frac{d^3k_2}{(2\pi)^3} e^{-i\mathbf{k}_1 \cdot \mathbf{x}_1} e^{-i\mathbf{k}_2 \cdot \mathbf{x}_2} \cdot \\ &\quad \times (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2) \exp\left(-\mathbf{k}_1^2/4m_{\text{P}}^2\right) \exp\left(-\mathbf{k}_2^2/4m_{\text{P}}^2\right) \\ &= g_{4\text{P}} g_{\text{P}}^4 \frac{\rho_{\text{NM}}^2}{\mathcal{M}^8} \cdot \int \frac{d^3k_1}{(2\pi)^3} e^{-i\mathbf{k}_1 \cdot (\mathbf{x}_1 - \mathbf{x}_2)} \cdot \exp\left(-\mathbf{k}_1^2/2m_{\text{P}}^2\right) \\ &= g_{4\text{P}} g_{\text{P}}^4 \frac{\rho_{\text{NM}}^2}{\mathcal{M}^8} \cdot \frac{1}{4\pi} \frac{4}{\sqrt{\pi}} \left(\frac{m_{\text{P}}}{\sqrt{2}}\right)^3 \exp\left(-\frac{1}{2} m_{\text{P}}^2 r_{12}^2\right) \cdot \end{aligned}$$

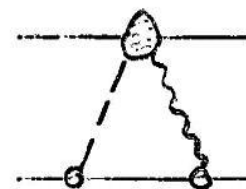
$g_{3\text{P}}/g_{\text{P}}$  and  $g_{4\text{P}}/g_{\text{P}}$  can be estimated from the data

# Two-Meson-Exchange Three-Nucleon Potentials



2体力 (ESC) の pair term から  
ユニークに決められる

Meson-Pair-Exchanges:



## nucleon-nucleon-meson Hamiltonians

$$\mathcal{H}_{PV} = \frac{f_P}{m_\pi} \bar{\psi} \gamma_5 \gamma_\mu \psi \cdot \partial^\mu \phi_P,$$

$$\mathcal{H}_V = g_V \bar{\psi} \gamma_\mu \tau \psi \cdot \phi_V^\mu - \frac{f_V}{2M} \bar{\psi} \sigma_{\mu\nu} \tau \psi \cdot \partial^\nu \phi_V^\mu,$$

$$\mathcal{H}_S = g_S \bar{\psi} \tau \psi \cdot \phi_S,$$

## nucleon-nucleon-meson-meson ( $NNm_1m_2$ ) Hamiltonians

$$J^{PC} = 0^{++} : \mathcal{H}_S = \bar{\psi}\psi [g_{(\pi\pi)_0}\pi\cdot\pi + g_{(\sigma\sigma)}\sigma^2] / m_\pi,$$

$$\mathcal{H}_E = \bar{\psi}\tau\psi \cdot \pi [g_{(\pi\eta)}\eta + g_{(\pi\eta')}\eta'] / m_\pi,$$

$$J^{PC} = 1^{--} : \mathcal{H}_V = g_{(\pi\pi)_1}\bar{\psi}\gamma_\mu\tau\psi \cdot (\pi \times \partial^\mu \pi) / m_\pi^2 \\ - \frac{f_{(\pi\pi)_1}}{2M}\bar{\psi}\sigma_{\mu\nu}\tau\psi\partial^\nu \cdot (\pi \times \partial^\mu \pi) / m_\pi^2,$$

$$J^{PC} = 1^{++} : \mathcal{H}_A = g_{(\pi\rho)_1}\bar{\psi}\gamma_5\gamma_\mu\tau\psi \cdot (\pi \times \rho^\mu) / m_\pi,$$

$$\mathcal{H}_P = g_{(\pi\sigma)}\bar{\psi}\gamma_5\gamma_\mu\tau\psi \cdot (\pi\partial^\mu\sigma - \sigma\partial^\mu\pi) / m_\pi^2$$

$$+ g_{(\pi P)}\bar{\psi}\gamma_5\gamma_\mu\tau\psi \cdot (\pi\partial^\mu P - P\partial^\mu\pi) / m_\pi^2,$$

$$J^{PC} = 1^{+-} : \mathcal{H}_H = -ig_{(\pi\rho)_0}\bar{\psi}\gamma_5\sigma_{\mu\nu}\psi\partial^\nu (\pi \cdot \rho^\mu) / m_\pi^2,$$

$$\mathcal{H}_B = -ig_{(\pi\omega)}\bar{\psi}\gamma_5\sigma_{\mu\nu}\tau\psi \cdot \partial^\nu (\pi \omega^\mu) / m_\pi^2.$$

## LNR approximation

$$V_{12;3}^{(eff)} = \frac{1}{4} \rho_{MN} \text{Tr} \int d^3 x_3 V(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$$

neglecting spin-flip and charge change  
for particle 3

$(\pi \pi)_1$  and  $(\pi \rho)_1$  potentials vanish completely

$$\text{Tr} \tau_3 = \text{Tr} \sigma_3 = 0.$$

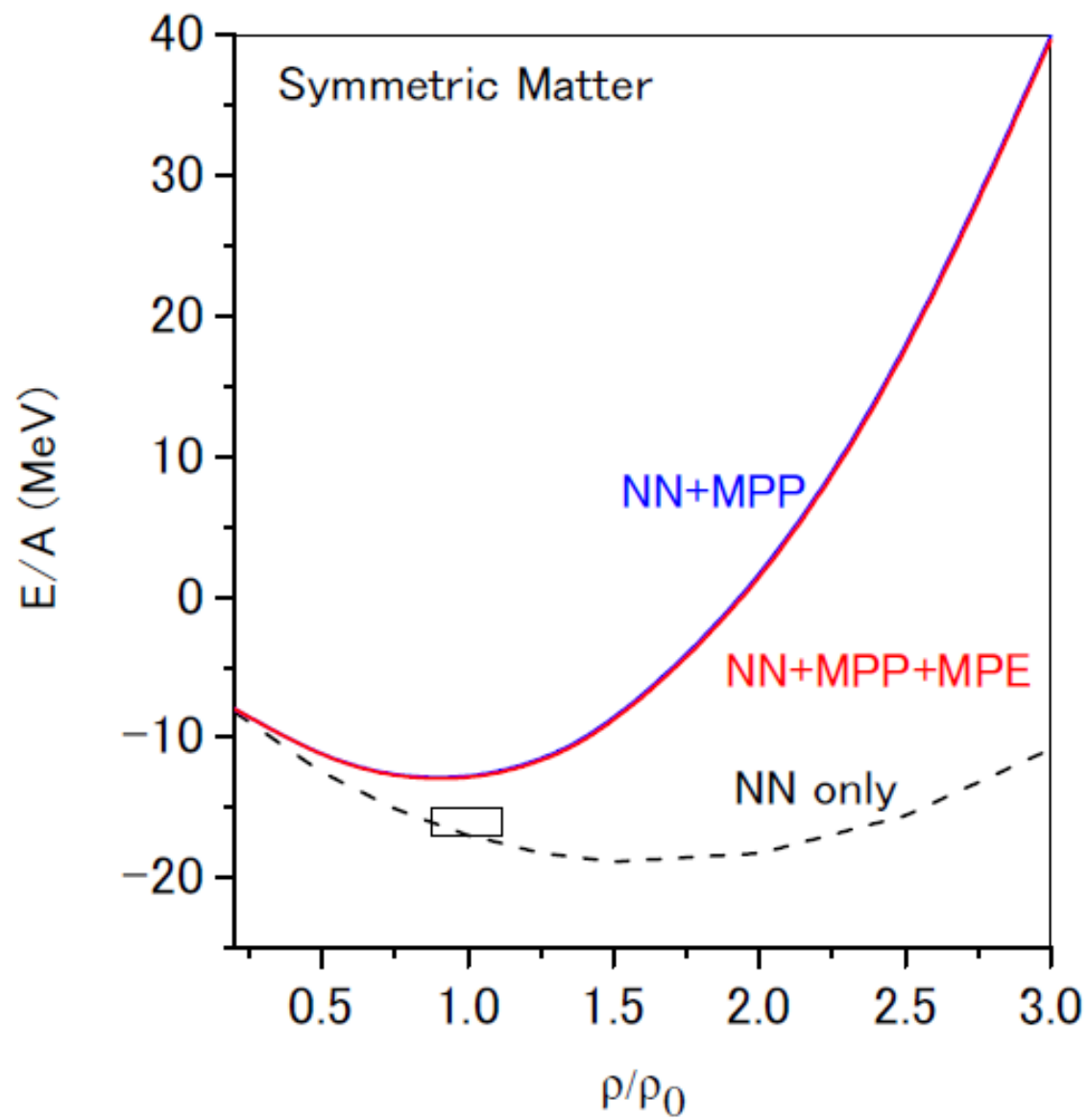
**MPE contribution in nuclear matter is very small**

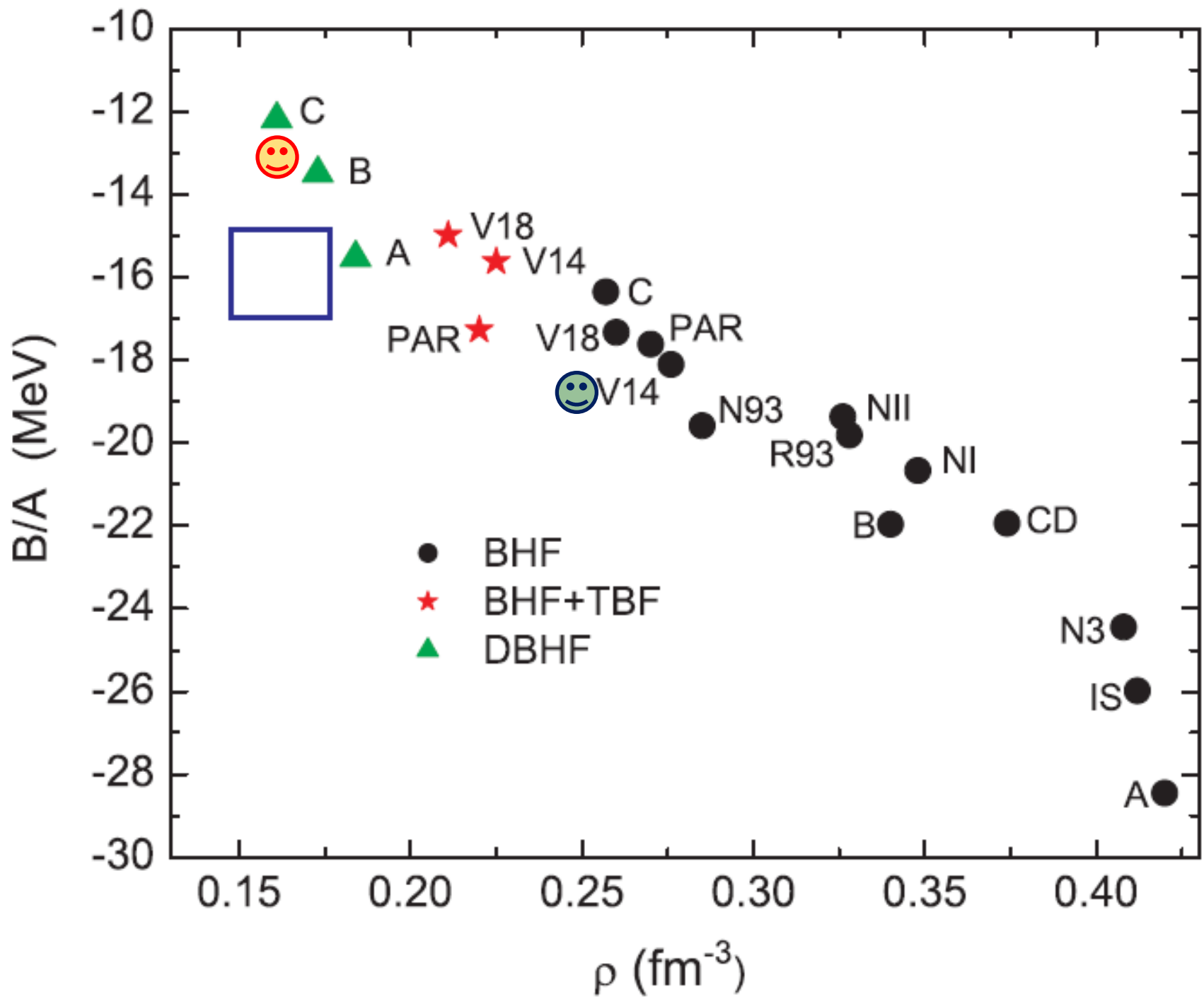
OK ???

**以下の結果は ESC08c+MPP+MPE**

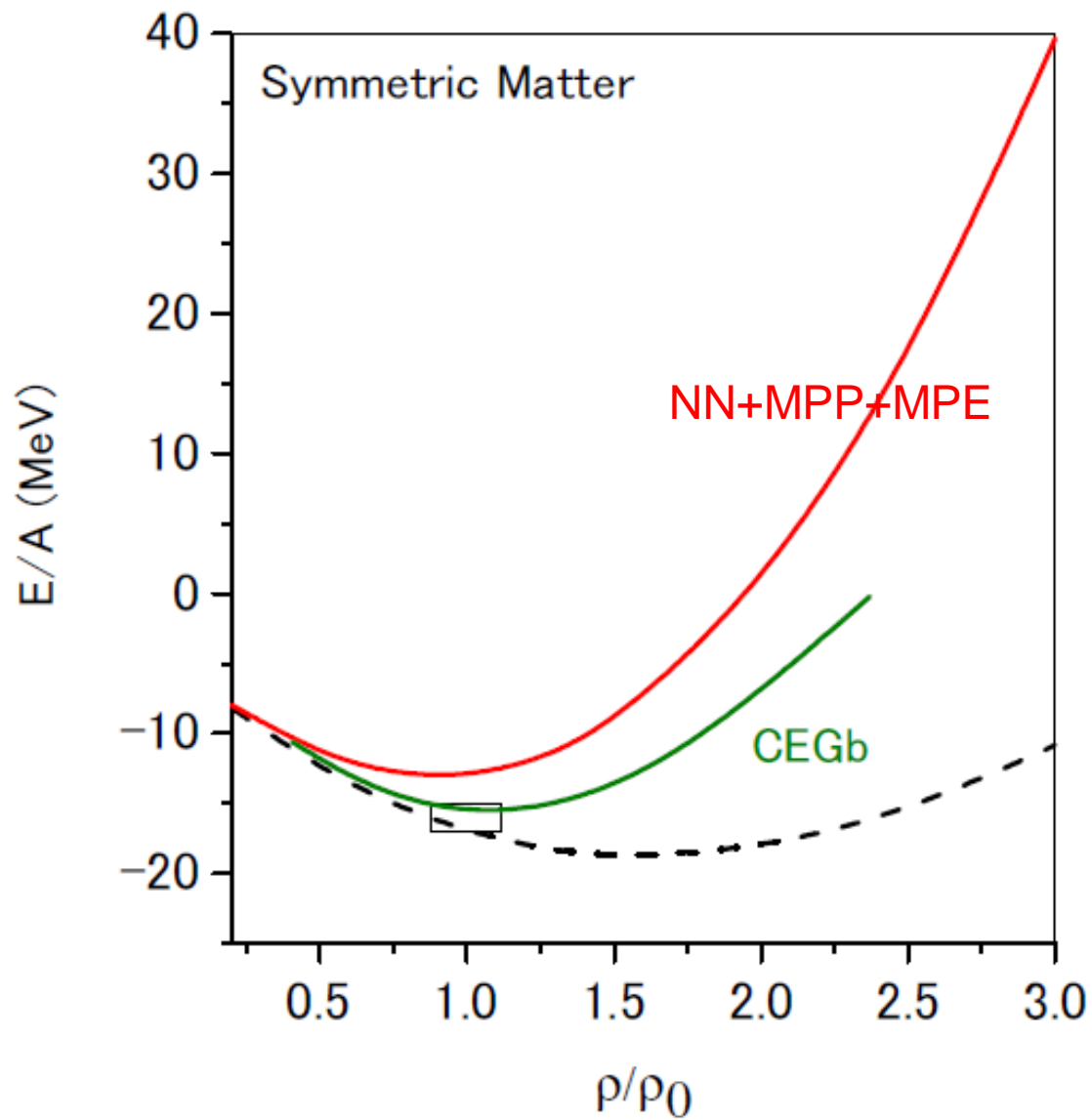
MPP/MPEはNN部分 (ESC08c) に応じてきまる

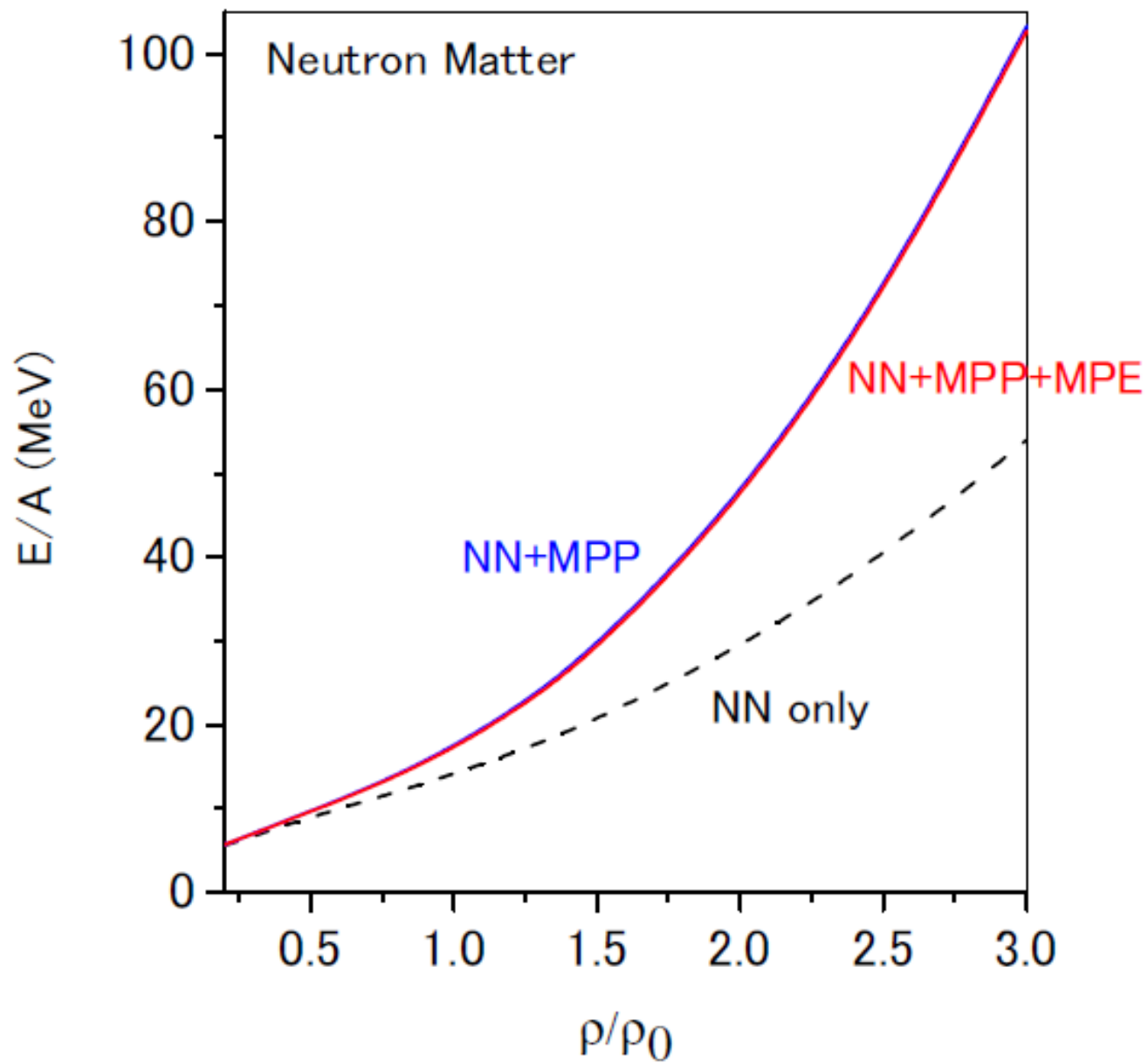
ESC08cはESC08a/bよりもquark-model coreの  
特徴をより忠実に反映している

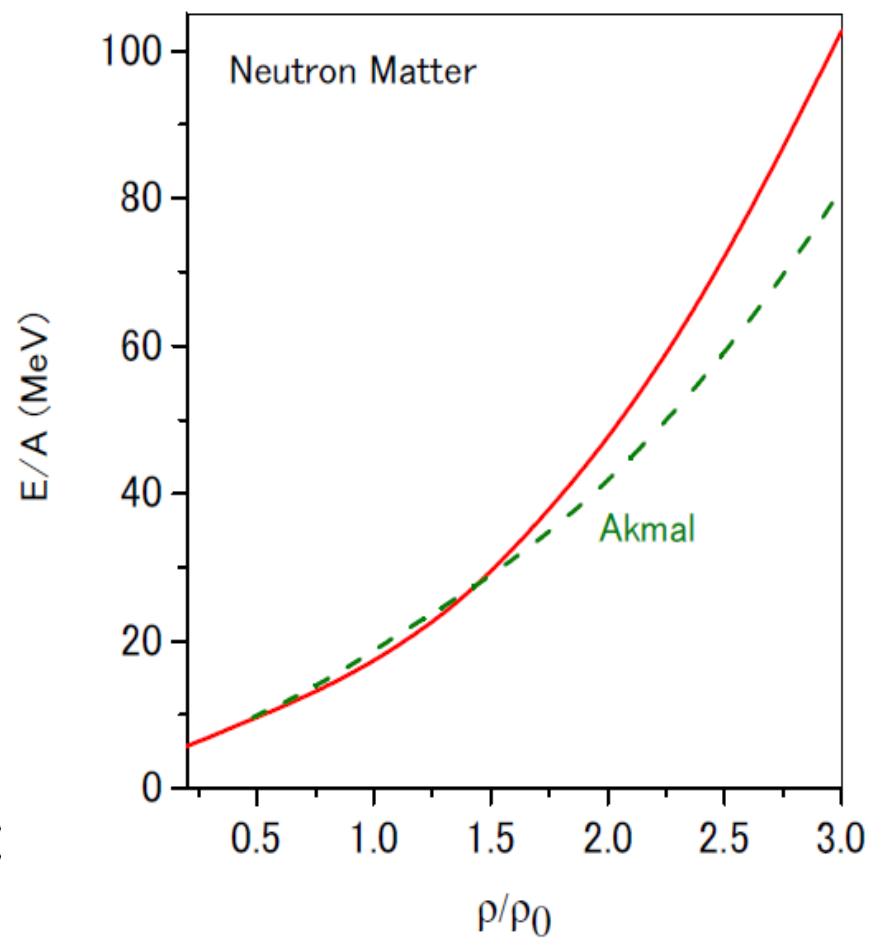
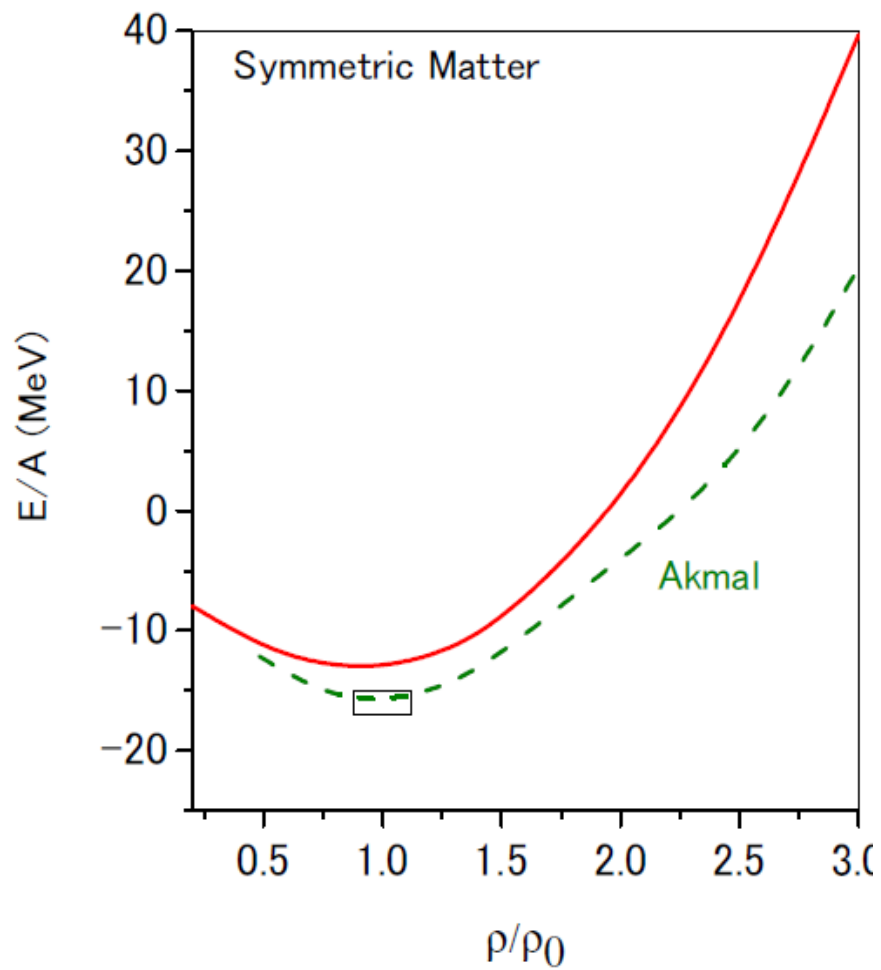




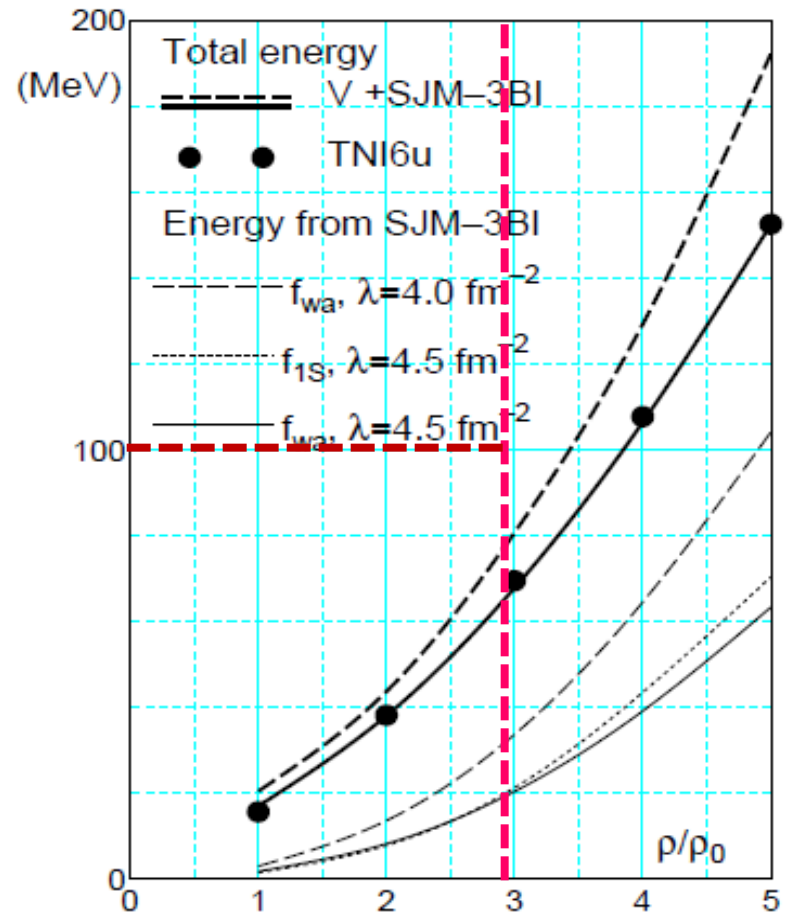
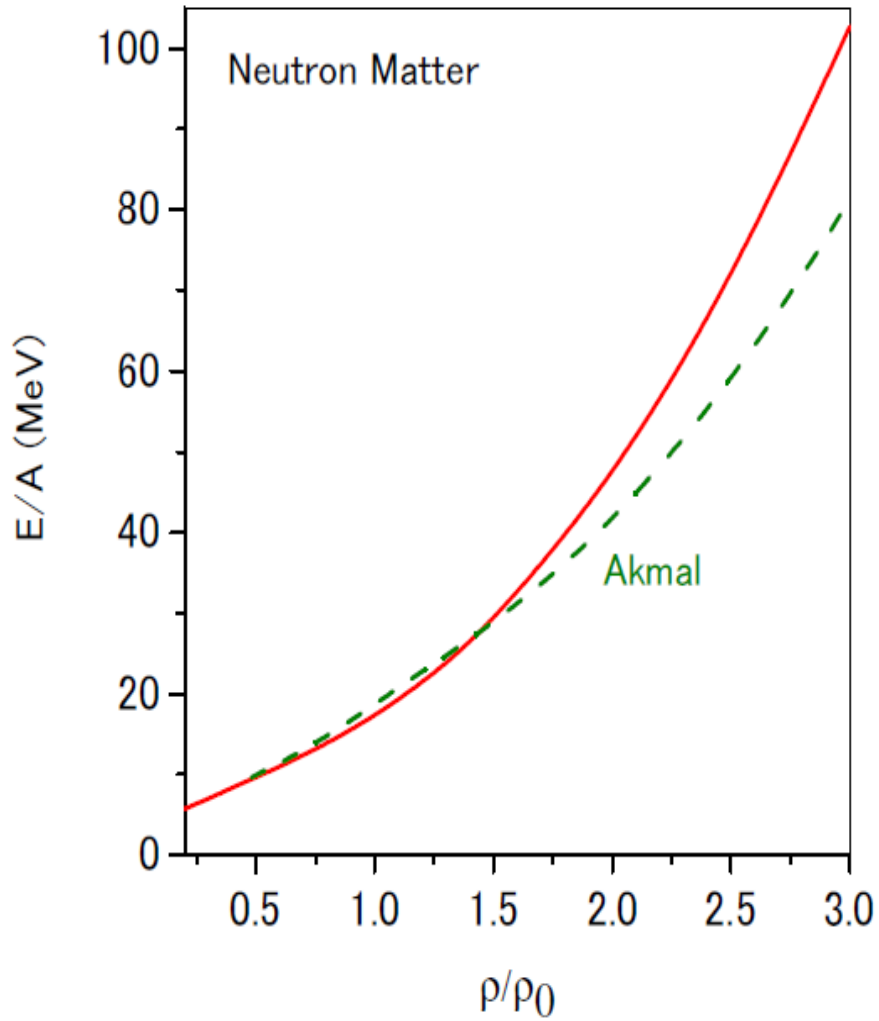


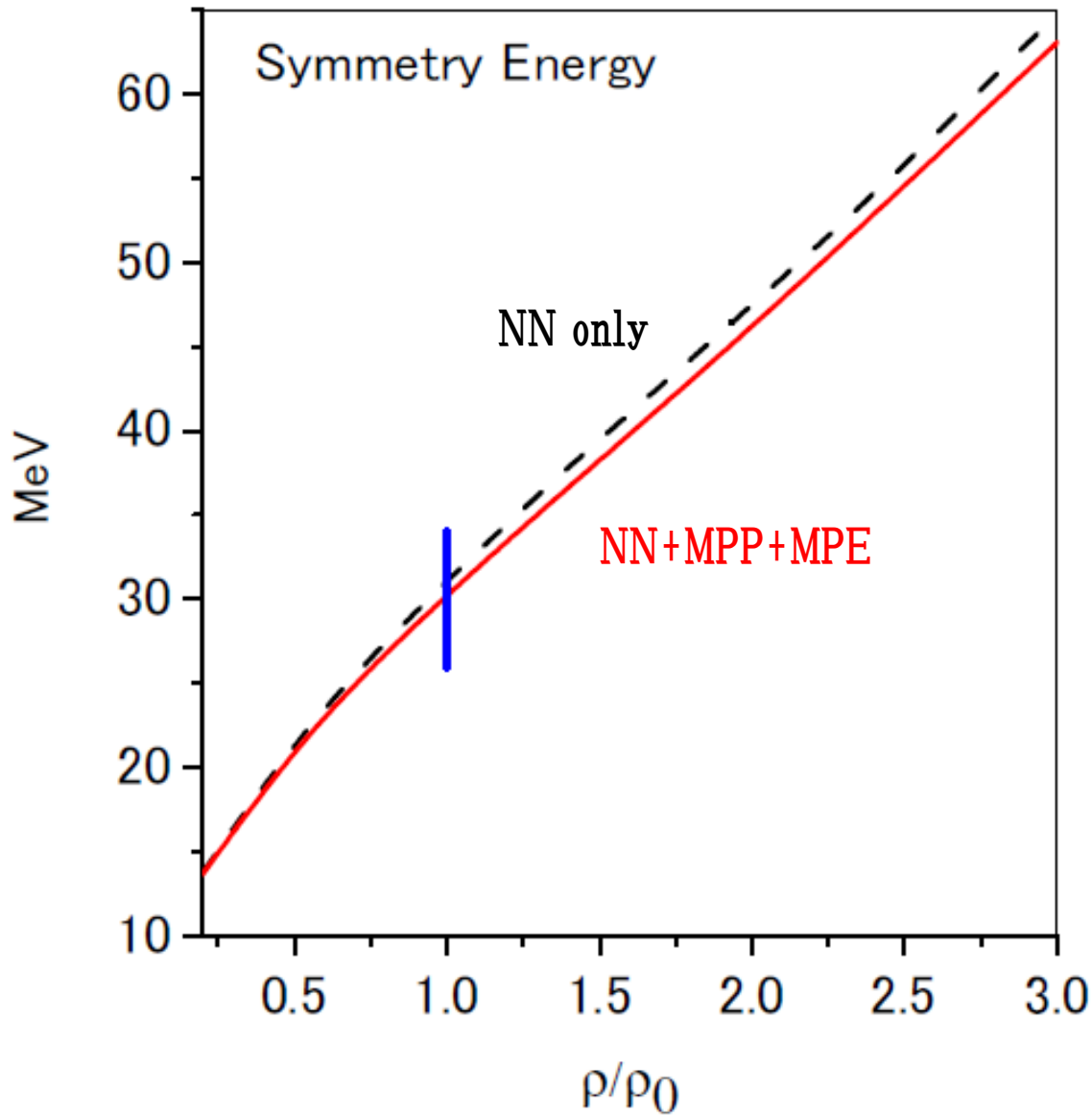






# Tamagaki SJM





$$E_{\text{sym}}(\rho) = -\frac{1}{4} \frac{\partial(E/A)}{\partial x}(\rho, 0) \approx \frac{E}{A}(\rho, 0) - \frac{E}{A}(\rho, 0.5)$$

neutron symmetric

MPP/MPE contributions in  $U_\Lambda$  &  $U_\Sigma$

Toward Universal TBR

Table 1:  $U_\Lambda$  at normal density and partial wave contributions in  $^{2S+1}L_J$  states (in MeV) with Continuous choice. Contributions of  $S$ -state spin-spin interactions are by  $U_{\sigma\sigma} = (U(^3S_1) - 3U(^1S_0))/12$ .

	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	$D$	$U_\Lambda$	$U_{\sigma\sigma}$
ESC08c	-14.6	-28.9	2.8	0.2	1.3	-3.3	-1.6	-44.0	1.20
+MPP	-12.5	-22.8	3.2	0.3	1.7	-2.5	-1.4	-33.9	1.22
+MPP+MPE	-12.5	-25.1	3.3	0.3	1.5	-2.6	-1.5	-36.5	1.05
ESC08a(r)	-11.6	-24.4	2.4	0.0	1.2	-3.3	-1.5	-37.2	0.88

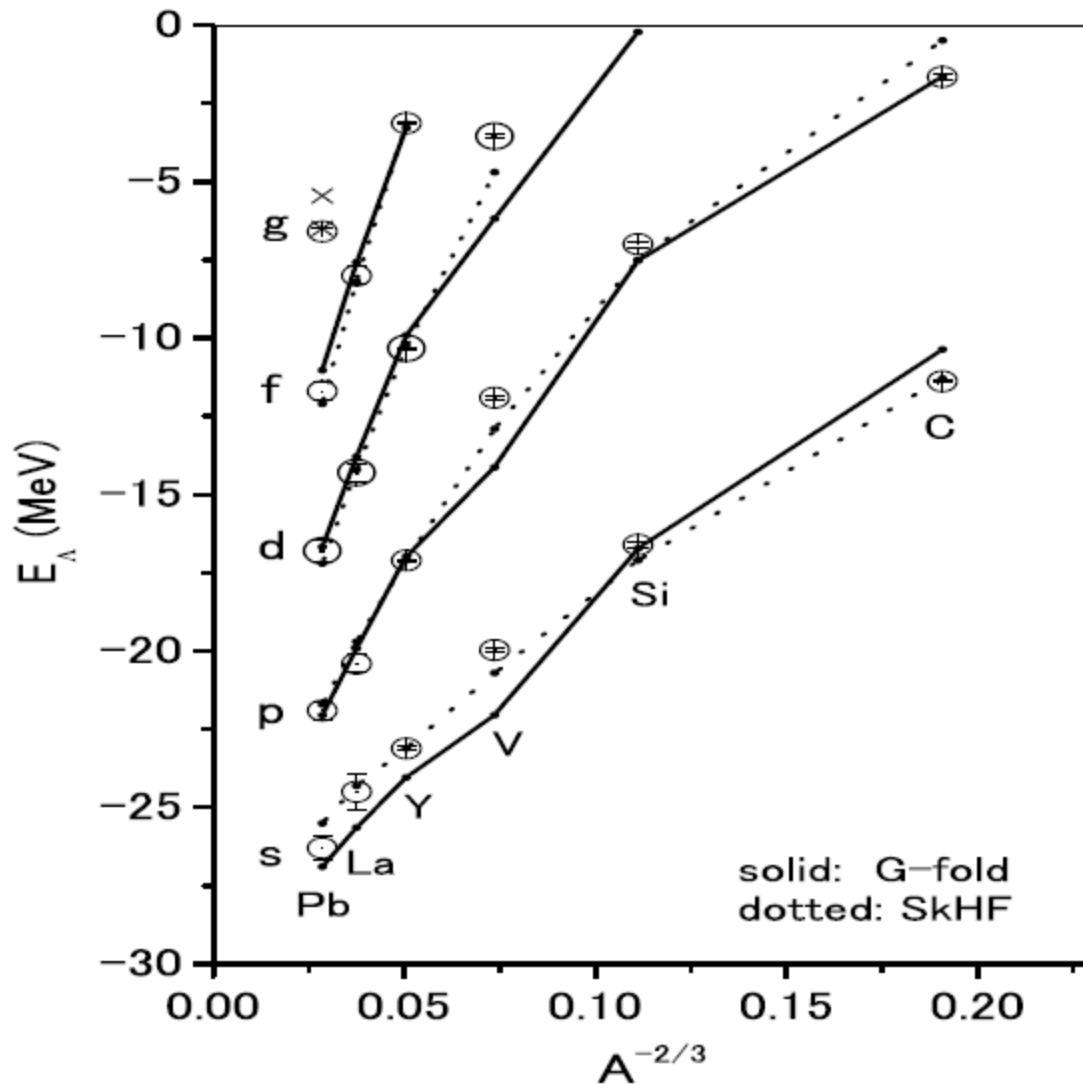


Fig. 1. Energy spectra of  ${}_{\Lambda}^{13}\text{C}$ ,  ${}_{\Lambda}^{28}\text{Si}$ ,  ${}_{\Lambda}^{51}\text{V}$ ,  ${}_{\Lambda}^{89}\text{Y}$ ,  ${}_{\Lambda}^{139}\text{La}$  and  ${}_{\Lambda}^{208}\text{Pb}$  are given as a function of  $A^{-2/3}$ ,  $A$  being mass numbers of core nuclei. Solid (dashed) lines show calculated values by the  $G$ -matrix folding model derived from ESC08a (the Skyrme-HF model). Open circles denote the experimental values taken from Ref. 17).



Table 1: Values of  $U_\Sigma$  at normal density and partial wave contributions.

model	$T$	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	$D$	$U_\Sigma$
ESC08c	1/2	11.2	-18.5	2.2	1.6	-5.5	-1.0	-0.7	6.3
	3/2	-13.5	34.6	-5.6	-1.9	5.6	-1.9	-0.3	
+MPP	1/2	11.7	-16.3	2.3	1.7	-5.4	-0.8	-0.6	15.6
	3/2	-12.1	37.8	-5.2	-1.8	5.8	-1.4	-0.2	
+MPP+MPE	1/2	11.5	-18.1	2.3	1.7	-5.7	-0.9	-0.7	14.4
	3/2	-11.0	37.5	-5.0	-2.0	6.1	-1.4	-0.1	
ESC08a	1/2	11.3	-23.6	1.7	1.9	-5.0	0.0	-0.7	13.6
	3/2	-11.5	44.4	-4.0	-2.2	5.4	-3.6	-0.2	
ESC08b	1/2	10.3	-25.5	1.4	2.5	-5.9	0.3	-0.8	19.8
	3/2	-10.4	52.4	-3.0	-2.7	5.9	-4.4	-0.1	

# Conclusion (Outlook)

$U_N, U_\Lambda, U_\Sigma, U_E$  に対するMPPのcontributionsは同じ  
(約10MeV at normal density)

MPP (NNN) の強さは観測量に基づいて決められる  
中性子星の最大質量、重イオン散乱 (FSY effect)

MPP (YNN) もハイパー核データで決められる(かも)

MPEについては still in progress