Non-perturbative renormalisation of minimally doubled fermions

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Simulations with canonical Karsten-Wilczek operator

- Study of Karsten-Wilczek action

$$\begin{aligned} S^f_{KW} = a^4 \sum_x \Big\{ \frac{1}{2a} \sum_{\mu=1}^{4} \Big[& \overline{\psi}(x) \left(\gamma_\mu (1 + d(\beta)\delta_{\mu,4}) - i\,\gamma_4 (1 - \delta_{\mu,4})\right) U_\mu(x) \psi(x + a\hat{\mu}) \\ & - \overline{\psi}(x + a\hat{\mu}) \left(\gamma_\mu (1 + d(\beta)\delta_{\mu,4}) + i\,\gamma_4 (1 - \delta_{\mu,4})\right) U^\dagger_\mu(x) \psi(x) \Big] \\ & + \overline{\psi}(x) \left(m_0 + c(\beta) \frac{i\gamma_4}{a} \right) \psi(x) \\ & + \beta \sum_{\mu<\nu} \left(1 - \frac{1}{N_c} \Re tr P_{\mu,\nu} \right) (1 + d_p(\beta)) \end{aligned} \qquad (1)$$

- Perturbation theory: 3 counterterms break hypercubic symmetry
- Due to quenched approximation no sensitivity to parameter $d_p(\beta)$
- Parameter $c(\beta)$ has dimension three conterterm
- Parameter $d(\beta)$ numerically small

Hypercubic symmetry breaking effects

Hypercubic symmetry of physical observables must be restored in order to have the *right continuum limit*. Concerns even naïvely unaffected quantities like **hadron masses**.

- KW Dirac operator explicitly selects time direction
- Correlation function measurement process selects time direction

Both origins of symmetry breaking clash!

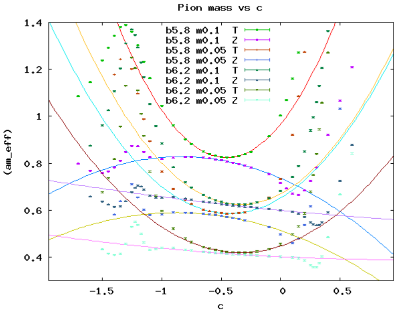
How to disentangle? Two possible solutions:

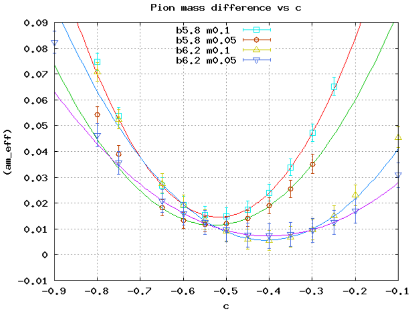
- either two different *KW Dirac operators*
- different *measuring directions*

Restoration of hypercubic symmetry means tuning the counterterms

Pion modes along the c axis

Data on 32^4, 30 configurations reveals dependence of effective mass on c.

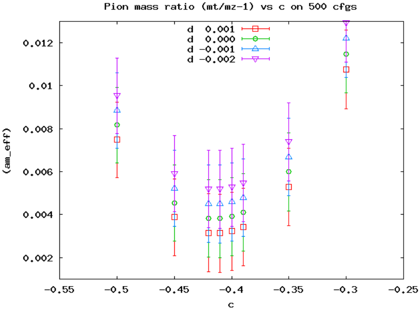


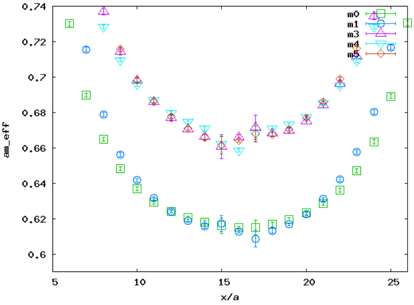


- reveals phase transitions at extreme values of c
- temporal correlators have a minimum in the middle

Pion mass cont'd: more statistics

Larger sample shows that effective masses do not match.



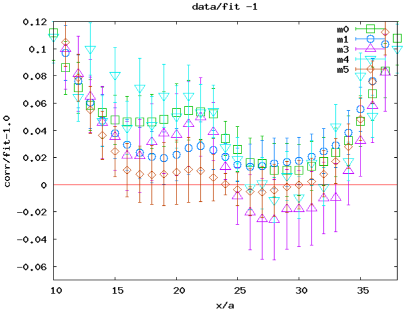


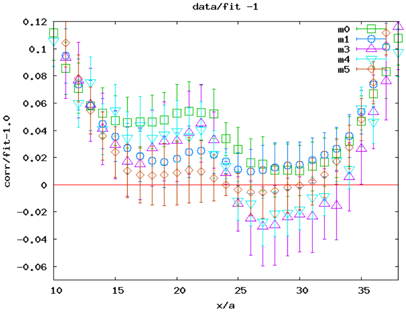
- Effective mass extraction is questionable here
- Central region cannot be considered a plateau here

Low lying excited states? Slopes at the central points?

Pion mass cont'd: larger volume

Larger volume removes excited states. However, **oscillations in temporal rho** mode are clearly visible.





- Oscillations in the pion can be removed with modified fit function:

$$C(n) \propto (1 + B(-1)^t) \cosh \left(m(t - \frac{T}{2} \right)$$

- Oscillating factor is due to second zero mode at $k_4 = \pi/2$

Summary and open questions

- Simulations with minimally doubled valence quarks
- Hadronic masses reveal phase transitions at extreme parameter values
- Hadronic modes in different measuring directions: symmetry restoration as renormalisation condition
- **Oscillations in temporal rho** due to second zero mode: study in HPE on the way
- Further problem remains: effective mass plateaus split, not yet understood
- **Safe extraction** of effective masses necessary for renormalisation