Lattice QCD with physical quark masses

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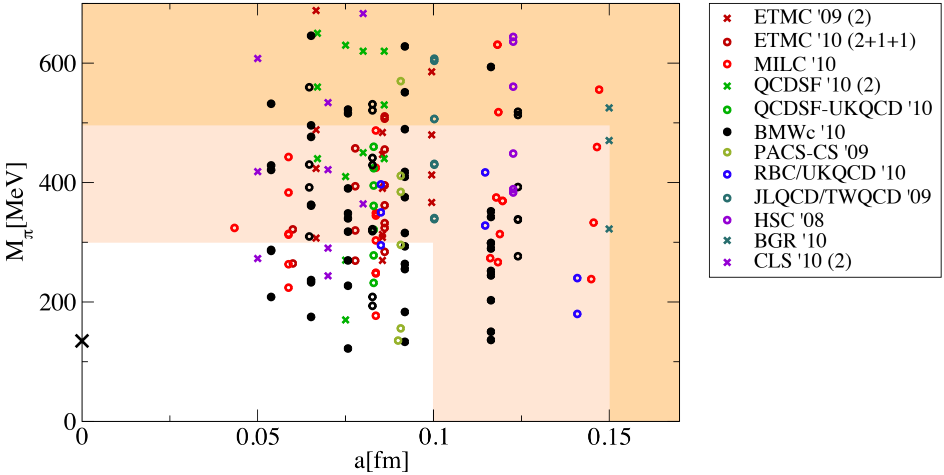
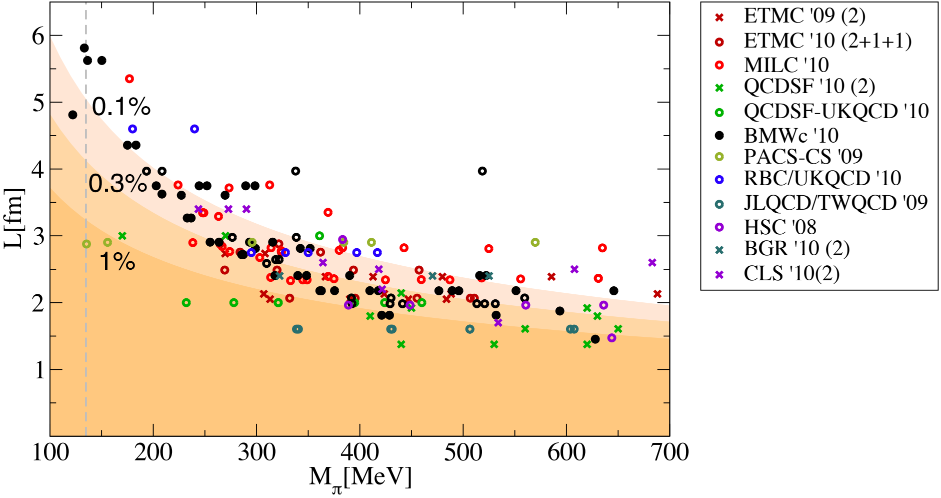
SFB TR55

(PLB 705:477,2011, JHEP 1108:148,2011; PLB 701:265,2011)

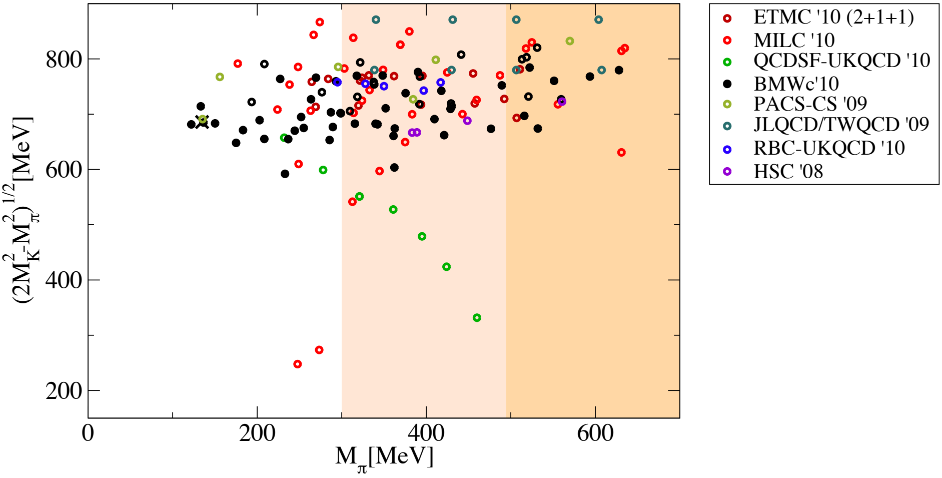
Landscape M_π vs. a



Landscape L vs. M_π



Landscape M_K vs. M_π



Quark mass definitions

- Lagrangian mass m^{bare}
- $m^{\text{ren}} = \frac{1}{Z_S}(m^{\text{bare}} - m^{\text{bare}}_{\text{crit}})$

- m^{PCAC} from $\frac{\langle \partial_0 A_0 P \rangle}{\langle P(t) P(0) \rangle}$
- $m^{\text{ren}} = \frac{Z_A}{Z_P} m^{\text{PCAC}}$

Better use

- $d = m^{\text{bare}}_s - m^{\text{bare}}_{ud}$
- $d^{\text{ren}} = \frac{1}{Z_S} d$

- $r = m^{\text{PCAC}}_s / m^{\text{PCAC}}_{ud}$
- $r^{\text{ren}} = r$

and reconstruct

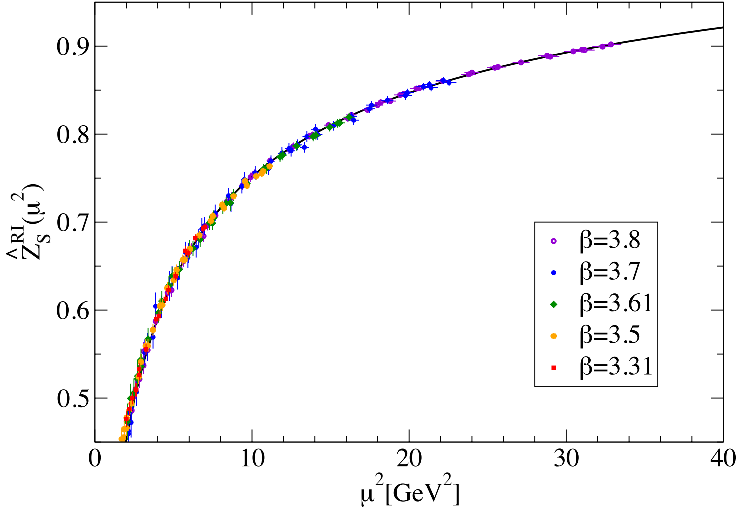
- $m^{\text{ren}}_s = \frac{1}{Z_S} \frac{rd}{r-1}$
- $m^{\text{ren}}_{ud} = \frac{1}{Z_S} \frac{d}{r-1}$

✓ No additive mass renormalization and ambiguity in m_{crit}

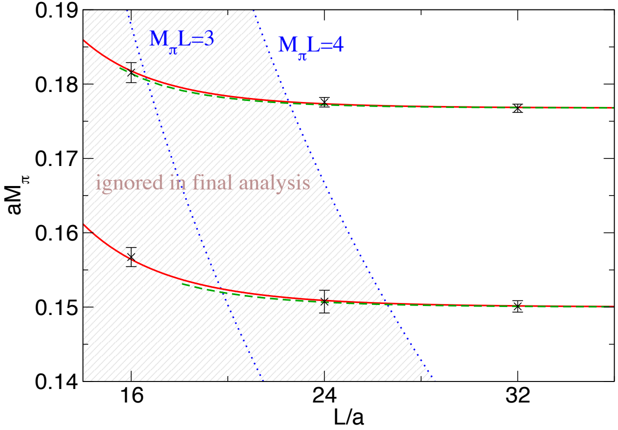
✓ Only Z_S multiplicative renormalization (no pion poles)

☞ Works with $O(a)$ improvement (we use this)

Nonperturbative running

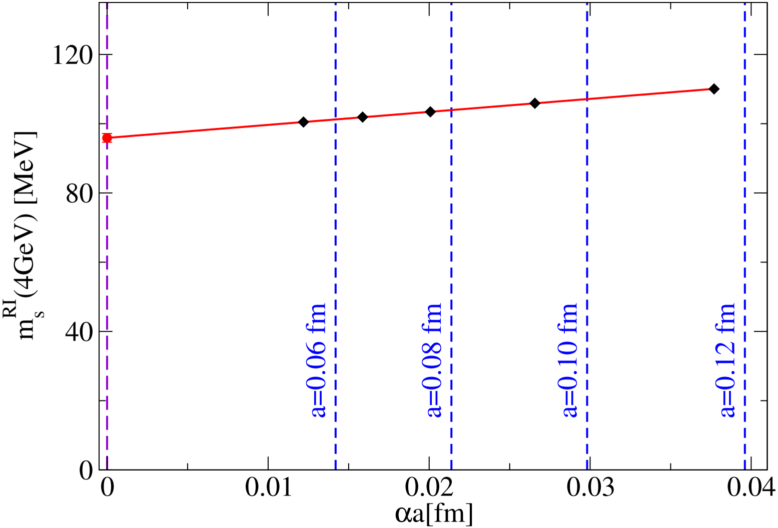


Tiny finite volume effects

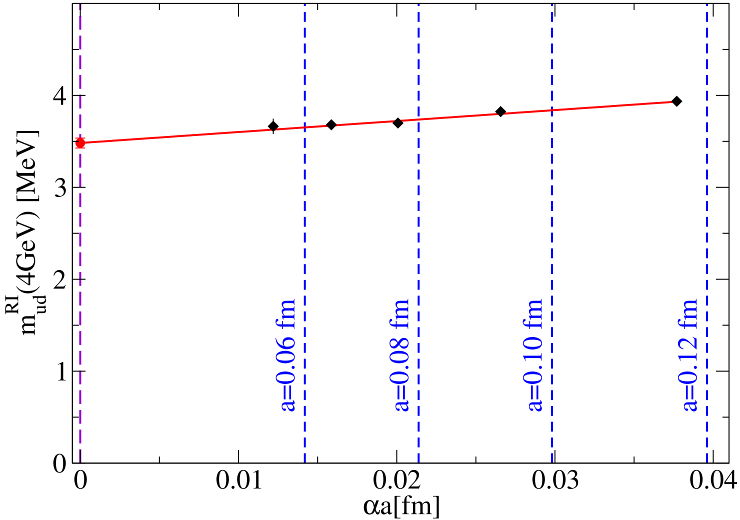


- FV effects tiny
- Dedicated FV runs
- Perfect agreement with FV χPT (Colangelo et. al. 2005)

Strange quark mass



Light quark masses



Systematic error treatment

- Goal:
  - Reliably estimate total systematic error
- Method:
  - 288 full analyses (2000 bootstrap on each)
    - 2 plateaux regions
    - 2 continuum forms: $O(\alpha_s a)$, $O(a^2)$
    - 3 chiral forms: $2 \times SU(2)$, Taylor
    - 2 chiral ranges: $M_\pi < 340, 380$ MeV
    - 3 renormalization matching procedures
    - 2 NP continuum running forms
    - 2 scale setting procedures
  - All analyses weighted by fit quality
    - Mean gives final result
    - Stdev gives systematic error
  - Statistical error from 2000 bootstrap samples

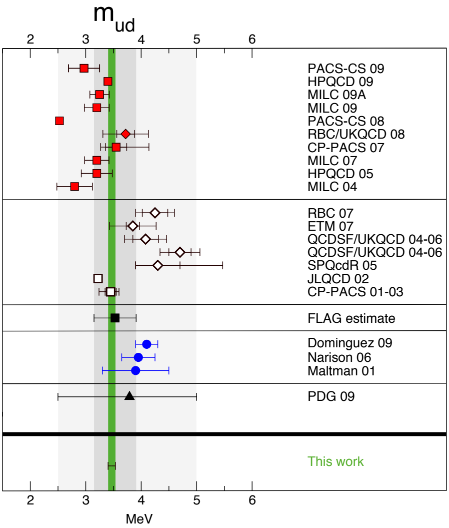
Final result

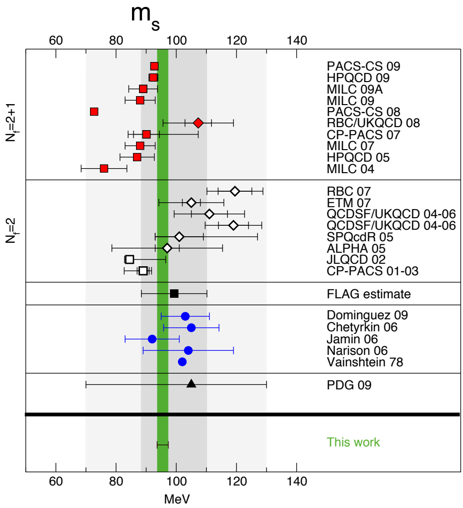
	RI @ 4 GeV	RGI	$\overline{\text{MS}}$ @ 2 GeV
m_s	96.4(1.1)(1.5)	127.3(1.5)(1.9)	95.5(1.1)(1.5)
m_{ud}	3.503(48)(49)	4.624(63)(64)	3.469(47)(48)
m_u	2.17(04)(10)	2.86(05)(13)	2.15(03)(10)
m_d	4.84(07)(12)	6.39(09)(15)	4.79(07)(12)

Additional consistency checks:

- ✓ Use m^{PCAC} only, no ratio-difference method
  ☞compatible, slightly larger error
- ✓ Unweighted final result and systematic error
  ☞negligible impact
- ✓ Additional Continuum, chiral and FV terms
  ☞all compatible with 0

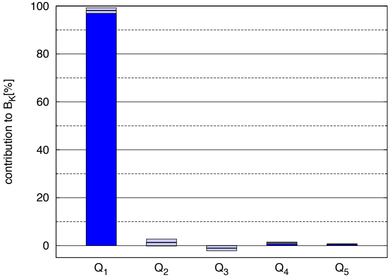
Comparison

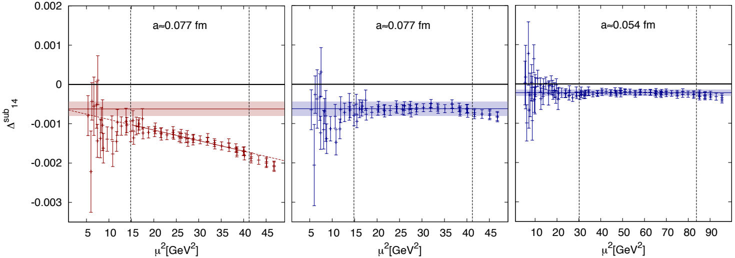

m_{ud}
2
3
4
5
6
PACS-CS 09
HPQCD 09
MILC 09A
MILC 09
PACS-CS 08
RBC/UKQCD 08
CP-PACS 07
MILC 07
HPQCD 05
MILC 04
RBC 07
ETM 07
QCDSF/UKQCD 04-06
QCDSF/UKQCD 04-06
SPQcdR 05
JLQCD 02
CP-PACS 01-03
FLAG estimate
Dominguez 09
Narison 06
Maltman 01
PDG 09
This work
MeV


m_s
60
80
100
120
140
$N_f=2+1$
$N_f=2$
PACS-CS 09
HPQCD 09
MILC 09A
MILC 09
PACS-CS 08
RBC/UKQCD 08
CP-PACS 07
MILC 07
HPQCD 05
MILC 04
RBC 07
ETM 07
QCDSF/UKQCD 04-06
QCDSF/UKQCD 04-06
SPQcdR 05
ALPHA 05
JLQCD 02
CP-PACS 01-03
FLAG estimate
Dominguez 09
Chetyrkin 06
Jamin 06
Narison 06
Vainshtein 78
PDG 09
This work
MeV

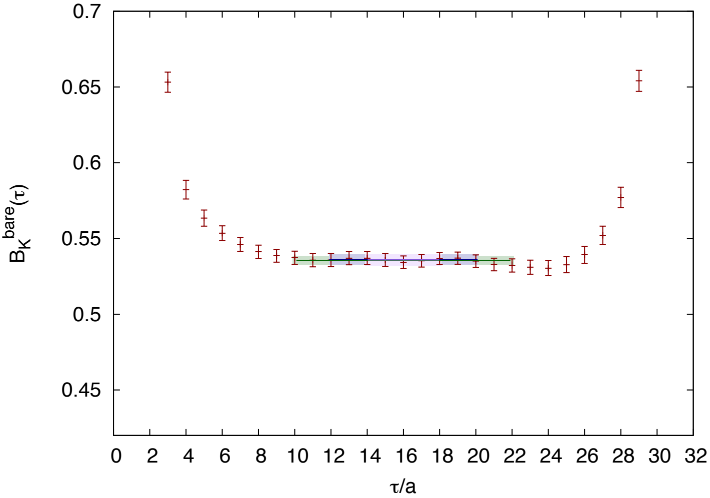
Wilson B_K: Unphysical operator mixing

- ☞ χSB induces mixing with 4 unphysical operators
- ☞ Mixing terms chirally enhanced
- ✓ Small even below physical m_π
- ✓ Good chirality of our action

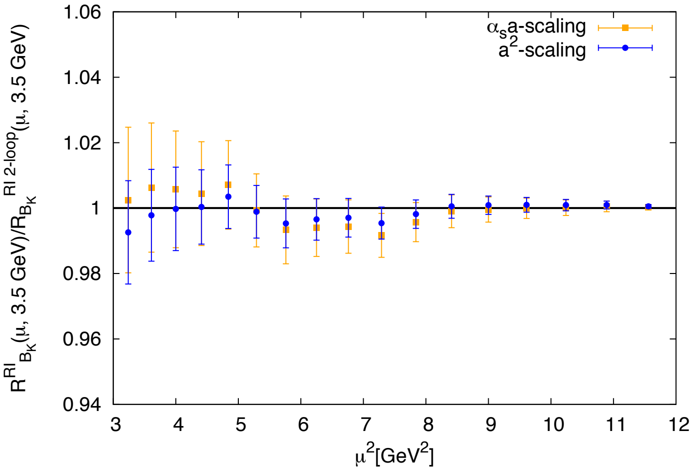




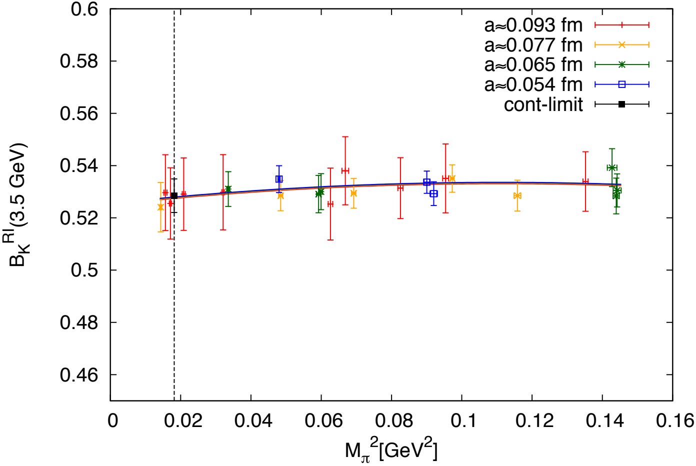
Signal


0.7
0.65
0.6
0.55
0.5
0.45
$B_K^{bare}(\tau)$
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
τ/a

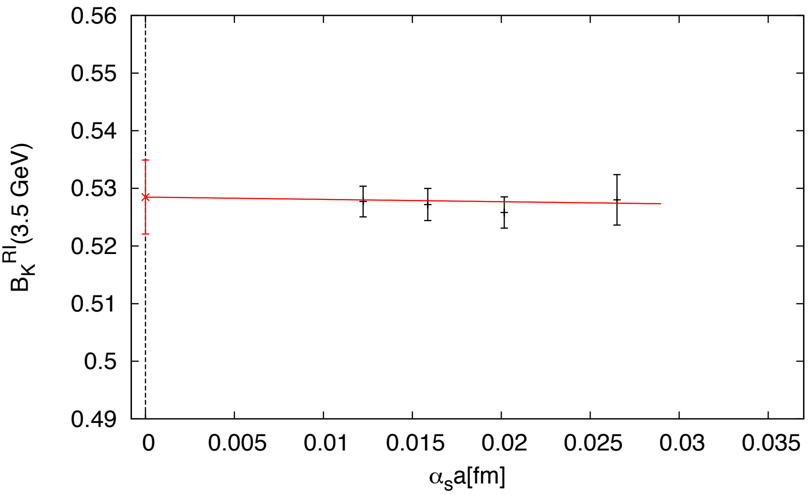
Running


1.06
1.04
1.02
1
0.98
0.96
0.94
3
4
5
6
7
8
9
10
11
12
$\alpha_s a$-scaling
a^2-scaling
$R^{RI}_{B_K}(\mu, 3.5\ \mathrm{GeV})/R^{RI\ 2\text{-}loop}_{B_K}(\mu, 3.5\ \mathrm{GeV})$
$\mu^2[\mathrm{GeV}^2]$

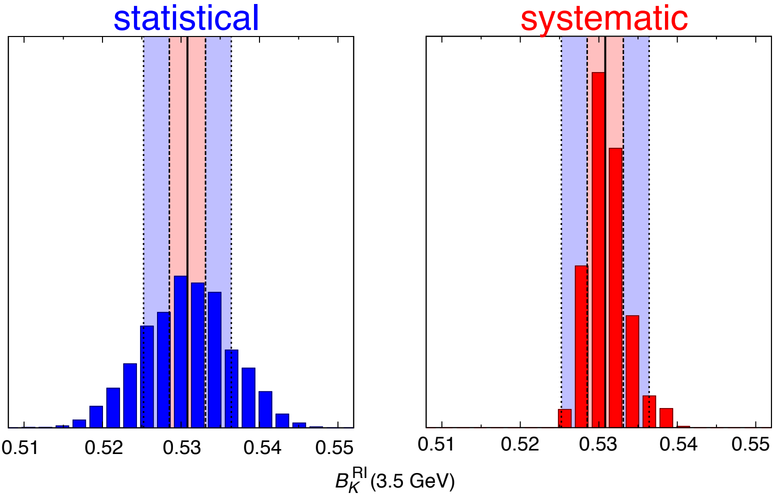
Physical point



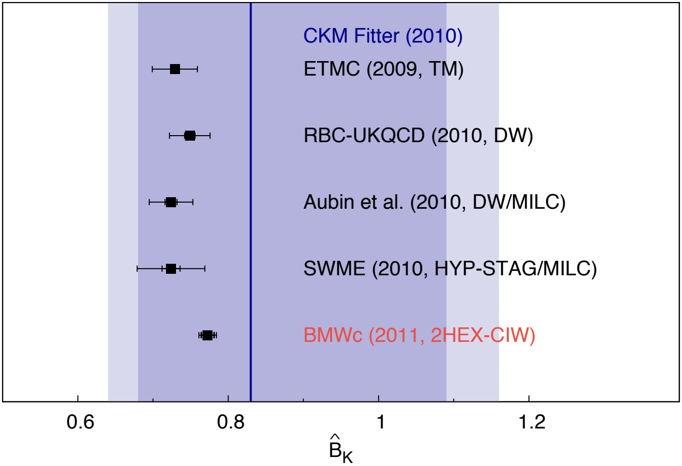
Continuum extrapolation



Errors


statistical
systematic
0.51
0.52
0.53
0.54
0.55
0.51
0.52
0.53
0.54
0.55
B_K^{RI}(3.5 GeV)

Comparison


CKM Fitter (2010)
ETMC (2009, TM)
RBC-UKQCD (2010, DW)
Aubin et al. (2010, DW/MILC)
SWME (2010, HYP-STAG/MILC)
BMWc (2011, 2HEX-CIW)
0.6
0.8
1
1.2
$\hat{B}_K$

BACKUP

Action details

Goal:

- Optimize physics results per CPU time
- Conceptually clean formulation

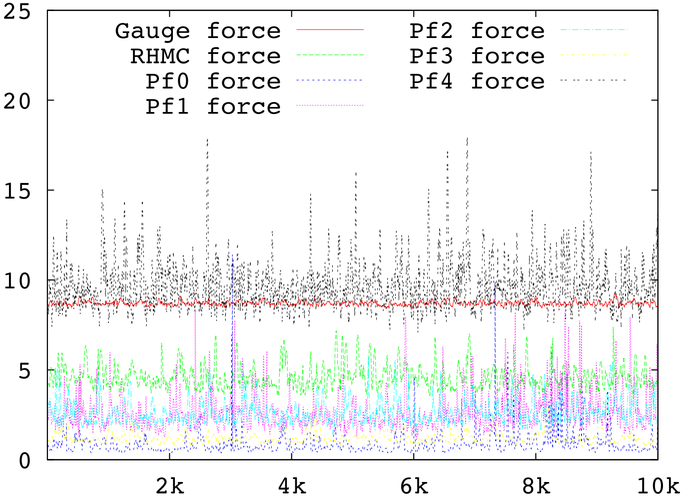
Method:

- Dynamical $2+1$ flavor, Wilson fermions at physical M_π
- 3-5 lattice spacings $0.053\ \text{fm} < a < 0.125\ \text{fm}$
- Tree level $O(a^2)$ improved gauge action (Lüscher, Weisz, 1985)
- Tree level $O(a)$ improved fermion action (Sheikholeslami, Wohlert, 1985)
  - Why not go beyond tree level?
    - Keeping it simple (parameter fine tuning)
    - No real improvement, UV mode suppression took care of this
  - This is a crucial advantage of our approach
- UV filtering (APE coll. 1985; Hasenfratz, Knechtli, 2001; Capitani, Durr, C.H., 2006)

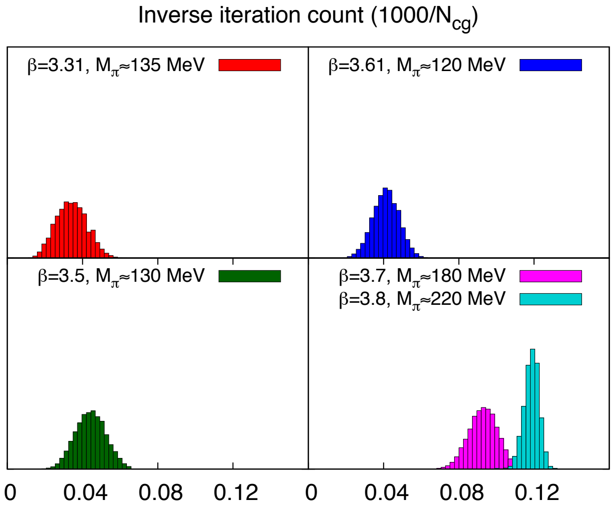
➛ Discretization effects of $O(\alpha_s a, a^2)$

✓ We include both $O(\alpha_s a)$ and $O(a^2)$ into systematic error

Algorithm stability

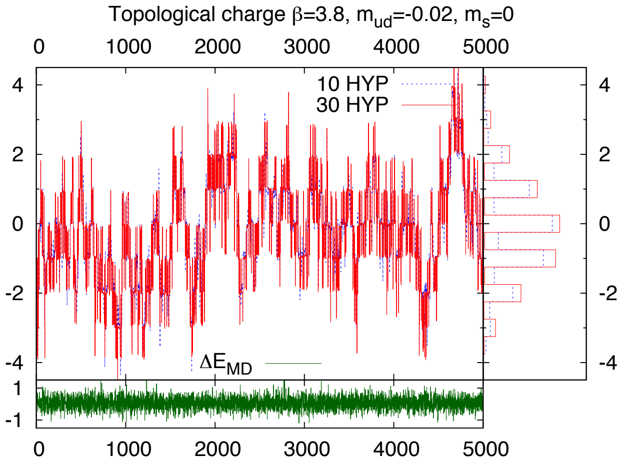


No exceptional configs



Topological sector sampling

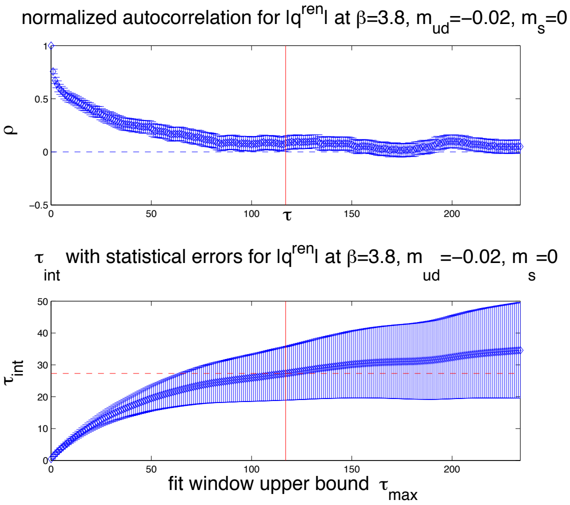
worst case



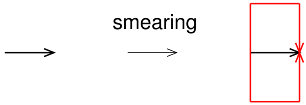
Autocorrelation time (finest lattice, small mass)

$\tau_{\text{int}} = 27.3(7.4)$

(MATLAB code from Wolff, 2004-7)



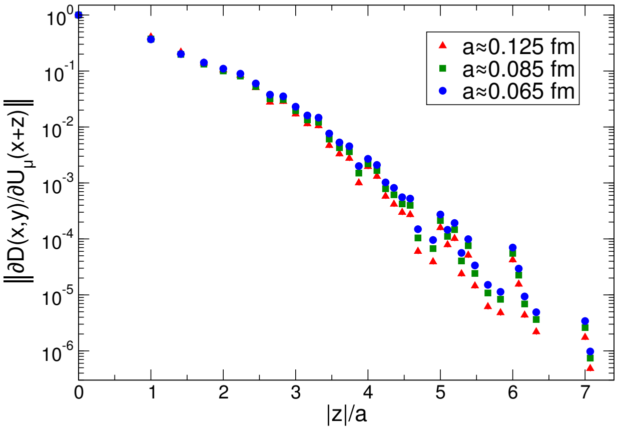
Locality properties



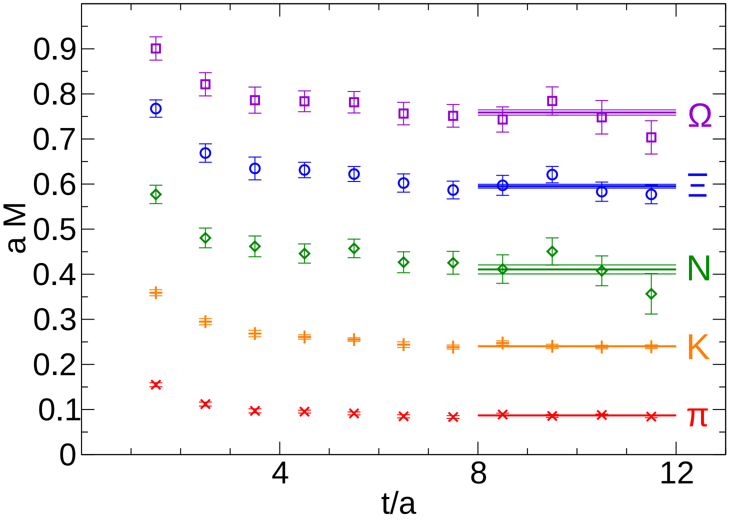
- locality in position space:
  $|D(x,y)| < \text{const}\, e^{-\lambda|x-y|}$ with $\lambda = O(a^{-1})$ for all couplings.
  Our case: $D(x,y) = 0$ as soon as $|x-y| > 1$
  (despite smearing)
- locality of gauge field coupling:
  $|\delta D(x,y)/\delta A(z)| < \text{const}\, e^{-\lambda|(x+y)/2-z|}$ with $\lambda = O(a^{-1})$ for all couplings.
  Our case: $\delta D(x,x)/\delta A(z) < \text{const}\, e^{-\lambda|x-z|}$ with $\lambda \simeq 2.2a^{-1}$ for $2 \leq |x-z| \leq 6$

Gauge field coupling locality

6-stout case:



Effective masses and correlated fits



Individual m_u and m_d

- Goal:
  - Compute m_u and m_d separately
- Method:
  - Need QED and isospin breaking effects in principle
  - Alternative: use dispersive input -Q from $\eta \to \pi\pi\pi$

    $$Q^2 = \frac{1}{2}\left(\frac{m_s}{m_{ud}}\right)^2 \frac{m_d - m_u}{m_{ud}}$$

  - ✓ Transform precise m_s/m_{ud} into $(m_d - m_u)/m_{ud}$
  - We use the conservative $Q = 22.3(8)$ (Leutwyler, 2009)